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## CORRIGENDA TO VOLS. 75 and 76.

Vol. 75, page 494, col. 1, line 12: *For* "$8 \times 10^{-3}$ mm" read "$8 \times 10^{-4}$ mm."

Vol. 75, page 528, Fig. 4: Corrections by Mr. Metz to this Figure will be found in his reply to the discussion (*see* vol. 76, page 456, col. 2).

Vol. 75, page 721, col. 2: Corrections to the particulars in regard to the Union d'Électricité, Paris, will be found in Mr. Lapeyre's communication on page 716 of vol. 76.

Vol. 75, page 834, col. 1, line 5 from bottom: *For* "1891" *read* "1888."

Vol. 75, page 834, col. 1, line 4 from bottom: *For* "Professors Ayrton and Perry" *read* "Professors Perry and Thompson, and between 1888 and 1891 at the City and Guilds Central Institution, South Kensington, under the late Professors Ayrton and Unwin."

Vol. 76, page 108, col. 1, line 10 from bottom: *For* "$\theta_0$, $\theta_1$, and $\theta_2$" *read* "$\phi_0$, $\phi_1$, and $\phi_2$."

Vol. 76, page 164, col. 2, line 18 from bottom: *For* "supplied" *read* "supplied more economically."

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EXPLANATIONS OF ABBREVIATIONS.

(P) indicates a reference to the title or subject of a paper or address.
(p) indicates a reference to a subject dealt with in a paper or address of which the title is not quoted.
(D) indicates a reference to a discussion upon a paper of which the title is quoted.
(d) indicates a reference to a subject dealt with in a discussion on a paper of which the title is not quoted.

A.



B.

## Q.



## R.

## S.

## T.



## U.



## V.

from these figures, it can be said that a 132 000-volt line in that particular district would be liable to 3 surges per annum of the order of 1 000 000 volts. Using this as a typical example, and taking into account the fact that there are many more surges of smaller magnitude and that those exceeding 1 000 000 volts might rise to considerably higher voltages on over-insulated lines, it will be realized that transformers connected to systems in such lightning districts have to operate under severe conditions. In countries (such as Malaya*) where the

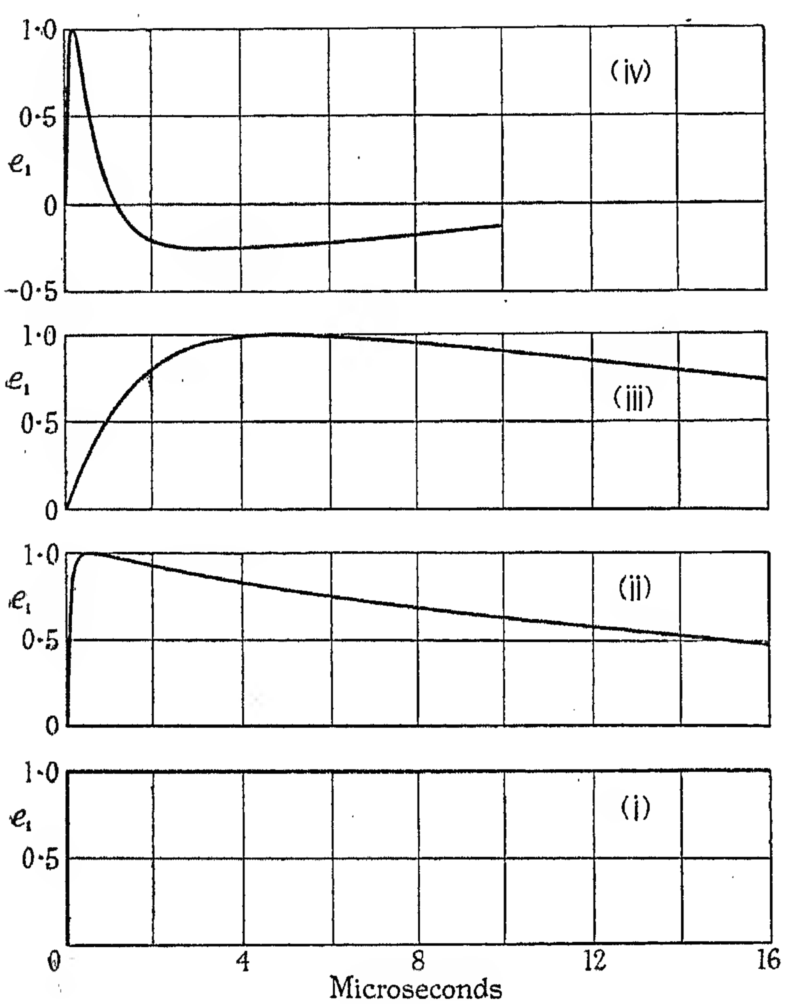



FIG. 1.

(i) Infinite rectangular wave.
(ii) 0·4–14 microsecond wave,
$$e_1 = 1\cdot025(\varepsilon^{-0\cdot05\times10^6 t} - \varepsilon^{-15\times10^6 t}).$$
(iii) 5–30 microsecond wave,
$$e_1 = 1\cdot22(\varepsilon^{-0\cdot03\times10^6 t} - \varepsilon^{-0\cdot65\times10^6 t}).$$
(iv) Insulator-chopped wave,
$$e_1 = 2\varepsilon^{-1\cdot5\times10^6 t} - 1\cdot61\varepsilon^{-15\times10^6 t} - 0\cdot39\varepsilon^{-0\cdot1\times10^6 t}.$$

lightning season is more rigorous than in the case considered, the position is even worse. In isolated instances very high voltages indeed have been measured, and Pittman and Torok† show an oscillogram of a surge which rose to 5 000 000 volts in less than 2 microseconds.

Evidence shows that lightning transients can be positive, negative, or oscillatory, though generally the unidirectional (as shown in Fig. 1) or mainly unidirectional ones predominate. Positive waves are of more frequent occurrence and are not usually so severe as those of negative polarity.

In general, the time taken to pass a point on the line varies between a few microseconds and 50 microseconds, and the time to reach maximum value ranges from a fraction of a microsecond to several microseconds, a rate of voltage-rise of 1 000 kV per microsecond being typical. As an extreme example, for the 5 000-kV surge already mentioned the rate of voltage-rise was approximately 4 000 kV per microsecond, and it was estimated that the rate must have been of the order of 10 000 kV per microsecond at the point of origin.

Regarding switching surges, an analysis of the data obtained on the five systems already referred to shows that, out of 724 surges, only 422 reached twice or more than twice normal voltage, and only 40 reached four times or more than four times normal. The maximum voltage recorded was 5·5 times normal, and this only applied to two surges. This type of surge is usually oscillatory, with a magnitude depending on whether the line is being energized or de-energized.*

Surges due to arcing earths† have been shown to reach from three to four times normal line voltage. Like the switching surges they are usually oscillatory.

From this very brief résumé of our present experimental knowledge regarding surges or travelling waves on transmission lines, which is typical of all analyses made on other systems in other parts of the world, it is evident that lightning plays the most important role.

Lightning surges can originate either from a direct stroke on to the line or from an induced charge. (An approaching charged cloud induces a charge of opposite polarity on the line, which is released in the form of a travelling wave when the cloud discharges to earth or to another cloud.‡) There appears to be a great deal of conflicting evidence as to the relative magnitudes and the frequency of occurrence of these two sources, but engineers are practically agreed that the direct stroke is by far the more important§ and that the induced stroke is important only on low-voltage lines. For instance, on one particular 220-kV line, 70 per cent of the trip-outs that occurred during one period were definitely due to direct strokes, the remainder being due either to direct strokes of low current magnitudes or to induced charges.||

Many systems are equipped with earth wires, and whilst the latter have the effect of reducing the magnitude of a wave resulting from an induced charge, the conductors are still open to direct hit (though naturally to a less extent), side flash from an earth wire, or flash-over across an insulator string. To obtain the maximum benefit from their use, earth wires must be mounted well above the conductors (in which case the expected increased freedom from direct strokes to a conductor is offset by the fact that the earth wires or the towers are open to a greater number of direct strokes), the span must be short, the lines must be low, and the tower footing resistance must be a minimum.¶ The degree of surge immunity obtainable is therefore to a large extent a question of economics.

It is not the purpose of this paper, however, to discuss the origin of lightning surges; the analysis given later deals with the waves when they reach the circuit under examination, and it is quite immaterial here how they originated. The immediate requirement, therefore, the general characteristics of these waves having been con-

* See Bibliography, (6).
† *Ibid.*, (7).

* See Bibliography, (3).
† *Ibid.*, (8), (9), and (10).
‡ *Ibid.*, (11) and (12).
§ *Ibid.*, (13), (14), (15), and (16).
|| *Ibid.*, (5).
¶ *Ibid.*, (11).

sidered, is to express them mathematically, and it will be realized at once that they can be written down as functions either of time or of distance. Since we are concerned with the voltage variations at a transition point on the line, it is necessary to express them as time functions (as they are measured by the high-speed cathode-ray oscillograph), and methods of effecting this are considered in Section (3).

### (3) Representation of Lightning Waves by Empirical Equations.

It is obvious, as a first approximation, that the simplest mathematical representation is the infinite rectangular wave, as shown in Fig. 1(i). This wave is the simplest to use in any analysis and it has the advantage that, by reason of its infinitely steep front and infinitely long back, any calculations based on it will give results which will be more severe than those met with in practice.* Since the characteristic shapes of true lightning waves are known, however, it becomes expedient to include expressions which take account of the fronts and tails of finite waves.

Teszner and Barbillion represented the front of a lightning wave by means of a Fourier series.† They used a sine series of odd terms, but the amount of computation required was excessive, because at least six terms of the series had to be taken, and probably this number would have been insufficient for very steep waves. Simpler methods have been used. Certain writers, for example, have employed wedge-shaped waves, the fronts being approximately represented by the straight-line relation $e = kt$.‡ Others have used a single exponential function to represent a wave having a rapidly rising front and an infinitely long back, or a rectangular front and a falling back. An extension of the latter method to two exponential functions allows both the front and the back of the wave to be represented, and Bewley has used this system as the basis of all his transient work.§ The great beauty of the method lies in the susceptibility of the exponential function to treatment by operational methods, which are used by Bewley and are adopted as the basis of this paper. The expression

$$e_1 = E(\epsilon^{-at} - \epsilon^{-bt}) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $t > 0$, and where $a$ and $b$ are real, is therefore used to represent the characteristic shape of the lightning waves.

The method may be extended to the use of three exponential terms, in order to simulate special wave-shapes. Such a case is represented by

$$e_1 = E_1\epsilon^{-at} - E_2\epsilon^{-bt} - E_3\epsilon^{-ct} \quad . \quad . \quad . \quad (1a)$$

If $a$, $b$, and $c$, are real, this equation can be made to represent a wave which rises rapidly to a maximum, falls away rapidly to zero, and then reverses its polarity for a relatively long time. Such waves are frequently recorded on cathode-ray oscillograms when the voltage has been sufficiently high to cause a flash-over on a line insulator.

In this paper four types of travelling waves are used and hereafter they will be called incident waves, the distinguishing figure (i), (ii), (iii), or (iv), being added to denote which one is being considered. They are shown graphically in Fig. 1.*

Fig. 1(i) shows the infinite rectangular wave. Apart from the fact that it imposes the severest test on equipment connected to the line, it is included because it is the wave which has been used a good deal by previous workers, and thus direct comparison with earlier results can be made. It will also be realized that the amount of computation required when using equations derived from it is less than half that necessary when the equations are derived for waves (ii), (iii), and (iv). For this wave $E = 1$, $a = 0$, and $b = \infty$.

Fig. 1(ii) shows a wave which rises to its maximum value in 0·4 microsecond and falls away to half value in 14 microseconds approximately. In this case

$$E = 1\cdot025, \quad a = 0\cdot05 \times 10^6, \quad \text{and} \quad b = 15 \times 10^6.$$

Fig. 1(iii) shows a wave of quite a different type. It has a relatively long front (rising to its maximum value in 5 microseconds) and a longer back. The length from the maximum value to half value is 30 microseconds. In this case

$$E = 1\cdot22, \quad a = 0\cdot03 \times 10^6, \quad \text{and} \quad b = 0\cdot65 \times 10^6.$$

Fig. 1(iv) shows the insulator-chopped type of wave, represented by equation (1$a$). In this case $E_1 = 2$, $E_2 = 1\cdot61$, $E_3 = 0\cdot39$, $a = 1\cdot5 \times 10^6$, $b = 15 \times 10^6$, and $c = 0\cdot1 \times 10^6$. This wave rises to its maximum value in 0·16 microsecond, falls away to zero in approximately 1 microsecond, and then reverses its polarity for several microseconds.

### (4) General Equations.

When any voltage wave $e_1$ (which, as shown in the previous section, is to be represented by exponential time-functions throughout this paper) and its associated current wave $i_1$, travelling along a line, encounter a change in circuit constants at a transition point, such as another line or cable of different surge impedance, a division of wave energy occurs. Part of the wave, $e_3$ and $i_3$, is reflected† and part is transmitted through such series impedance as exists at the transition point into the next section. When shunt admittance is present, such as condensers, transformers, and substations, part is transmitted to earth; the voltage across this admittance, in general, being the same as

* In a transformer, for instance, the greater the steepness of wave-front, other things being equal, the greater is the concentration of voltage stress. Further, owing to the fact that a finite though small time is required to cause insulation failure, the chance of such failure is naturally greater the longer the back of the wave.

† See Bibliography, (17).

‡ *Ibid.*, (18).

§ *Ibid.*, (12).

* It will be seen that the damped oscillatory travelling wave has not been dealt with, as it is only necessary to consider it when discussing the less important switching surges or possibly surges in distribution systems induced from the h.t. side. Actually it can be formed from (1) by making the constants complex, but usually it is simpler to use an expression of the form $e_1 = E\epsilon^{-at}\sin\omega t$. The effect of condensers on this type of wave has already been investigated by the author in another paper (see Bibliography, 19).

† The line is assumed long enough to enable the return of this reflected wave from the far end to be neglected. This assumption is quite justifiable because, even in a relatively short line, the return of this wave will be delayed by many microseconds, and because rapid attenuation (see Bibliography, 20 and 21) will make it more or less innocuous when it does arrive.

the transmitted voltage. If the line terminates at the point considered, there is, of course, no transmitted wave, the incident wave giving rise only to the reflected wave and the terminal voltage to earth.

In this paper it is the transmitted voltage-wave and the voltage to earth that are considered. General equations, which are derived in the Appendix, will now be given which will enable these voltages to be quickly obtained for the circuits dealt with later.* In nearly all cases these circuits are assumed concentrated, so that only one transition point is considered.

Fig. 2 represents two lines of surge impedances, $Z_1$ and $Z_2$ respectively, connected as shown to an arrangement of concentrated circuits whose separate operational impedances are designated $Z_a(p)$, $Z_b(p)$, and $Z_c(p)$, these being functions of the resistances $r$ and $R$, the inductance $L$, the capacitance $C$, and the operator $p$. The wave $e_1 i_1$, entering from the line $Z_1$, gives rise to a transmitted voltage-wave $e_2$ along the line $Z_2$, which, in operational or symbolic form, is given by the equation

$$e_2 = \frac{2Z_2 Z_a(p) Z_c(p)}{[Z_1 + Z_2]Z_a(p)Z_c(p) + Z_1 Z_2[Z_a(p) + Z_b(p) + Z_c(p)] + Z_b(p)[Z_1 Z_c(p) + Z_2 Z_a(p)] + Z_a(p)Z_b(p)Z_c(p)} e_1 \quad . \quad (2)$$

The symbolic equations giving the transmitted voltage-waves for all the circuits used in the paper can be obtained from (2) by assigning the appropriate operational impedances to $Z_a(p)$, $Z_b(p)$, and $Z_c(p)$. After such substitution and simplification, the numerator and denominator reduce to rational integral functions of $p$, whose coefficients are functions of $r$, $R$, $L$, $C$, and $Z$.

Only two cases are considered here; that where the denominator is linear in $p$, and that where it is quadratic, the former applying to circuits having one degree of freedom and the latter to circuits having two degrees of freedom. Equations for circuits having three degrees of freedom, which are less frequently required, are given in the Appendix.

* At this stage it is desirable to draw attention to factors which have some bearing on the equations to follow. While formulæ for the reflected waves from the transition point are not given (and they are only of interest when discussing the behaviour of waves on the line), the effect of reflection is inherently taken into account in the derivation of the equations. It is incorporated therein (as shown in the Appendix) by means of the well-known expressions $e_1 = Z_1 i_1$ and $e_3 = -Z_1 i_3$, the minus sign indicating movement in the opposite sense. $Z_1$, the surge impedance of the line, is equal to $\sqrt{(L'/C')}$ (where $L'$ and $C'$ are the inductance and capacitance per unit length of line respectively), and is expressed in ohms. These two simple relationships, which are derived from the classical transmission-line theory, presume that the line is distortionless or that the leakage and resistances are small compared with capacitance admittance and reactance. Apart from the fact that at high effective frequencies the conductor resistance increases by a varying amount, the assumption is in error on account of corona. This increases the effective diameter of a wire, so that its surge impedance is decreased and the coupling between it and other wires increased; and this effect is complicated by the fact that corona, varying with voltage, differs throughout the wave. In addition, the power loss associated with it also affects the results. Easily computable solutions, taking into account resistance and leakage, when relatively complicated circuits are connected to the line, cannot be derived, however, and probably when the resistance and leakage are variable, as obtains with any high-voltage wave, to formulate any solution at all is impossible. It therefore becomes necessary to assume that the respective voltage and current waves are simply related as stated, and it has been found by various workers that equations based on this assumption are in good agreement with practice.

It should also be pointed out that the equations developed in the paper are based on a single-wire system—the earth being the other conductor—so that coupling between wires of a multi-phase overhead equipment is neglected (see Bibliography, 22). Except when the effects of earth wires or the transposition of wires on wave propagation are being studied (see Bibliography, 23), the assumption is, in general, valid, although it must be remembered that the presence of other wires causes a reduction in the surge impedance of a single conductor. This reduction depends on the line configuration and the magnitudes of the waves on the other wires, and as a maximum may be of the order of 50 per cent. Actually, waves on several conductors are related through the self surge impedances and the inter-conductor surge impedances by a set of simultaneous equations (see Bibliography, 24), but it is impracticable to obtain general solutions using these.

In the first case, after the appropriate substitutions have been made, (2) reduces to the form

$$e_2 = 2K\left[\frac{\mu p + \nu}{\beta p + \gamma}\right] e_1 \quad . \quad . \quad . \quad . \quad (3)$$

and in the second case it becomes

$$e_2 = 2K\left[\frac{\lambda p^2 + \mu p + \nu}{\alpha p^2 + \beta p + \gamma}\right] e_1 \quad . \quad . \quad . \quad (4)$$

where the coefficients $\alpha, \beta, \gamma, \lambda, \mu, \nu$, are known functions of $r$, $R$, $L$, $C$, and $Z$, and $K$ is a constant.

Formulæ (3) and (4) are general symbolic equations* giving the transmitted voltage-waves in terms of the coefficients and the operator $p$. All the circuits analysed here, with two exceptions, lead to symbolic equations for the transmitted voltage-waves which reduce to either of these two forms.

The actual solutions, which express the transmitted voltage-waves in terms of the coefficients $\alpha, \beta, \gamma, \lambda, \mu, \nu$, and the time $t$, will now be given.

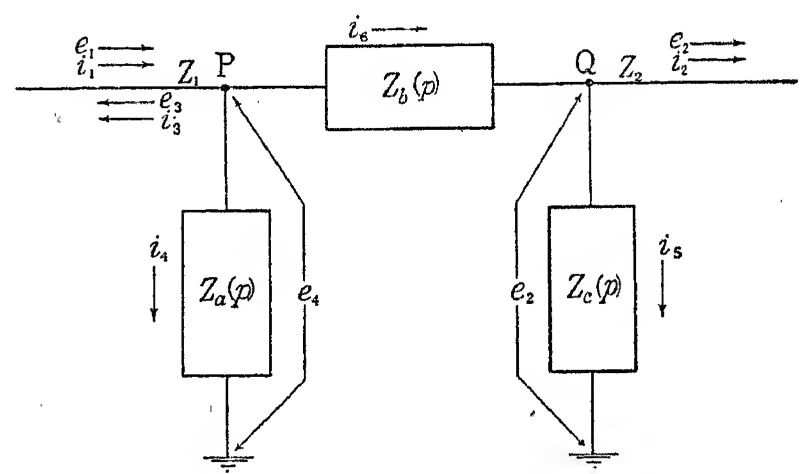



FIG. 2.

*Solutions for Circuits having One Degree of Freedom.*

When the incident wave is given by (1), namely

$$e_1 = E(\epsilon^{-at} - \epsilon^{-bt})$$

the actual solution of the general symbolic equation (3) is

$$e_2 = \frac{2KE}{\beta(\gamma - a\beta)}\left[\beta(\nu - a\mu)\epsilon^{-at} - (\nu\beta - \mu\gamma)\epsilon^{-\frac{\gamma t}{\beta}}\right] - \frac{2KE}{\beta(\gamma - b\beta)}\left[\beta(\nu - b\mu)\epsilon^{-bt} - (\nu\beta - \mu\gamma)\epsilon^{-\frac{\gamma t}{\beta}}\right] \quad (5)$$

When a third exponential is incorporated in the incident wave, such as occurs in wave (iv), a third similar term will be subtracted in (5).

* In some respects the phrase "general symbolic equations giving the transmitted voltage-waves" is a misnomer when applied to (3) and (4). These, though special cases of (2), and therefore less general than the latter, are still perfectly general symbolic equations. They arise in many other problems associated with operational methods (the solutions given later will obviously apply in such cases). On the other hand, (2) only applies to transmitted voltage-waves in the circuit shown in Fig. 2. In order to avoid confusion, however, (2) has been termed a "general operational impedance equation," and (3) and (4) are called "general symbolic equations." The phrase "symbolic equation" is applied when the coefficients $\alpha, \beta, \gamma, \lambda, \mu, \nu$, have been replaced by the functions of $R$, $L$, $C$, and $Z$, appropriate to the particular problem at hand, though the term "operational solution" is frequently employed in this connection.

When the incident wave is infinite rectangular, by making $a = 0$ and $b = \infty$, (5) reduces to

$$e_2 = \frac{2KE}{\beta\gamma}\left[\nu\beta - (\nu\beta - \mu\gamma)\epsilon^{-\frac{\gamma t}{\beta}}\right] \quad . \quad . \quad (6)$$

If a coefficient in the numerator of (3) is absent, it is only necessary to put $\mu$ or $\nu$ equal to zero in (5) or (6).

*Solutions for Circuits having Two Degrees of Freedom.*

In the case of the general symbolic equation (4), two actual solutions must be considered, corresponding to the aperiodic and oscillatory conditions respectively.

If the circuit is aperiodic, so that $\beta^2 - 4\alpha\gamma$ is real, then, for the case where the incident voltage-wave is given by (1), the actual solution is

$$\theta = \text{arc tan}\,\frac{\gamma(\beta\lambda - \alpha\mu) - \alpha(\gamma\mu - \beta\nu)}{\psi(\alpha\nu - \gamma\lambda)}$$

Where a numerator coefficient in (4) is absent, the corresponding symbol $\lambda$, $\mu$, or $\nu$, is put equal to zero in (7), (8), (9), or (10). On the other hand, the denominator coefficients $\alpha$ and $\gamma$ cannot be neglected, so that equations based on (3) cannot be derived from those based on (4). Further, the solutions do not apply when $\beta^2 = 4\alpha\gamma$.

The succeeding sections of the paper are devoted to the development and solution by means of the above equations of the special cases of the general operational impedance equation which correspond to the several circuit arrangements, and in all cases the mathematical development is introduced by remarks regarding their practical application. Owing to space restrictions, actual solutions when the incident wave is infinite rectangular are, except in a few isolated instances, not included. Such solutions can always be derived from those obtained with the finite incident waves by proceeding to the limiting value when $a = 0$ and $b = \infty$; the former thus being special cases of the latter. For reasons stated earlier in the paper, however, curves based on the infinite rectangular incident wave are

$$e_2 = 2KE\left[\frac{a^2\lambda - a\mu + \nu}{a^2\alpha - a\beta + \gamma}\epsilon^{-at} - \frac{b^2\lambda - b\mu + \nu}{b^2\alpha - b\beta + \gamma}\epsilon^{-bt}\right.$$
$$+\frac{\lambda\left(\frac{-\beta+\phi}{2\alpha}\right)^2 + \mu\left(\frac{-\beta+\phi}{2\alpha}\right) + \nu}{\phi\left(a + \frac{-\beta+\phi}{2\alpha}\right)}\epsilon^{\frac{-\beta+\phi}{2\alpha}t} - \frac{\lambda\left(\frac{-\beta-\phi}{2\alpha}\right)^2 + \mu\left(\frac{-\beta-\phi}{2\alpha}\right) + \nu}{\phi\left(a + \frac{-\beta-\phi}{2\alpha}\right)}\epsilon^{\frac{-\beta-\phi}{2\alpha}t}$$
$$\left.-\frac{\lambda\left(\frac{-\beta+\phi}{2\alpha}\right)^2 + \mu\left(\frac{-\beta+\phi}{2\alpha}\right) + \nu}{\phi\left(b + \frac{-\beta+\phi}{2\alpha}\right)}\epsilon^{\frac{-\beta+\phi}{2\alpha}t} + \frac{\lambda\left(\frac{-\beta-\phi}{2\alpha}\right)^2 + \mu\left(\frac{-\beta-\phi}{2\alpha}\right) + \nu}{\phi\left(b + \frac{-\beta-\phi}{2\alpha}\right)}\epsilon^{\frac{-\beta-\phi}{2\alpha}t}\right] \quad . \quad (7)$$

where $\phi^2 = \beta^2 - 4\alpha\gamma$.

If the circuit is oscillatory, so that $4\alpha\gamma - \beta^2$ is real, then for the case where the incident voltage-wave is given by (1), the actual solution is

$$e_2 = 2KE\left[\frac{a^2\lambda - a\mu + \nu}{a^2\alpha - a\beta + \gamma}\epsilon^{-at} - \frac{b^2\lambda - b\mu + \nu}{b^2\alpha - b\beta + \gamma}\epsilon^{-bt}\right.$$
$$\left.-\frac{2}{\alpha^{\frac{1}{2}}\psi}\left\{(\alpha\nu - \gamma\lambda)^2 + (\alpha\mu - \beta\lambda)(\gamma\mu - \beta\nu)\right\}^{\frac{1}{2}}\left\{\frac{\cos\left(\frac{\psi t}{2\alpha} - \theta_1\right)}{(a^2\alpha - a\beta + \gamma)^{\frac{1}{2}}} - \frac{\cos\left(\frac{\psi t}{2\alpha} - \theta_2\right)}{(b^2\alpha - b\beta + \gamma)^{\frac{1}{2}}}\right\}\epsilon^{-\frac{\beta}{2\alpha}t}\right] \quad . \quad (8)$$

where

$$\psi^2 = 4\alpha\gamma - \beta^2$$
$$\theta_1 = \text{arc tan}\,\frac{(\gamma - a\beta)(\beta\lambda - \alpha\mu) - \alpha(\gamma\mu - \beta\nu) - 2a\alpha(\alpha\nu - \gamma\lambda)}{\psi[a(\beta\lambda - \alpha\mu) + (\alpha\nu - \gamma\lambda)]}$$
$$\theta_2 = \text{arc tan}\,\frac{(\gamma - b\beta)(\beta\lambda - \alpha\mu) - \alpha(\gamma\mu - \beta\nu) - 2b\alpha(\alpha\nu - \gamma\lambda)}{\psi[b(\beta\lambda - \alpha\mu) + (\alpha\nu - \gamma\lambda)]}$$

As before, the incorporation of a third exponential in the incident wave requires the subtraction of a third similar term in (7) or (8).

When the incident wave is infinite rectangular, (7) and (8) reduce to (9) and (10) respectively, e.g.

$$e_2 = 2KE\left[\frac{\nu}{\gamma} + \frac{\lambda\left(\frac{-\beta+\phi}{2\alpha}\right)^2 + \mu\left(\frac{-\beta+\phi}{2\alpha}\right) + \nu}{\phi\left(\frac{-\beta+\phi}{2\alpha}\right)}\epsilon^{\frac{-\beta+\phi}{2\alpha}t} - \frac{\lambda\left(\frac{-\beta-\phi}{2\alpha}\right)^2 + \mu\left(\frac{-\beta-\phi}{2\alpha}\right) + \nu}{\phi\left(\frac{-\beta-\phi}{2\alpha}\right)}\epsilon^{\frac{-\beta-\phi}{2\alpha}t}\right] \quad (9)$$

and

$$e_2 = 2KE\left[\frac{\nu}{\gamma} - \frac{2}{\alpha^{\frac{1}{2}}\gamma^{\frac{1}{2}}\psi}\left\{(\alpha\nu - \gamma\lambda)^2 + (\alpha\mu - \beta\lambda)(\gamma\mu - \beta\nu)\right\}^{\frac{1}{2}}\epsilon^{-\frac{\beta t}{2\alpha}}\cos\left(\frac{\psi t}{2\alpha} - \theta\right)\right] \quad . \quad . \quad . \quad (10)$$

plotted and discussed. Further, in view of their greater simplicity, it has been thought desirable to include in this section general solutions for this wave, so that actual solutions can be directly derived therefrom for cases not considered by the author. No actual solutions are included for wave (iv), as they can be written down, after some manipulation, from the other solutions in the manner already indicated.

It has already been stated that in most of the circuits dealt with only one transition point is considered, so that connecting leads and the conductors comprising the inductances are, in general, assumed short. While this is usually admissible with waves (iv), (ii), and (iii), whose fronts range from 160 to 5 000 ft. approximately, the assumption obviously cannot theoretically hold for wave (i), whose front is of zero length. Making this assumption, however, for usual circuit constants, the difference between results using an infinitely steep front and one of the order of 0·1 microsecond, say, is small, so that wave (i) can conveniently be regarded as having this very steep front. More is said about this later.

### (5) The Junction of Lines.

The junction of two lines or cables, with no series impedance or shunt admittance at the junction, is shown in Fig. 3. The relationship between the incident voltage-wave $e_1$ on the incoming line $Z_1$, and the transmitted voltage-wave $e_2$ on the outgoing* line $Z_2$ (which is the voltage to earth at the junction) is obtained immediately from (2) by substituting therein $Z_b(p) = 0$ and $Z_a(p) = Z_c(p) = \infty$. This gives the well-known expression

$$e_2 = \frac{2Z_2}{Z_1 + Z_2} e_1$$

As $p$ does not appear in this equation, the latter is an actual solution, and not a symbolic one. For the same reason the incident voltage-wave is transmitted without change in shape, although there will be an alteration in its amplitude. As overhead lines have surge impedances varying from 300 to 600 ohms (a value of 350 ohms is used for illustrative purposes throughout the paper), and cables have surge impedances varying from 50 to 100 ohms, it is seen that in the case of the junction of a line and a cable the amplitude of the voltage wave along the latter is considerably less than that of the incident voltage-wave.

If there are $n$ outgoing lines of surge impedances $Z_2, Z_3, Z_4, \ldots Z_n$, the effective outgoing surge impedance is conveniently computed according to the parallel-resistance rule, so that $e_2$, the voltage at the junction or the separate voltage-wave on each outgoing line, is given by

$$e_2 = \frac{2(Z_2Z_3Z_4 \ldots Z_n)}{(Z_2Z_3Z_4 \ldots Z_n) + Z_1\{(Z_3Z_4Z_5 \ldots Z_n) + (Z_2Z_4Z_5 \ldots Z_n) + (Z_2Z_3Z_5 \ldots Z_n) + \ldots\}} e_1$$

Thus, the greater the number of lines connected to a transition point, the smaller is the amplitude of the voltage wave transmitted into each of them. This statement also applies to the cases where other circuits are connected at the junction. In what follows then, it is to be understood that $Z_2$ can represent the equivalent surge impedance of any number of outgoing lines, so long as there is no series impedance in them.

The analysis of the junction of surge impedance has an important application. It has been shown by Boehne* that each phase of a rotating machine under the influence of travelling waves behaves as a finite transmission line. He carried out experiments on two motors operating at 2 200 volts and 6 600 volts, and a 24 000-volt synchronous condenser. These machines had different conductor and slot dimensions, and it was found that the effective surge impedance varied in the different designs from 685 to 1 000 ohms per phase, the machine with the largest slot having the greatest surge impedance.

It is to be expected that a machine having one conductor per slot would behave as a finite line. Such a machine would offer no other alternative to the wave than to travel round the metallic path provided by the conductors, so that, except for the end connections, coupling between different parts of the winding would be small. In the case of windings having several conductors per slot, the coupling between turns will be appreciable, though the coupling between coils will still be small. Boehne's investigations were carried out with machines of this type, and apparently the turn-to-turn coupling did not have sufficient effect to prevent the winding as a whole behaving as a line. But it is reasonable to suppose that when the number of conductors per slot is large (a mush-wound motor is an extreme case), the relatively great distance of the inner wires from the iron and the close proximity of the wires to each other would cause such increase in coupling that the analogy would no longer hold. The tendency then would be for a steep-fronted wave to give rise to a concentration of voltage stress between the first turns of a coil in a manner similar to that occurring in a transformer winding. Neglecting effective terminal capacitance, it can be assumed as a good approximation, however, that, in general, a rotating-machine winding can behave as a finite line, and on this assumption the voltages at the line terminals and the neutral can be estimated.

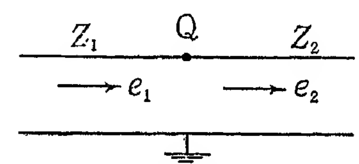



Fig. 3.

A 3-phase star-connected machine, each phase having a surge impedance $Z_2$ connected to three incoming lines each of surge impedance $Z_1$, will be dealt with. Since most of the circuits dealt with in this paper are assumed concentrated, only one transition point exists,

* In describing the lines, the terms incoming and outgoing refer to the direction of wave propagation, and are in no way concerned with the 50-cycle conditions.

* See Bibliography, (25).

and hence only one reflection occurs. A winding acting as a finite line of the order of a fraction of a mile in length cannot be assumed concentrated, however, and yet it is too short to allow of reflection from the far end being neglected. Hence two transition points must here be taken into account.

An extreme case of equal infinite rectangular waves $e_1$ on each incoming line will be considered. Dealing first with an open neutral, each wave $e_1$ on striking the appropriate junction between the line and a phase of the winding, gives rise to a terminal voltage $2e_1Z_2/(Z_1 + Z_2)$. Neglecting attenuation, a wave of this magnitude then travels to the neutral, where part is transmitted into the other phases and part reflected back. At the same time two similar waves are transmitted into the phase in question, and these, together with the reflected wave in this phase, give rise to a neutral-to-earth voltage $4e_1Z_2/(Z_1 + Z_2)$ and an effective return wave $2e_1Z_2/(Z_1 + Z_2)$. This wave, on reaching the terminal some time later (for winding lengths ranging from $0 \cdot 01$ to $0 \cdot 3$ mile, at the propagation velocity of 10 000 miles per sec. in the slot found by Boehne, the time of traverse of a wave varies between 1 and 30 microseconds) gives rise to an extra voltage of $4e_1Z_1Z_2/(Z_1 + Z_2)^2$.* Thus, at some instant of time, the terminal voltage attains the value

$$4e_1Z_1Z_2/(Z_1 + Z_2)^2 + 2e_1Z_2/(Z_1 + Z_2)$$

Further reflections take place which manifest themselves as oscillations in time at the points considered. (In this ideal case, of course, they will be square-topped oscillations.) Since the reflections and transmissions are equivalent to effective reflected waves of alternative polarities, and since each reflection entails loss to the line at the transition point, the terminal and neutral voltages fall in an oscillatory manner to a value equal to twice the voltage of the incident wave. Thus the formulæ stated give the maximum voltages attained. Earthing the neutral through a resistance $R_g = Z_2/3$ has been suggested. This causes the reflected voltage-wave in one phase and the two waves transmitted from the other phases to cancel out, so that the neutral voltage is held down to the same value as the terminal voltage, that is $2e_1Z_2/(Z_1 + Z_2)$. When the neutral is solidly earthed, there is no transmission from one phase to another and the reflections from the neutral are of opposite sign to that of the incident wave. Thus the maximum terminal voltage is $2e_1Z_2/(Z_1 + Z_2)$, and each double traverse of the reflections causes this to decrease with time.

These considerations show that under open-neutral conditions, with a wave on each phase (which is a practicable assumption), and for $Z_1 = 350$ ohms and $Z_2 = 1\,000$ ohms, the maximum terminal and neutral voltages are $2 \cdot 25$ and 3 times the incident voltage respectively; and these figures do not alter much for other reasonable values of the constants. They assume an infinite rectangular wave and neglect attenuation. When the useful length of the back of the incident wave in microseconds is less than the time required for a double traverse of the winding, or the front longer than this time, by virtue of the additive and subtractive effects of the internal reflections, the voltages are less than those indicated. Attenuation also has the effect of giving a further diminution in these voltages.

The case where there is a wave $e_1$ on one phase only, whatever the neutral connections, does not give rise to such high voltages. An investigation similar to that just indicated shows that always the maximum voltage is found at the line terminal and is equal to $2e_1Z_2/(Z_1 + Z_2)$.

Another interesting point arises from this similarity between a machine winding and a finite transmission line. The voltage stress between turns under any conditions is controlled by the length of the front of the incident wave, not only by virtue of the shunt and series capacitances of the winding as in a transformer, but also more directly. Taking an infinite rectangular wave as an extreme case, it will be seen that at any instant of time a particular turn will be stressed at full voltage while the next turn, neglecting induction, will have zero potential. Thus full voltage appears progressively between individual turns. This inter-turn stress decreases with decreasing wave-front slope, and in practice, owing to losses, there is progressive flattening as the wave penetrates the winding.

These remarks show, therefore, that it is desirable to employ some wave-front flattening device to reduce the inter-turn stresses in cases where the machine is subjected to surges.* It should also be realized that while the insulation to earth in the slot is always stronger than the between-turn insulation, breakdown to earth is more likely to occur in a rotating machine than in a transformer, and on this account it is desirable also to obtain amplitude reduction.

### (6) Condenser and Condenser-Resistance Circuits.

The various circuit arrangements to be discussed in this section are shown in Fig. 4. Their consideration is important, firstly because condensers are frequently used as protection against travelling waves, and secondly because they represent for transient purposes a wide range of engineering equipments such as the busbar structures and switchgear of substations and transformers.

Regarding the representation of a substation by a condenser, it is fairly evident that the switchgear tanks, the steelwork in close proximity to the line, and the considerable number of insulators, have the effect of a lumped capacitance to earth provided that the lines in the substation are not long. Experimental evidence is

* These expressions, and those to follow in this section, are simply obtained as follows. Let three waves $e_2i_2$ travel through the winding $Z_2$ towards the neutral, which is earthed through a resistance $R_g$ Let each reflected wave per phase be $e_{2r}i_{2r}$, each transmitted wave in the opposite phases $e_2'i_2'$, and the current flowing to earth through $R_g$, $3i_2''$. Then, as will be evident from the Appendix, a consideration of the conditions at a transition point, in this case the neutral, gives: $e_2 = i_2Z_2$, $e_{2r} = -i_{2r}Z_2$, $e_2 + e_{2r} = e_2'$, $i_2''R_g = e_2'$, $e_2' = i_2'Z_2$, and $i_2 + i_{2r} = 2i_2' + i_2''$. Manipulation of these equations will show that, for effective non-reflection, $3R_g = Z_2$, and remembering that $e_2$, the initial internal voltage-wave, and $e_1$, the incident voltage-wave on the line, are related through $e_2 = 2e_1Z_2/(Z_1 + Z_2)$, the equations in the text can be obtained. Considerations with $R_g$ absent or a wave on one phase only are special cases of this analysis.

* See Bibliography, (26).

available in support of this view. Southgate,* for instance, found as a result of oscillograph tests that the fronts of waves incident on substations become sloped and their amplitudes reduced, and that this effect is increased somewhat by the addition of large transformers to the busbars.

The statement that a transformer behaves as a capacitance to travelling waves may, however, need some explanation. A transformer winding can be regarded as a circuit consisting of self-inductance, mutual inductance, series capacitance, earth capacitance, and resistance. To abrupt impulses the inductance acts initially as an open circuit, and the voltage distribution is then decided by the series and earth capacitances. From the instant of the initial distribution (which gives rise to dangerous inter-turn voltages), the inductive impedance falls while the capacitive impedances rise, so that an internal redistribution of potential proceeds. This is evidenced as internal oscillations which are eventually damped out as the final distribution, determined by the resistance and leakage (assuming an impulse with a maintained back), is approached. Experiment shows, however, that after the initial charging of the earth and series capacitances (which decreases the rate of voltage-rise across the transformer) the effect at the terminal of these oscillations is slight, and for periods of approximately 20 to 40 microseconds, so far as external circuits are concerned, the transformer behaves essentially as a capacitance to earth.

Blume and Boyajian† have shown that the effective transient earth capacitance of a transformer is given by $C_{eff.} = \sqrt{(C_g C_s)}$, where $C_g$ is the total earth capacitance of the winding and $C_s$ is the series capacitance from one end of the winding to the other. To obtain absolute values of the effective earth capacitance from this formula in any particular case needs care,‡ and it is very desirable to solve the problem by measuring, for example, the flattening of the front of a wave incident on the transformer.

*(a) Condensers Earthing the Junction of Two or More Lines.*

The circuit shown in Fig. 4(*a*) can represent:—

(i) A condenser earthing the junction of two or more lines or cables.

(ii) A transformer with or without a busbar and switchgear system to which two or more lines or cables are connected. In this case the transformer might be preceded by a static condenser, and if the line joining the two is short it is legitimate to assume that $C_1$ is the sum of the two individual capacitances.

(iii) A condenser protecting a large rotating machine at the end of a single line. In this case $Z_2$ must be assigned the appropriate value of approximately 600 to

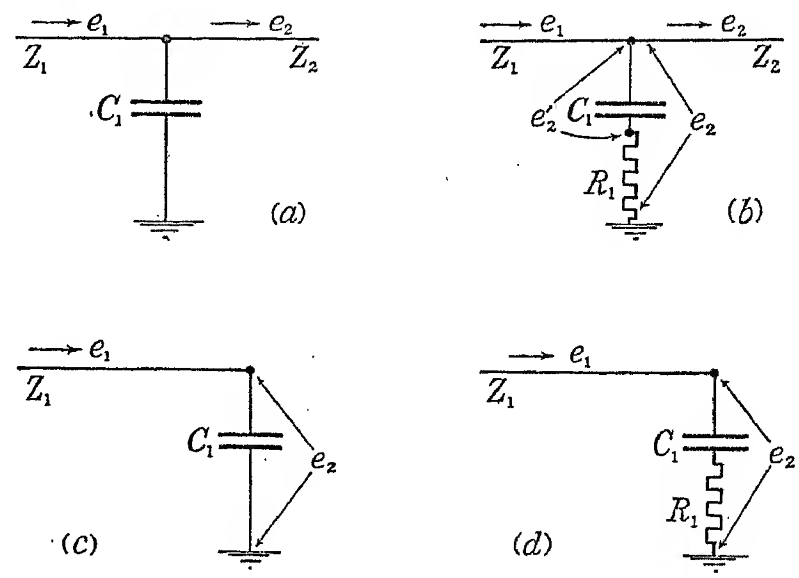



FIG. 4.

1 000 ohms. The equations to be given do not incorporate the effect of reflection from the far end of $Z_2$. Where necessary this can be taken into account in the manner outlined in Section (5).

The symbolic equation for the voltage across $C_1$ or the voltage wave along $Z_2$ is obtained from (2) by substituting $Z_a(p) = 1/(pC_1)$, $Z_b(p) = 0$, and $Z_c(p) = \infty$, so that

$$e_2 = \frac{2Z_2}{Z_1 + Z_2 + pC_1Z_1Z_2}e_1$$

This particular symbolic equation is of the type given by (3), so that when $\mu = 0$, $\nu = 1$, $\beta = Z_1Z_2C_1$, $\gamma = Z_1 + Z_2$, and $K = Z_2$, are substituted in (5), the actual solution is obtained for all cases where the incident waves are given by (1). It is

$$e_2 = \frac{2Z_2E}{Z_1 + Z_2 - aC_1Z_1Z_2}\left[\epsilon^{-at} - \epsilon^{-\frac{Z_1+Z_2}{C_1Z_1Z_2}t}\right] - \frac{2Z_2E}{Z_1 + Z_2 - bC_1Z_1Z_2}\left[\epsilon^{-bt} - \epsilon^{-\frac{Z_1+Z_2}{C_1Z_1Z_2}t}\right] \quad . \quad . \quad . \quad (11)$$

This equation is plotted for various values of $C_1$ and for wave (i) in Fig. 5. It will be seen that a capacitance of 200 micromicrofarads gives a negligible amount of wave-front flattening. As the capacitance to earth of the terminal of a large 100 000-volt transformer is only of the order of 200 micromicrofarads, the claims frequently put forward that condenser terminals are useful for wave-front flattening are optimistic. Condensers having a capacitance of 1 000 micromicrofarads have often been suggested as satisfactory for this purpose, but it will be seen that they can only give a flattening up to 0·75 microsecond. It is only when the value of a pure capacitance exceeds 3 000 micromicrofarads that

* See Bibliography, (27).

† *Ibid.*, (28).

‡ There are many factors which contribute to the difficulties in the numerical evaluation of this expression. The determination of $C_s$, which is usually of the order of a few micromicrofarads only, is rendered difficult by the fact that the dielectric is non-homogeneous, that the electric field is complicated by conductors, and that end rings may or may not be employed. In the case of $C_g$, which is of the order of 0·001 microfarad, extraneous capacitances have to be partly allocated to $C_g$ itself and partly to $C_{eff.}$ Unless considerations such as these are carefully taken into account it will be found that the calculated value of $C_{eff.}$ will differ considerably from that determined by the oscillograph Similar remarks apply to the expression

$$\frac{E}{l}\sqrt{\frac{C_g}{C_s}}$$

which gives the voltage gradient at the line end of the winding for infinite rectangular incident waves. The whole question of internal phenomena in transformer windings, which is just touched upon in this section, has been dealt with by many writers. For representative present-day opinion see Bibliography, (29) to (36) inclusive.

there is any considerable flattening, and with incident waves having less steep fronts (like wave iii) this value may not have much effect.

It is also evident that any outgoing h.t. cables, by virtue of their lower surge impedance, will have a voltage considerably less than the incident voltage impressed upon them. It thus follows that the connection of such cables to a substation will reduce the total voltage developed on it.

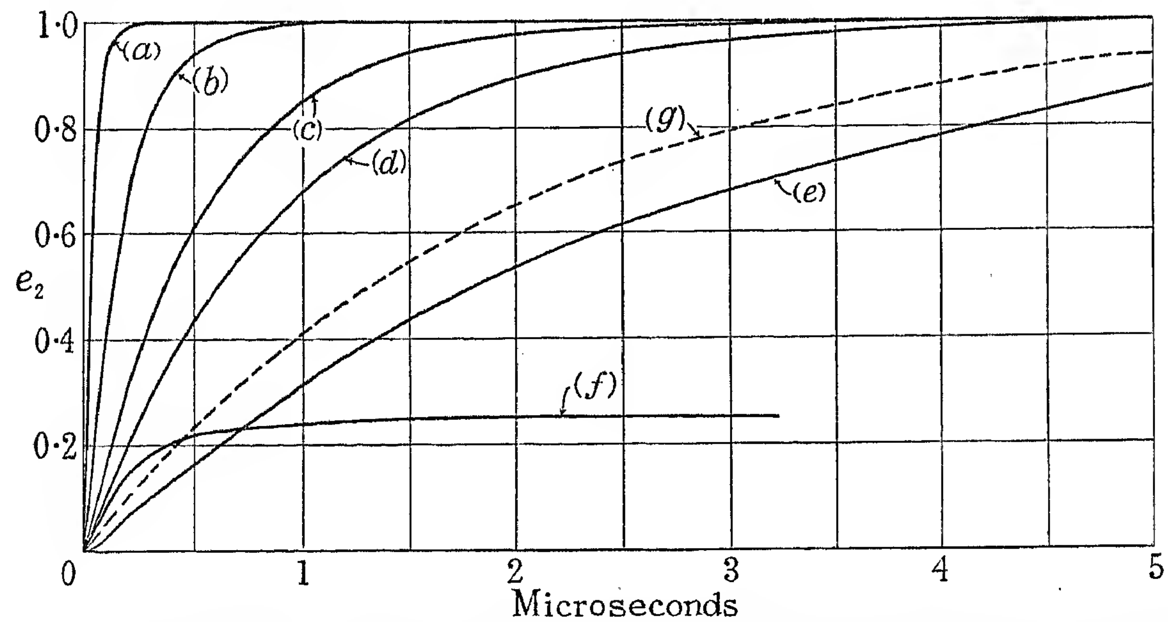



FIG. 5.—Transmitted voltage-waves when the junction of two lines is earthed through a condenser as in Fig. 4(*a*). Infinite rectangular incident wave.

(*a*) $C_1 = 200\ \mu\mu F$, $Z_1 = Z_2 = 350\ \Omega$.
(*b*) $C_1 = 1\,000\ \mu\mu F$, $Z_1 = Z_2 = 350\ \Omega$.
(*c*) $C_1 = 3\,000\ \mu\mu F$, $Z_1 = Z_2 = 350\ \Omega$.
(*d*) $C_1 = 5\,000\ \mu\mu F$, $Z_1 = Z_2 = 350\ \Omega$.
(*e*) $C_1 = 15\,000\ \mu\mu F$, $Z_1 = Z_2 = 350\ \Omega$.
(*f*) $C_1 = 5\,000\ \mu\mu F$, $Z_1 = 350\ \Omega, Z_2 = 50\ \Omega$.
(*g*) $e_2'$ in Fig. 4(*b*). $C_1 = 5\,000\ \mu\mu F, R_1 = 200\ \Omega, Z_1 = Z_2 = 350\ \Omega$.

substation can be more readily neglected when the longer-fronted waves are used. This statement is confirmed by Southgate, who definitely found that in small substations, at any rate, no appreciable difference in wave-shape exists in different parts of the station. In addition, by employing incident waves with finite backs, comparisons between the crest values can now be made. The most important point to note is that it requires very large values of capacitance to reduce the

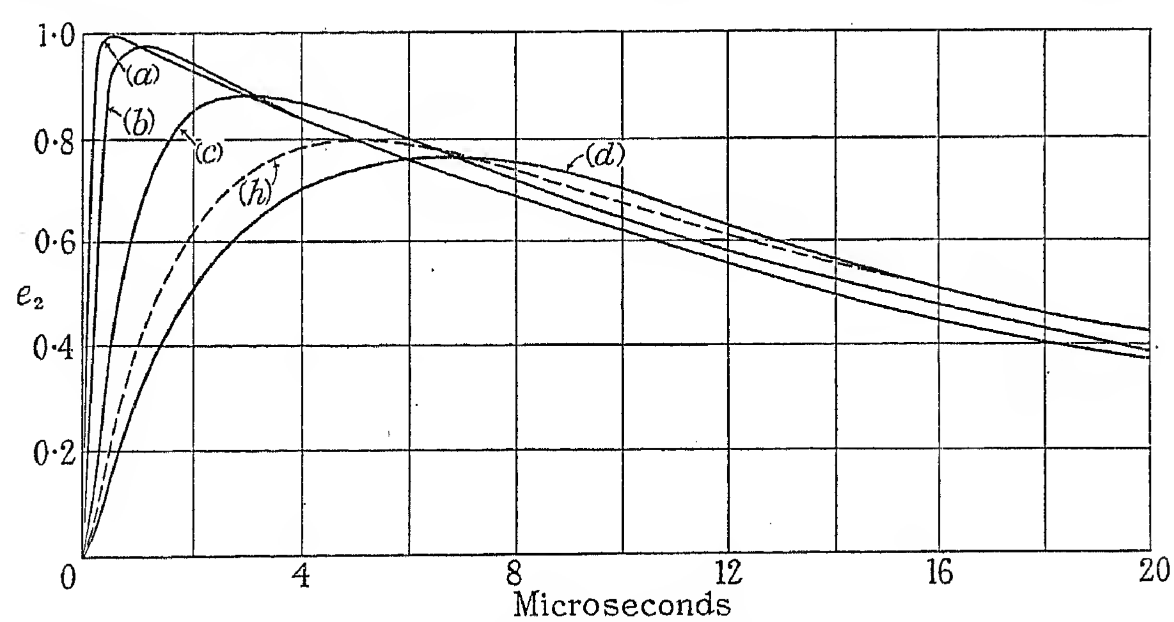



FIG. 6.—Transmitted voltage-waves when junction of two lines is earthed through a condenser as in Fig. 4(*a*). 0·4–14 microsecond incident wave.

$Z_1 = Z_2 = 350\ \Omega$. (*a*) $C_1 = 200\ \mu\mu F$. (*b*) $C_1 = 1\,000\ \mu\mu F$. (*c*) $C_1 = 5\,000\ \mu\mu F$. (*d*) $C_1 = 15\,000\ \mu\mu F$. (*h*) $e_2'$ in Fig. 4(*b*). $C_1 = 5\,000\ \mu\mu F$; $R_1 = 200\ \Omega$.

For waves (ii), (iii), and (iv), and for various values of $C_1$, (11) is plotted in Figs. 6 and 7. The remarks previously made regarding wave-front flattening apply here, but the curves are much more useful for several reasons. For instance, when a substation is regarded as a condenser, the finite lengths of the lines inside the crest value of the surge by any appreciable amount, except in the case of the insulator-chopped wave. In addition, the back of the wave, even with quite appreciable values of the earth capacitance, is not lengthened a great deal.

So far, the author has been unable to trace much

experimental data on the effective capacitance to surges of transformers and substations. Scattered records do exist, however, and it is interesting to make some comparisons between the experimental results of Southgate and the figures here obtained mathematically. Southgate's tests were carried out on a 132/220-kV substation, the particular bank of transformers under investigation being three 30 000-kVA single-phase units with solidly earthed neutrals. With all the busbars, circuit breakers, and the transformer bank, in circuit, it was found that the 5·5-microsecond front of the incident wave was increased to approximately 10·5 microseconds with a reduction in amplitude of 35 per cent. With the transformer bank and its busbar alone in circuit, the front was increased to 6·5 microseconds. Making an approximate comparison with the curves already given, it would appear that a complete substation of this type might have an effective earth capacitance per phase of the order of 0·01 to 0·02 microfarad, and that the corresponding figure for a large power transformer with its busbars would be 0·003 microfarad.

That the latter is of the right order of magnitude has been confirmed experimentally by the author. For an 11 000-volt 500-kVA transformer the effective earth capacitance per phase was 0·001 microfarad, and for an 11 000-volt 50-kVA transformer it was 0·0005 microfarad. McEachron* also quotes the case of an 8 000-kVA transformer having an earth capacitance of 0·0014 microfarad.

Other writers also have suggested figures,† and although no details are given regarding their derivation it may be interesting to quote some of them here. They are shown in Table 1.

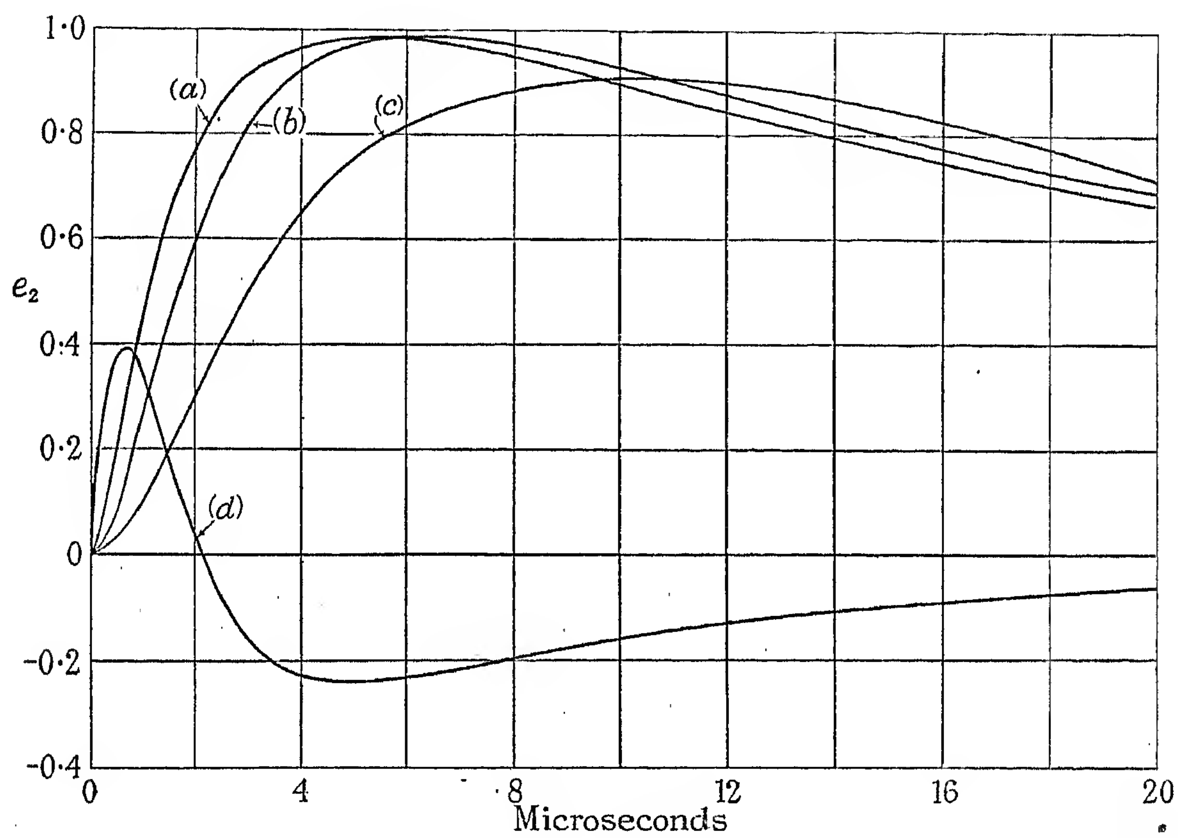



FIG. 7.—Transmitted voltage-waves when the junction of two lines is earthed through a condenser as in Fig. 4(a).

$Z_1 = Z_2 = 350\ \Omega$.
(a) $C_1 = 1\,000\ \mu\mu F$ } 5–30 microsecond incident wave.
(b) $C_1 = 5\,000\ \mu\mu F$
(c) $C_1 = 15\,000\ \mu\mu F$
(d) $C_1 = 5\,000\ \mu\mu F$, insulator-chopped wave.

TABLE 1.

| Apparatus | Capacitance, microfarads | | |
|---|---|---|---|
| | Maximum | Minimum | Average |
| Distribution transformers | 0·002 | 0·0004 | 0·001 |
| Power transformers .. | 0·001 | 0·0002 | 0·0005 |
| Busbar systems .. .. | 0·015 | 0·002 | 0·005 |

(b) *Condensers Closing the End of a Line.*

The circuit shown in Fig. 4(c) may represent the same equipments as those given in (i) and (ii) of Section (6a), except that the condenser, transformer, or substation, is connected at the end of a single line or cable.

The symbolic equation for the voltage across $C_1$ is obtained from (2) by substituting therein $Z_a(p) = 1/(pC_1)$, $Z_b(p) = 0$, and $Z_c(p) = \infty$. In addition, however, as $Z_2$ is now absent, the substitution $Z_2 = \infty$ must also be made. This leads to the equation

$$e_2 = \frac{2}{1 + pC_1Z_1}e_1$$

* See Bibliography, (37).

† *Ibid.*, (38).

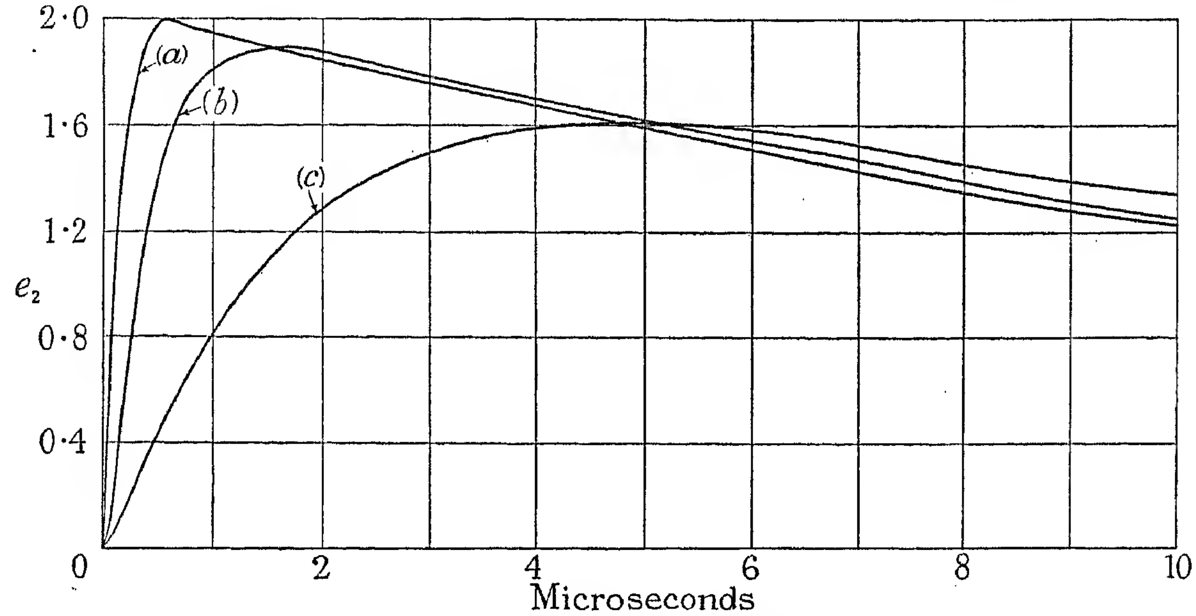



FIG. 8.—Voltages to earth at end of a line closed through a condenser as in Fig. 4($c$). 0·4–14 microsecond incident wave.

$Z_1 = 350\ \Omega$. ($a$) $C_1 = 200\ \mu\mu$F. ($b$) $C_1 = 1\,000\ \mu\mu$F. ($c$) $C_1 = 5\,000\ \mu\mu$F.

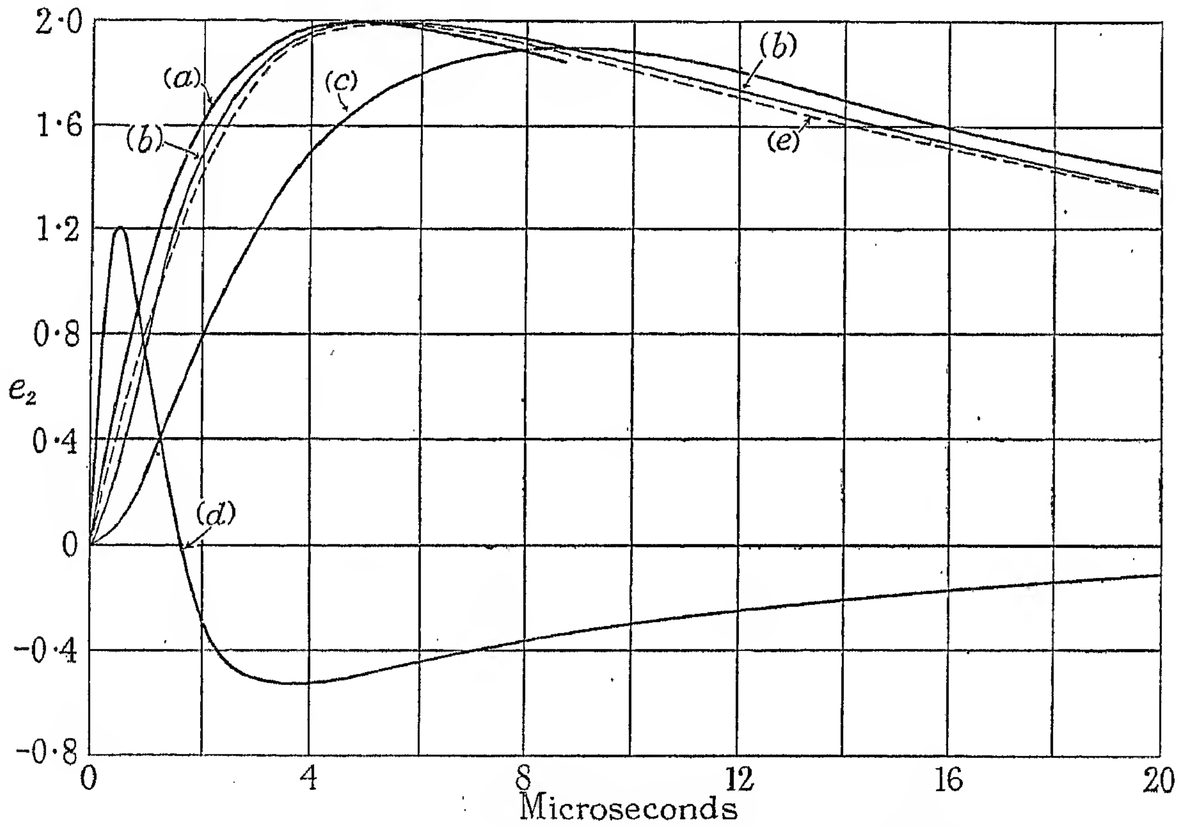



FIG. 9.—Voltages to earth at end of a line closed through a condenser as in Fig. 4($c$).

$Z_1 = 350\ \Omega$.
($a$) $C_1 = 200\ \mu\mu$F
($b$) $C_1 = 1\,000\ \mu\mu$F } 5–30 microsecond incident wave.
($c$) $C_1 = 5\,000\ \mu\mu$F
($d$) $C_1 = 1\,000\ \mu\mu$F, insulator-chopped wave.
($e$) Voltage to earth when the condenser is earthed through a resistance as in Fig. 4($d$). $C_1 = 1\,000\ \mu\mu$F, $R_1 = 200\ \Omega$, 5–30 microsecond incident wave.

the actual solution of which, when the incident wave is given by (1), is*

$$e_2 = \frac{2E}{1 - aC_1Z_1}\left[\epsilon^{-at} - \epsilon^{-\frac{t}{C_1Z_1}}\right] - \frac{2E}{1 - bC_1Z_1}\left[\epsilon^{-bt} - \epsilon^{-\frac{t}{C_1Z_1}}\right] \quad . \quad (12)$$

* Equation (12) could have been obtained immediately by putting $Z_2 = \infty$ in (11). It seemed desirable, however, to make the method quite clear at the outset, in order to cover problems in which the solutions for the cases where $Z_2$ is present are not obtained first. It should be pointed out that in this and similar simplifications it must always be ensured that the Expansion Theorem solution is not invalidated (see Appendix).

Using waves (ii), (iii), and (iv), (12) is plotted in Figs. 8 and 9 for various values of $C_1$. For long-backed waves the voltage across the condenser now approaches twice the incident voltage, and for wave-fronts and values of $C_1$ such that the amount of flattening is relatively small the average rate of rise of this voltage is also approximately twice that of the incident wave-front. On these accounts, therefore, this circuit is at a disadvantage compared with the one shown in Fig. 4($a$). When the amount of front flattening is relatively large (i.e. with steep-fronted waves and values of $C_1$ of the

order of 1 000 micromicrofarads, or long-fronted waves and very large values of $C_1$), other things being equal, it is approximately twice as great in this case as in Fig. 4(*a*). As the voltage amplitude is nearly doubled in the present case, however, the two average gradients are approximately the same.

(*c*) *Condensers Earthing the Junction of Two or More Lines through Resistance.*

Another aspect of the condenser problem which must be considered is the effect of series resistance. It is important for two reasons; firstly, condensers used as protection against surges are frequently earthed through resistance (more will be said of this later), and secondly, there is the inherent earth resistance which varies considerably with locality. In many places a value of 20 to 50 ohms is obtained, but a not unusual value is of the order of 200 to 300 ohms, and with certain soil conditions, unless artificial precautions are taken, earth resistances of the order of 2 000 ohms are possible.*

Two cases are considered, namely the voltage across the condenser and the resistance, which is the voltage wave ($e_2$ in Fig. 4*b*) transmitted along $Z_2$, and the voltage ($e_2'$ in Fig. 4*b*) across the condenser only.

Solutions for $e_2$ will be derived first. The symbolic equation can be obtained from (2) by putting $Z_b(p) = 0$, $Z_c(p) = \infty$, and $Z_a(p) = R_1 + 1/(pC_1)$, so that

$$e_2 = \frac{2Z_2(1 + R_1pC_1)}{Z_1 + Z_2 + pC_1(Z_1Z_2 + Z_1R_1 + Z_2R_1)}e_1$$

The actual solution is obtained by substitution in (5) where $\nu = 1$, $\mu = R_1C_1$, $\beta = C_1(Z_1Z_2 + Z_1R_1 + Z_2R_1)$, $\gamma = Z_1 + Z_2$, and $K = 2Z_2$, so that

$$e_2 = \frac{2EZ_2}{Z_1 + Z_2 - aC_1(Z_1Z_2 + Z_1R_1 + Z_2R_1)}\left[(1 - aR_1C_1)\epsilon^{-at} - \frac{Z_1Z_2}{Z_1Z_2 + Z_1R_1 + Z_2R_1}\epsilon^{-\frac{(Z_1+Z_2)t}{C_1(Z_1Z_2+Z_1R_1+Z_2R_1)}}\right]$$
$$- \frac{2EZ_2}{Z_1 + Z_2 - bC_1(Z_1Z_2 + Z_1R_1 + Z_2R_1)}\left[(1 - bR_1C_1)\epsilon^{-bt} - \frac{Z_1Z_2}{Z_1Z_2 + Z_1R_1 + Z_2R_1}\epsilon^{-\frac{(Z_1+Z_2)t}{C_1(Z_1Z_2+Z_1R_1+Z_2R_1)}}\right] \quad (13)$$

Fig. 10 shows (13) plotted for various values of $R_1$ and for waves (i) and (ii). It will be seen that the effect of the resistance is such that part of the front of the wave is transmitted unchanged, that is, initially the transmitted wave has the same gradient as the incident wave. On the other hand, owing to loss in $R_1$, some decrease in the amplitude of the finite wave is obtained over that which occurs when there is no resistance.

The actual solution for $e_2$ for the case shown in Fig. 4(*d*) can be immediately obtained from (13). It is shown dotted in Fig. 9 for one set of constants and for wave (iii).

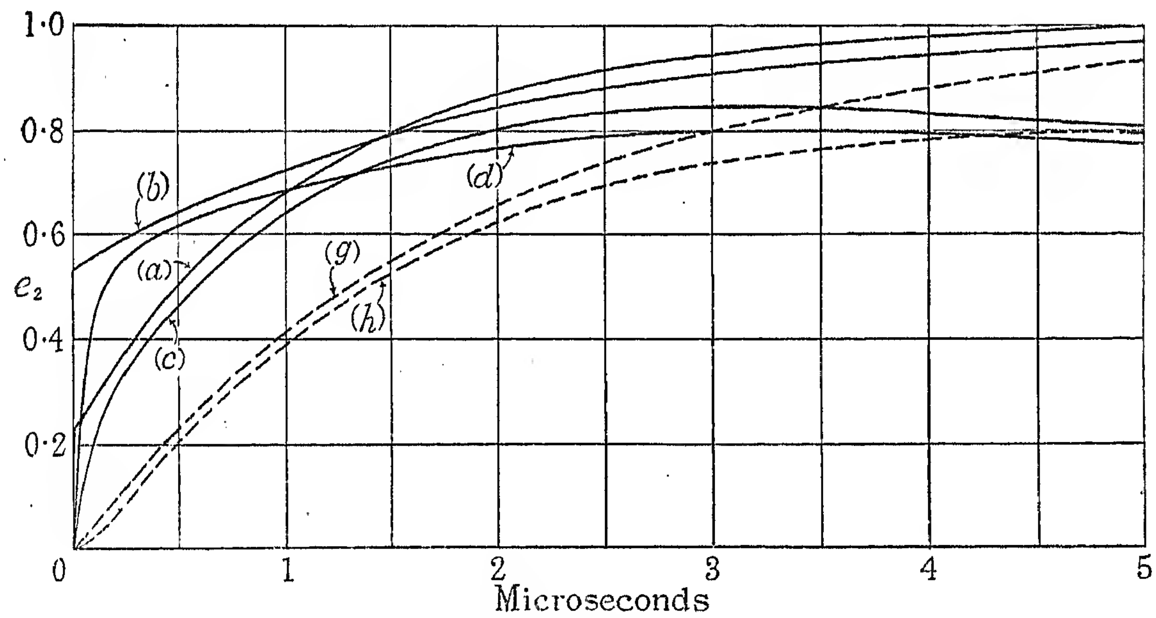



FIG. 10.—Transmitted voltage-waves when the junction of two lines is earthed through a condenser in series with a resistance and the voltages across the condenser.

$C_1 = 5\,000\ \mu\mu F$, $Z_1 = Z_2 = 350\ \Omega$.

$e_2$ in Fig. 4(*b*):
(*a*) $R_1 = 50\ \Omega$, infinite rectangular incident wave.
(*b*) $R_1 = 200\ \Omega$, infinite rectangular incident wave.
(*c*) $R_1 = 50\ \Omega$, 0·4–14 microsecond incident wave.
(*d*) $R_1 = 200\ \Omega$, 0·4–14 microsecond incident wave.

$e_2'$ in Fig. 4(*b*):
(*g*) $R_1 = 200\ \Omega$, infinite rectangular incident wave.
(*h*) $R_1 = 200\ \Omega$, 0·4–14 microsecond incident wave.

* See Bibliography, (39) and (40).

The symbolic equation for $e_2'$ is

$$e_2' = \frac{1/(pC_1)}{1/(pC_1) + R_1} e_2$$
$$= \frac{2Z_2}{Z_1 + Z_2 + pC_1(Z_1Z_2 + Z_1R_1 + Z_2R_1)} e_1$$

the actual solution of which is

$$e_2' = \frac{2EZ_2}{Z_1 + Z_2 - aC_1(Z_1Z_2 + Z_1R_1 + Z_2R_1)}\left[\epsilon^{-at} - \epsilon^{-\frac{(Z_1+Z_2)t}{C_1(Z_1Z_2+Z_1R_1+Z_2R_1)}}\right]$$
$$- \frac{2EZ_2}{Z_1 + Z_2 - bC_1(Z_1Z_2 + Z_1R_1 + Z_2R_1)}\left[\epsilon^{-bt} - \epsilon^{-\frac{(Z_1+Z_2)t}{C_1(Z_1Z_2+Z_1R_1+Z_2R_1)}}\right] \quad . \quad (14)$$

For comparison purposes (14) is shown dotted in Figs. 5 and 10 for wave (i), and in Figs. 6 and 10 for wave (ii). It is evident in this case that the rate of voltage-rise across the condenser alone is less than that obtained when there is no series resistance, and the potentiometer effect causes further amplitude reduction when the incident wave is finite.

### (7) Inductance and Inductance-Resistance Circuits.

The three different arrangements which will be discussed in this section of the paper are shown in Fig. 11. They deal with air-core inductances having series or shunt resistance at the junction of lines or air-core inductances at the end of a line. Fig. 11($a$) represents, therefore:—

(i) An inductance or current-limiting reactor inserted between lines or lines and cables.

(ii) An inductance or current-limiting reactor inserted in front of a rotating machine where the earth capacitance is small.

Fig. 11($b$) represents arrangements similar to (i) and (ii) with the addition of shunt resistance. Fig. 11($c$) is of minor importance only.

Regarding Fig. 11($a$), and referring to (i), while it is unlikely that two lines would be joined by an inductance for surge-protection reasons, such is often inserted between a line and a cable. Further, lines interconnected through reactors can frequently be simulated by this circuit. Referring to (ii), both reflection in the machine winding and the earth capacitance of the necessary switchgear (assumed concentrated at the line terminal) are neglected. The former can be taken care of, if necessary, in the manner indicated in Section (5), while it will be evident from Section (9) that small values of the earth capacitance are unimportant when an impedance $Z_2$ (in this case the winding) is present. Thus, this circuit is useful for estimating the voltage wave entering a machine preceded by an inductance when the earth capacitance is small. When the latter is large, the circuits discussed in Section (9) must be consulted.

Regarding Fig. 11($b$), the usual reasons given for connecting resistance across an inductance coil or reactor in a line are, firstly, to prevent oscillation with earth capacitance, and secondly, to obtain surge-energy loss. Such loss is, however, small. Further, when $Z_2$ is present, even if capacitance exists there is no oscillation, so that the use of shunt resistance is unnecessary.

The inductances are regarded as having no inter-turn or shunt capacitance, but their resistances, assumed constant, are taken into account. Theoretically this latter assumption is in error because the transient resistance of a coil differs from the d.c. resistance and depends on the equivalent frequency of the wave passing through it. Thus the resistance varies during the passage of the surge. In the present state of the art it is impossible to take into account this variation with frequency, but experiment has shown that the resistance is generally not high enough to make much difference in the results; in fact, ordinary coils ranging in inductance from 200 to 10 000 microhenrys have average transient-resistances ranging from 10 to 100 ohms.

An error also arises owing to the fact that the length of the wire comprising the coil may be appreciably longer than the front of the wave. With wave (ii), which has a front of approximately 400 ft., the error is not great and oscillograph records have confirmed this view. With a wave having a 5 000-ft. front there is practically no error on this account. With very steep-fronted waves, like (i), however, the error may be

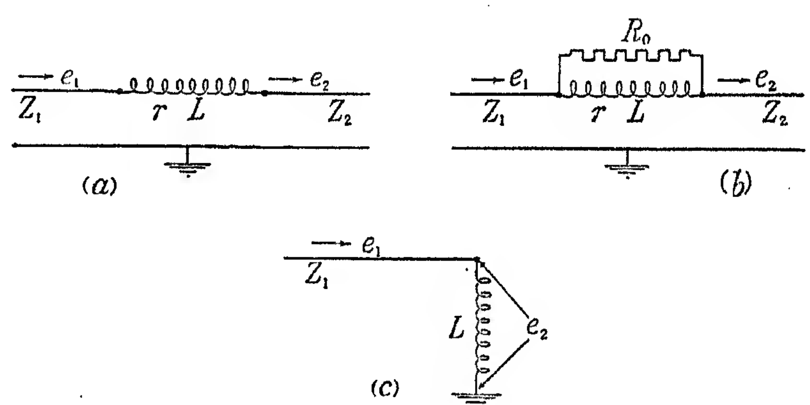



Fig. 11.

considerable. Under normal 50-cycle conditions at the entrance of the coil there is a counter e.m.f. due to the current in the exit turns (and in the others, of course), but with a current wave having a very steep front just entering the coil there will be no current in the exit turns, and hence no counter e.m.f. on this account. As the wave penetrates the coil this effect certainly decreases, but for a relatively short period of time the coil responds with a value of inductance considerably less than the measured value. Experiment shows that with practical steep waves having fronts of 0·1 microsecond, say, the surge or transient inductance is of the order of 85 per cent of the 50-cycle value.

#### ($a$) *Inductance, having Series Resistance, Joining Two or More Lines.*

The circuit being considered here is shown in Fig. 11($a$). The symbolic equation for the transmitted voltage-wave is obtained by substituting $Z_a(p) = \infty$, $Z_c(p) = \infty$, and $Z_b(p) = r + pL$, in (2).

Thus $$e_2 = \frac{2Z_2}{Z_1 + Z_2 + r + pL} e_1$$

When the incident wave is given by (1), the actual solution is

$$e_2 = \frac{2Z_2E}{Z_1 + Z_2 + r - aL}\left[\epsilon^{-at} - \epsilon^{-\frac{Z_1+Z_2+r}{L}t}\right] - \frac{2Z_2E}{Z_1 + Z_2 + r - bL}\left[\epsilon^{-bt} - \epsilon^{-\frac{Z_1+Z_2+r}{L}t}\right] \quad . \ (15)$$

This equation is plotted in Figs. 12, 13, and 14, for various values of the circuit constants and for all the incident waves. For values of inductance and capacitance which are economic and usual in practice, it is apparent that greater wave-front flattening and amplitude reduction are afforded by inductance than by capacitance. On the other hand, coils of 7 or 8 turns, which are frequently used in practice under certain circumstances, have an inductance of the order of 0·0001 henry as a maximum, and it will be realized that their effect must be negligible. It is also evident that series resistance has a beneficial effect in reducing the amplitude of the transmitted wave, but that it causes some decrease in the amount by which the front is flattened.

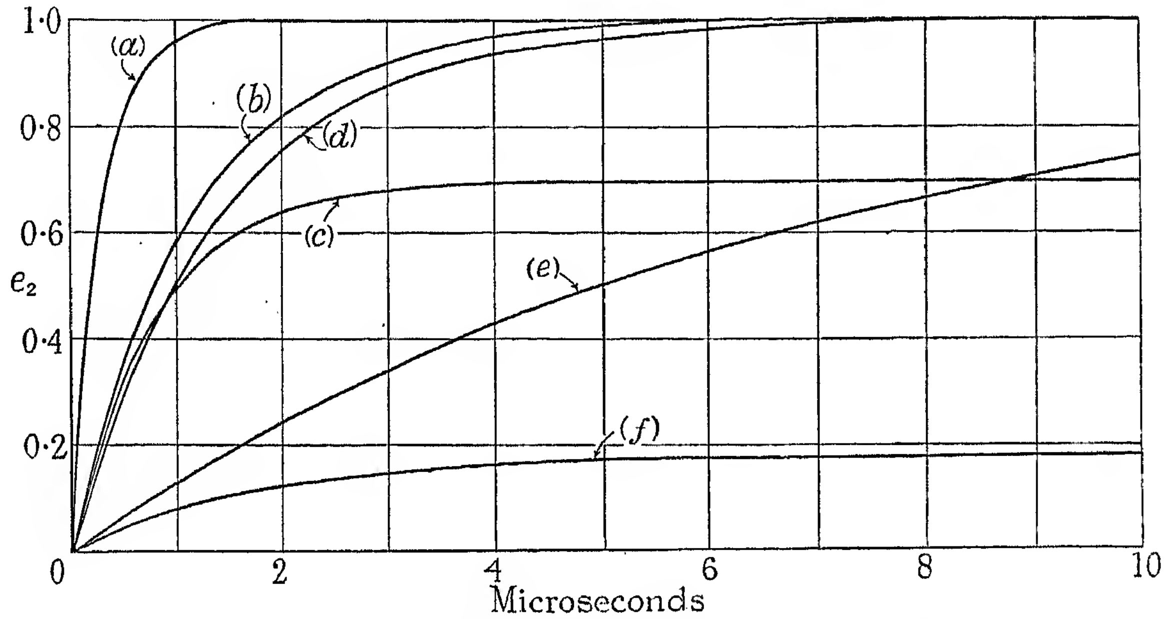



FIG. 12.—Transmitted voltage-waves when two lines are joined by an inductance as in Fig. 11(*a*). Infinite rectangular incident wave.

(*a*) $L = 200\ \mu H$, $r = 0$, $Z_1 = Z_2 = 350\ \Omega$.
(*b*) $L = 800\ \mu H$, $r = 0$, $Z_1 = Z_2 = 350\ \Omega$.
(*c*) $L = 800\ \mu H$, $r = 300\ \Omega$, $Z_1 = Z_2 = 350\ \Omega$.
(*d*) $L = 1\,000\ \mu H$, $r = 0$, $Z_1 = Z_2 = 350\ \Omega$.
(*e*) $L = 5\,000\ \mu H$, $r = 0$, $Z_1 = Z_2 = 350\ \Omega$.
(*f*) $L = 1\,000\ \mu H$, $r = 0$, $Z_1 = 500\ \Omega$, $Z_2 = 50\ \Omega$.

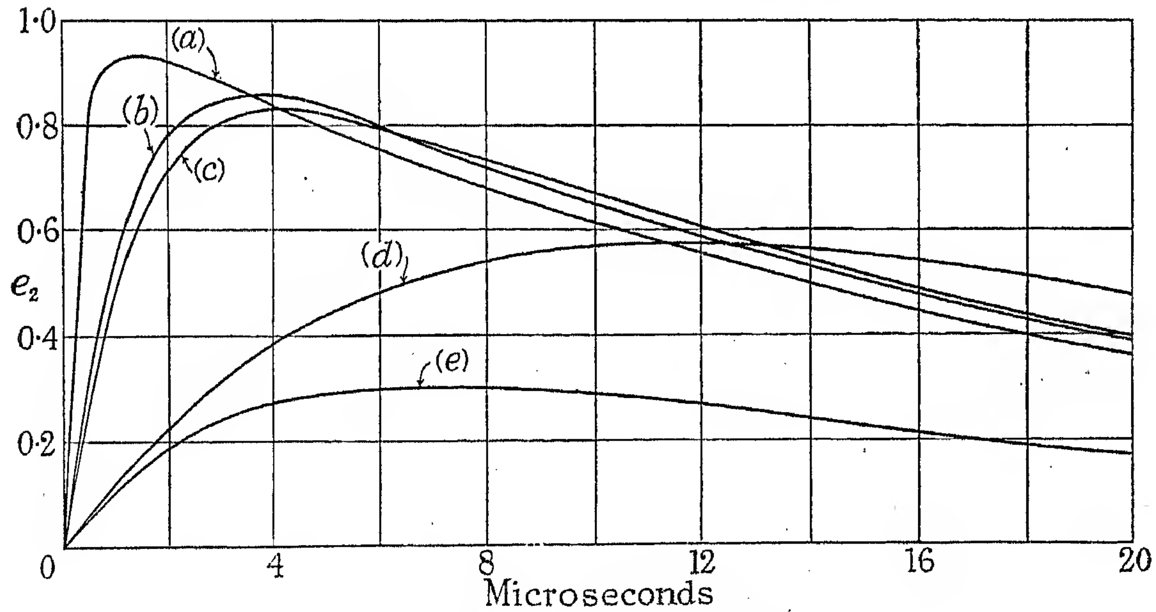



FIG. 13.—Transmitted voltage-waves when two lines are joined by an inductance as in Fig. 11(*a*). 0·4–14 microsecond incident wave.

$Z_1 = Z_2 = 350\ \Omega$.
(*a*) $L = 200\ \mu H$, $r = 0$.
(*b*) $L = 800\ \mu H$, $r = 0$.
(*c*) $L = 1\,000\ \mu H$, $r = 0$.
(*d*) $L = 5\,000\ \mu H$, $r = 0$.
(*e*) $L = 5\,000\ \mu H$, $r = 1\,000\ \Omega$.

(*b*) *Inductance, having Shunt and Series Resistance, Joining Two or More Lines.*

When the coil is shunted by a resistance, as in Fig. 11(*b*), the symbolic equation for the transmitted voltage-wave is obtained by substituting $Z_a(p) = \infty$, $Z_c(p) = \infty$, and $Z_b(p) = R_0(r + pL)/(R_0 + r + pL)$. Thus

$$e_2 = \frac{2Z_2(R_0 + r + pL)}{r(Z_1 + Z_2) + R_0(Z_1 + Z_2 + r) + pL(Z_1 + Z_2 + R_0)} e_1$$

For incident waves given by (1), the actual solution of this is

$$e_2 = \frac{2EZ_2}{Z_1+Z_2+r-aL+(r-aL)(Z_1+Z_2)/R_0}\left[\left(1+\frac{r-aL}{R_0}\right)\epsilon^{-at}-\frac{1}{1+(Z_1+Z_2)/R_0}\epsilon^{-\frac{1}{L}\left\{\frac{Z_1+Z_2+r+(Z_1+Z_2)r/R_0}{1+(Z_1+Z_2)/R_0}\right\}t}\right]$$
$$-\frac{2EZ_2}{Z_1+Z_2+r-bL+(r-bL)(Z_1+Z_2)/R_0}\left[\left(1+\frac{r-bL}{R_0}\right)\epsilon^{-bt}-\frac{1}{1+(Z_1+Z_2)/R_0}\epsilon^{-\frac{1}{L}\left\{\frac{Z_1+Z_2+r+(Z_1+Z_2)r/R_0}{1+(Z_1+Z_2)/R_0}\right\}t}\right] \quad (16)$$

value of the resistance for best all-round results, and in the particular case examined this value is approximately 500 ohms.

(*c*) *Inductance Earthing the End of a Line.*

Inductance at the end of a line, as shown in Fig. 11(*c*), is chiefly of academic interest. The symbolic equation for the voltage across the inductance is

$$e_2 = \frac{2pL}{Z_1 + pL} e_1$$

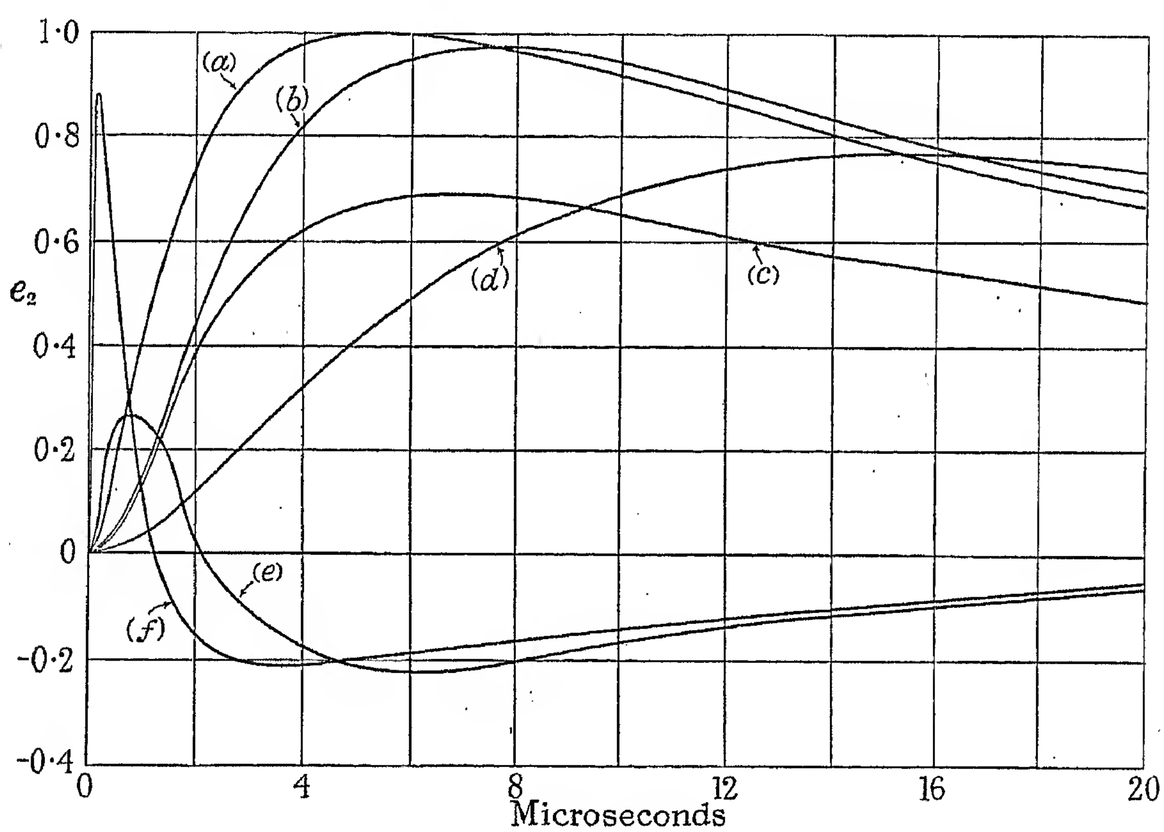



FIG. 14.—Transmitted voltage-waves when two lines are joined by an inductance as in Fig. 11(*a*).

$Z_1 = Z_2 = 350\ \Omega$.
(*a*) $L = 200\ \mu H$, $r = 0$. } 5–30 microsecond incident wave.
(*b*) $L = 1\,000\ \mu H$, $r = 0$.
(*c*) $L = 1\,000\ \mu H$, $r = 300\ \Omega$.
(*d*) $L = 5\,000\ \mu H$, $r = 0$.
(*e*) $L = 1\,000\ \mu H$, $r = 0$, insulator-chopped wave.
(*f*) $L = 1\,000\ \mu H$, $r = 0$, $R_0 = 300\ \Omega$. With shunt resistance as in Fig 11(*b*); insulator-chopped wave.

Using wave (i), (16) is plotted in Fig. 15 for a constant $L$ and varying values of $R_0$. As would be expected, an inductance with shunt resistance behaves in a manner similar to a condenser with series resistance; that is to say, part of the front of the wave is transmitted unchanged.

Using wave (ii) and a constant value of $L$, the relationships between $R_0$ and the maximum value of the transmitted wave, and $R_0$ and the length of the front of this wave, are shown in Fig. 16. It is seen that there is a

and the actual solution, for waves given by (1), is

$$e_2 = \frac{2E}{Z_1 - aL}\left[Z_1\epsilon^{-\frac{Z_1 t}{L}} - aL\epsilon^{-at}\right]$$
$$- \frac{2E}{Z_1 - bL}\left[Z_1\epsilon^{-\frac{Z_1 t}{L}} - bL\epsilon^{-bt}\right] \quad . \quad (17)$$

This equation is plotted in Fig. 17 for waves (i), (ii), and (iii). There is, of course, no wave-front flattening, and for small values of $L$ the voltage falls to zero in a

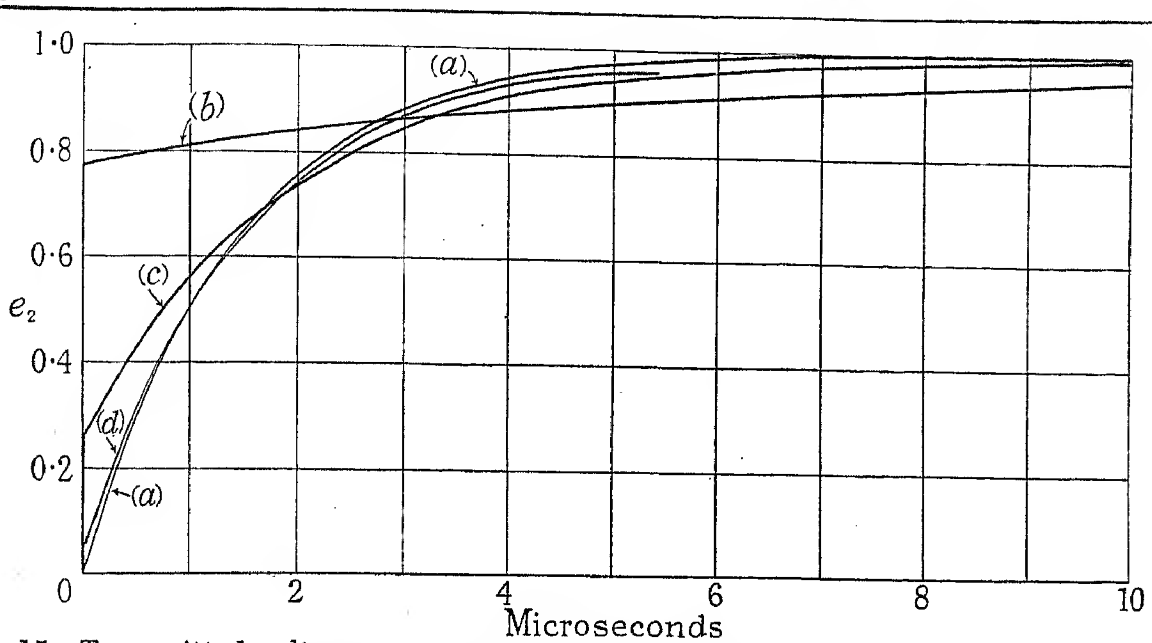



FIG. 15—Transmitted voltage-waves when two lines are joined by an inductance having shunt resistance as in Fig. 11(*b*). Infinite rectangular incident wave.

$Z_1 = Z_2 = 350\ \Omega, r = 0.$

(*a*) $L = 1\,000\ \mu H$, $R_0 = \infty$.
(*b*) $L = 1\,000\ \mu H$, $R_0 = 200\ \Omega$.
(*c*) $L = 1\,000\ \mu H$, $R_0 = 2\,000\ \Omega$.
(*d*) $L = 1\,000\ \mu H$, $R_0 = 20\,000\ \Omega$.

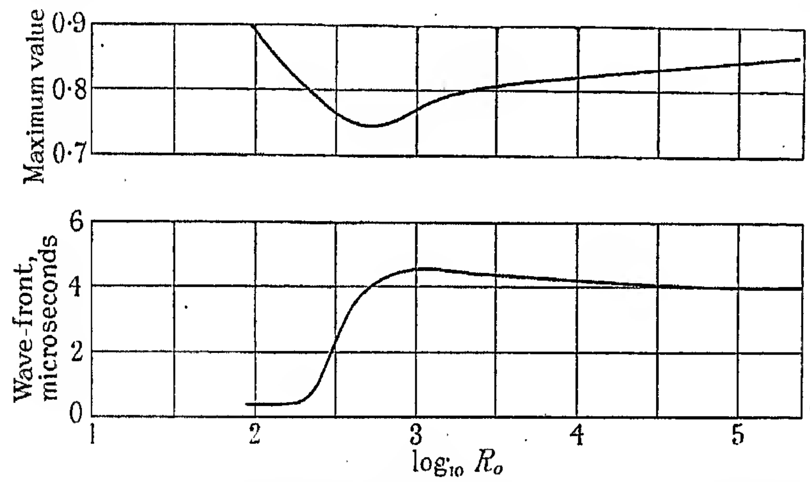



FIG. 16.—Maximum values, and the lengths of the wave-fronts in microseconds, of the transmitted voltage-waves for various values of the shunt resistance $R_0$, shunting an inductance of $1\,000\ \mu H$ at the junction of two lines. 0·4–14 microsecond incident wave, $Z_1 = Z_2 = 350\ \Omega$, $r = 0$.

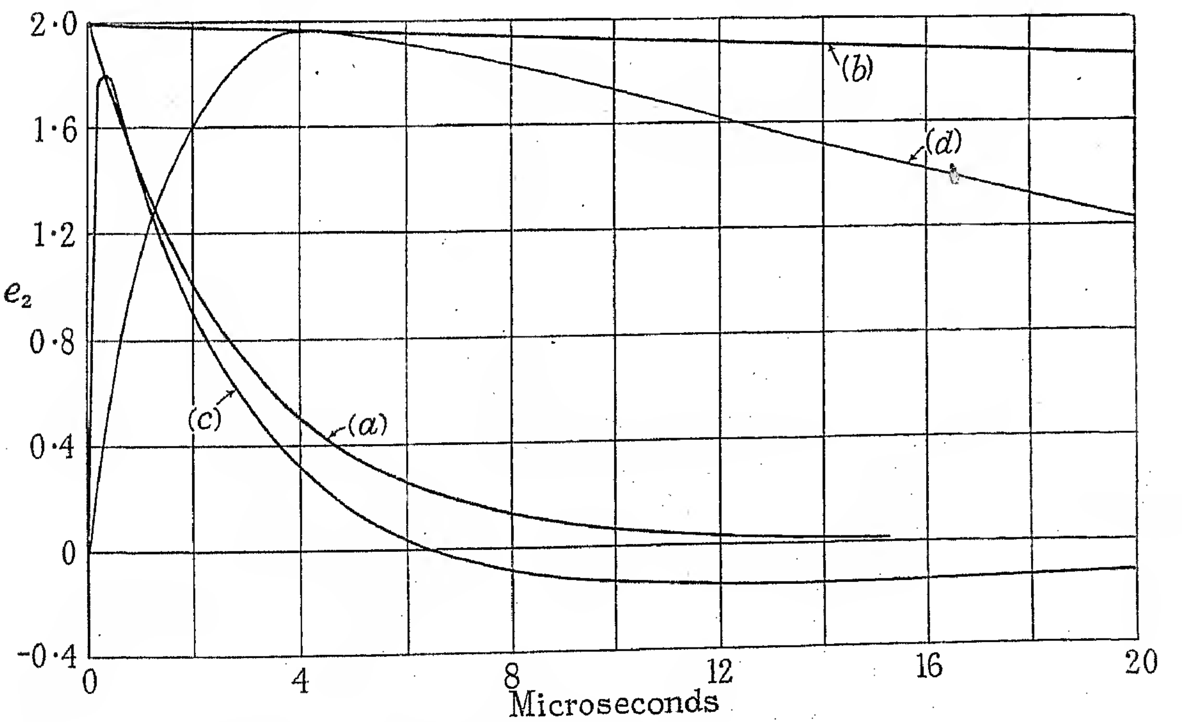



FIG. 17.—Voltages to earth at end of line closed by an inductance as in Fig. 11(*c*).

$Z_1 = 350\ \Omega.$

(*a*) $L = 1\,000\ \mu H$ } Infinite rectangular incident wave.
(*b*) $L = 0{\cdot}1\ H$ }
(*c*) $L = 1\,000\ \mu H$, 0·4–14 microsecond incident wave.
(*d*) $L = 0{\cdot}1\ H$, 5–30 microsecond incident wave.

time shorter than the total length of the incident wave, and then reverses its polarity. In addition, under some circumstances the maximum value of the voltage across the inductance is less than twice the incident voltage-wave, although its rate of rise is approximately the same as that of the latter. Large values of the inductance behave as open circuits for long periods of time; in the case of curve (*b*) in Fig. 17, for instance, the drop in voltage after 20 microseconds is only 7 per cent.

### (8) Circuits Involving Inductance and Capacitance in Parallel.

(*a*) *Inductance and Capacitance in Parallel Joining Two or More Lines.*

The circuit to be discussed in this section of the paper is shown in Fig. 18(*a*), and represents a series coil where

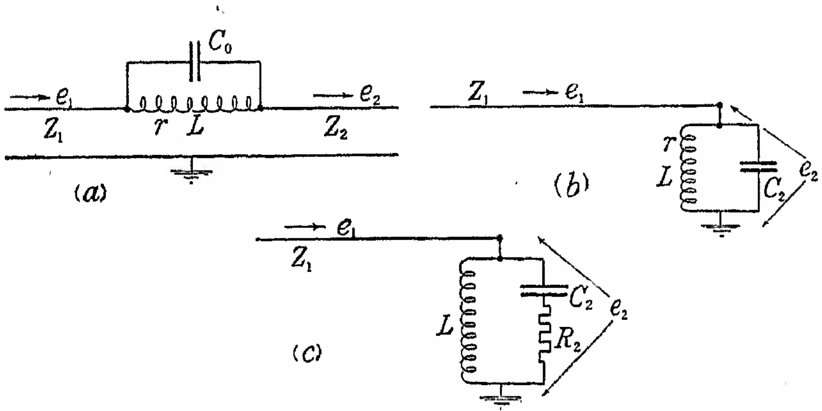



Fig. 18.

shunt capacitance exists between the ends of the winding. Since no advantage is to be gained by actually connecting a condenser across the coil, the arrangement considered only applies to accidental or inherent capacitance. The former may be due to the methods adopted for the mounting of the coil and the latter is due to the turn-to-turn or distributed capacitance, the circuit being an approximation for this case. With single-layer coils the shunt capacitance can be calculated directly with a fair degree of accuracy; but where the coil is a multi-layer one, calculation is unreliable. The capacitance must then be found by measuring the natural frequency of the coil, and such measurements, when applied to the formulæ to be given later, lead to results which are in good agreement with those found by oscillograph measurements. The shunt capacitance of normal inductance coils is usually so small as to have negligible effect, but as in special circumstances it may become important it is necessary to consider it.

The general tendency of this circuit is to transmit for a very short period of time the initial part of the wave unchanged,* so giving a peak at the beginning, as shown in Fig. 20. If appreciable earth capacitance exists at the junction of the coil and $Z_2$, as in Fig. 22(*a*), this peak is unimportant, so that the practical applications of this circuit are similar to those cases given under (i) and (ii) of Fig. 11(*a*), Section (7).

The symbolic equation for the transmitted voltage-wave is obtained from (2) by making the substitutions: $Z_a(p) = \infty$, $Z_c(p) = \infty$, $Z_b(p) = (r+pL)/(1+pC_0r+p^2C_0L)$, so that

$$e_2 = \frac{2Z_2(p^2C_0L+pC_0r+1)}{p^2(Z_1+Z_2)LC_0+p[L+rC_0(Z_1+Z_2)]+Z_1+Z_2+r}e_1$$

This equation is of the type given by (4), where $K = Z_2$, $\lambda = C_0L$, $\mu = C_0r$, $\nu = 1$, $\alpha = C_0L(Z_1 + Z_2)$, $\beta = L + C_0r(Z_1 + Z_2)$, and $\gamma = Z_1 + Z_2 + r$. Two

* See Bibliography, (41) and (42).

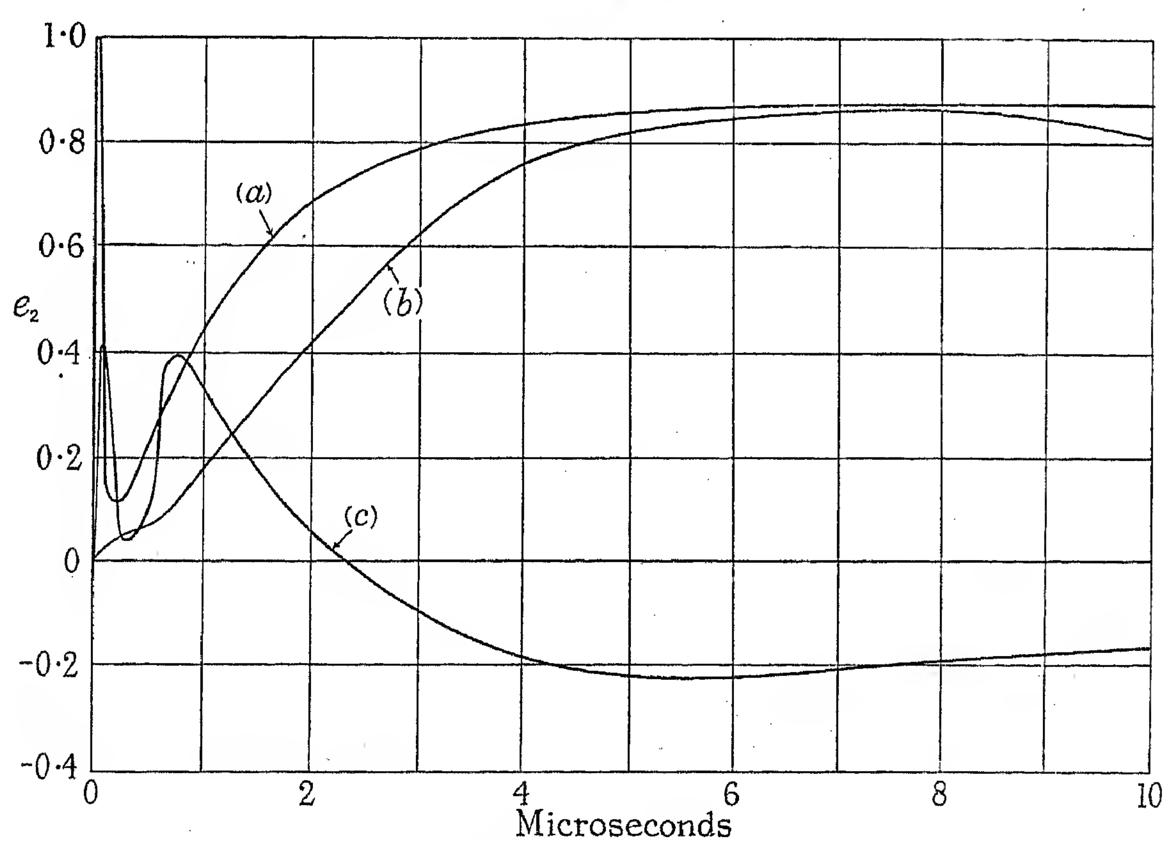



Fig. 19.—Transmitted voltage-waves when two lines are joined by an inductance having shunt capacitance, as in Fig. 18(*a*).

$L = 1\,000\ \mu$H, $C_0 = 100\ \mu\mu$F, $Z_1 = Z_2 = 350\ \Omega$.
(*a*) $r = 100\ \Omega$, infinite rectangular incident wave.
(*b*) $r = 100\ \Omega$, 5–30 microsecond incident wave.
(*c*) $r = 0$, insulator-chopped incident wave.

actual solutions must be considered. When $\beta^2 > 4a\gamma$ the equation for the transmitted voltage-wave is obtained by substitution of the above values in (7), so that, for waves given by (1),

$$e_2 = 2Z_2E\left[\frac{a^2C_0L - aC_0r + 1}{a^2C_0LZ - a(L + C_0rZ) + (Z + r)}\epsilon^{-at} - \frac{b^2C_0L - bC_0r + 1}{b^2C_0LZ - b(L + C_0rZ) + (Z + r)}\epsilon^{-bt} + \frac{(b - a)(C_0LK_1^2 - C_0rK_1 + 1)}{\phi(K_1 - a)(K_1 - b)}\epsilon^{-K_1t} - \frac{(b - a)(C_0LK_2^2 - C_0rK_2 + 1)}{\phi(K_2 - a)(K_2 - b)}\epsilon^{-K_2t}\right] \quad . \quad (18)$$

where $K_1 = \dfrac{L + C_0rZ - \phi}{2C_0LZ}$, $K_2 = \dfrac{L + C_0rZ + \phi}{2C_0LZ}$,

$$\phi = \sqrt{[(L - rC_0Z)^2 - 4Z^2LC_0]},$$

and

$$Z = Z_1 + Z_2.$$

When $4a\gamma > \beta^2$, substitution in (8) gives

$$e_2 = 2Z_2E\left[\frac{a^2C_0L - C_0r + 1}{a^2C_0LZ - a(L + C_0rZ) + (Z + r)}\epsilon^{-at} - \frac{b^2C_0L - bC_0r + 1}{b^2C_0LZ - b(L + C_0rZ) + (Z + r)}\epsilon^{-bt} - \frac{2L}{\psi Z^{\frac{1}{2}}}\epsilon^{-\frac{L+C_0rZ}{2C_0LZ}t}\left\{\frac{\cos\left(\frac{\psi t}{2C_0LZ} - \theta_1\right)}{\sqrt{[a^2C_0LZ - a(L + C_0rZ) + (Z + r)]}} - \frac{\cos\left(\frac{\psi t}{2C_0LZ} - \theta_2\right)}{\sqrt{[b^2C_0LZ - b(L + C_0rZ) + (Z + r)]}}\right\}\right] \quad . \quad (19)$$

where $\psi = \sqrt{[4Z^2LC_0 - (L - rC_0Z)^2]}$,

$$\theta_1 = \text{arc tan}\,\frac{(aL - r)(C_0Zr - L) + 2LZ}{\psi(aL - r)},$$

$$\theta_2 = \text{arc tan}\,\frac{(bL - r)(C_0Zr - L) + 2LZ}{\psi(bL - r)},$$

and

$$Z = Z_1 + Z_2.$$

Using the four incident waves, (18) and (19) are plotted, for various values of the circuit constants, in Figs. 19 and 20.

Referring to Fig. 19, where in one case the incident wave is infinite rectangular, it will be seen that the wave is transmitted nearly unchanged for a brief period of time, the condenser behaving as a short-circuit initially. Later, when the condenser is fully charged, it behaves as an open circuit and the inductance exerts its influence in the ordinary way. With wave (iv), which has a finite though very steep front, the initial peak is still very noticeable, and the gradient is excessive in four different parts of the wave. With wave (iii) the front is long enough to prevent any initial peak being transmitted with the circuit constants chosen.

In Fig. 20, different transmitted voltage-waves are plotted for various values of $C_0$ using wave (ii) as the incident wave and for constant $L$. It is seen that the initial peak is prominent for values of the shunt capacitance down to 100 micromicrofarads, and 20 per cent of the maximum value of the incident wave is transmitted initially, even when the shunt capacitance is as low as 50 micromicrofarads.

*(b) Inductance and Capacitance in Parallel Closing the End of a Line.*

The voltage to earth at the end of a line earthed through an inductance and a condenser in parallel will now be considered. The arrangement is shown in Fig. 18(*b*).

The symbolic equation for the voltage $e_2$ is obtained from (2) by substituting $Z_a(p) = r + pL$, $Z_b(p) = 0$, $Z_c(p) = 1/(C_2p)$, and $Z_2 = \infty$, so that

$$e_2 = \frac{2(pL + r)}{p^2LC_2Z_1 + p(L + C_2rZ_1) + (Z_1 + r)}e_1$$

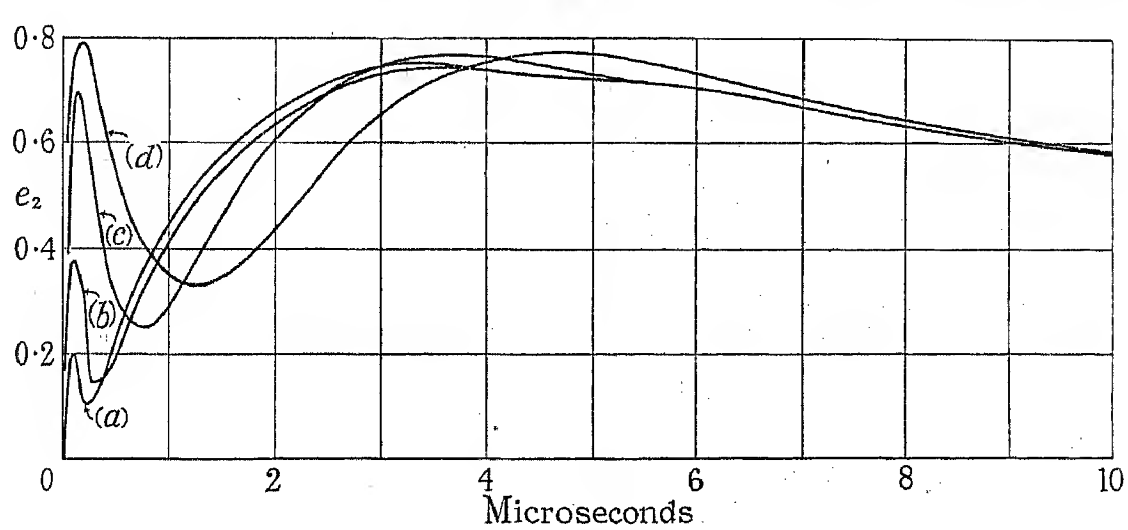



FIG. 20.—Transmitted voltage-waves when two lines are joined by an inductance having shunt capacitance, as in Fig. 18(*a*). 0·4–14 microsecond incident wave.
$L = 1\,000\,\mu$H, $Z_1 = Z_2 = 350\,\Omega$, $r = 100\,\Omega$. (*a*) $C_0 = 50\,\mu\mu$F. (*b*) $C_0 = 100\,\mu\mu$F. (*c*) $C_0 = 500\,\mu\mu$F. (*d*) $C_0 = 1\,000\,\mu\mu$F.

Therefore, for the aperiodic case,

$$e_2 = 2E\left[\frac{r-aL}{Z_1LC_2a^2-(L+Z_1rC_2)a+(r+Z_1)}\epsilon^{-at}-\frac{r-bL}{Z_1LC_2b^2-(L+Z_1rC_2)b+(r+Z_1)}\epsilon^{-bt}\right.$$
$$\left.+\frac{(b-a)(r-LK_1)}{\phi(K_1-a)(K_1-b)}\epsilon^{-K_1t}-\frac{(b-a)(r-LK_2)}{\phi(K_2-a)(K_2-b)}\epsilon^{-K_2t}\right] \quad . \quad (20)$$

and for the oscillatory case

$$e_2 = 2E\left[\frac{r-aL}{Z_1LC_2a^2-(L+Z_1rC_2)a+(Z_1+r)}\epsilon^{-at}-\frac{r-bL}{Z_1LC_2b^2-(L+Z_1rC_2)b+(Z_1+r)}\epsilon^{-bt}\right.$$
$$\left.-\frac{2L\sqrt{Z_1}}{\psi}\epsilon^{-\frac{L+C_2Z_1r}{2Z_1LC_2}t}\left\{\frac{\cos\left(\frac{\psi t}{2Z_1LC_2}-\theta_1\right)}{\sqrt{[Z_1LC_2a^2-(L+Z_1rC_2)a+(Z_1+r)]}}-\frac{\cos\left(\frac{\psi t}{2Z_1LC_2}-\theta_2\right)}{\sqrt{[Z_1LC_2b^2-(L+Z_1rC_2)b+(Z_1+r)]}}\right\}\right] \quad . \quad (21)$$

where $K_1 = \frac{L+C_2Z_1r-\phi}{2Z_1LC_2}$, $K_2 = \frac{L+C_2Z_1r+\phi}{2Z_1LC_2}$,

$$\phi = \sqrt{[(L-Z_1rC_2)^2-4Z_1^2LC_2]},$$
$$\psi^2 = -\phi^2,$$
$$\theta_1 = \text{arc tan}\,\frac{(Z_1rC_2-L)(r-aL)-2Z_1L}{\psi(r-aL)},$$

and $$\theta_2 = \text{arc tan}\,\frac{(Z_1rC_2-L)(r-bL)-2Z_1L}{\psi(r-bL)}.$$

Using waves (i) and (iii), (20) and (21) are plotted in Fig. 21. It is seen that with large values of $L$ the initial gradient of the transmitted wave is similar to that obtained with the condenser alone, while later the gradual decrease in amplitude is almost identical with that resulting from the inductance alone. With the value of 0·1 henry used in the calculations, this latter effect is noticeable after approximately 10 microseconds. Therefore, while this circuit is theoretically a closer approximation to a transformer than the simple condenser circuit given in Section (6), it is evident that, for an examination of the rate of rise of voltage across a transformer, incorporation of the inductance is unnecessary. Similarly Fig. 11($c$) is a sufficiently good approximation when considering the longer time-intervals.

It has been stated in the past that oscillations can occur between the inductance of a transformer and any condenser connected in parallel with it for surge-protection reasons, and in order to prevent such oscillations the connection of resistance (of the order of a few hundred ohms) in series with the condenser has been suggested. Apart from the fact that such resistance nullifies to some extent the wave-front flattening effect of the condenser, it can be readily seen that its use is unnecessary.

The approximate circuit is shown in Fig. 18($c$), the transformer earth capacitance being assumed small in comparison with the external condenser $C_2$. Elementary

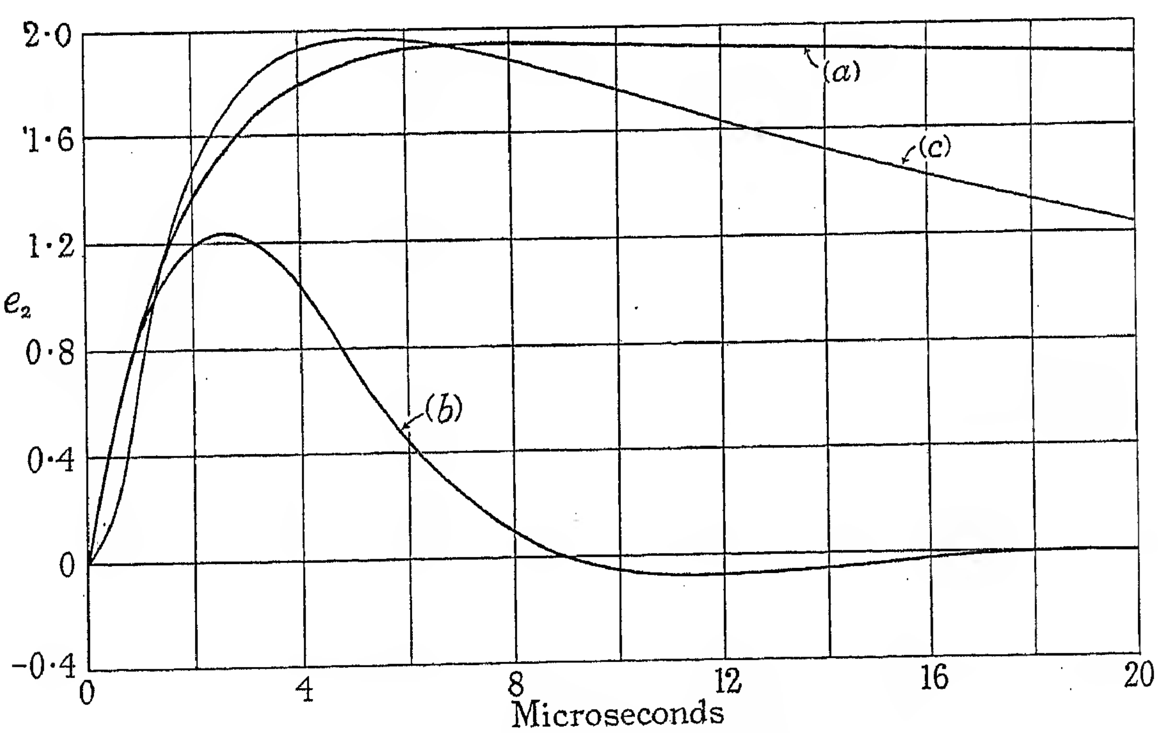



FIG. 21.—Voltages across an inductance and condenser in parallel at the end of a line as in Fig. 18($b$).

$Z_1 = 350\,\Omega, r = 0.$
($a$) $L = 0·1$ H, $C_2 = 5\,000\,\mu\mu$F } Infinite rectangular incident wave.
($b$) $L = 1\,000\,\mu$H, $C_2 = 5\,000\,\mu\mu$F }
($c$) $L = 0·1$ H, $C_2 = 1\,000\,\mu\mu$F, 5–30 microsecond incident wave.

considerations immediately show that the line surge-impedance $Z_1$ is equivalent to a very high series damping resistance, so that it might be expected that normal values of $R_2$ would have little effect on the critical state of the circuit.

The condition for oscillation is

$$4Z_1^2LC_2 > (L - R_2Z_1C_2)^2$$

and examination of this expression shows that for usual values of $Z_1$, $L$, and $C_2$, and for any value of $R_2$, it is impracticable to obtain this condition. In addition, since the internal oscillations in the transformer have very little effect on the terminal voltage (these oscillations have nothing to do with the external condenser and cannot be prevented by external series resistance), it would appear that the only trouble which could occur is due either to oscillation between the condenser and some stray inductance, or to repeated reflections between the condenser and the transformer when the connecting line is not short.

The position is quite different, however, when choking coils are connected in series with the line, and this aspect of the problem will be dealt with in the next section of the paper.

### (9) Circuits Involving Series Inductance and Earth Capacitance.

These circuits, which are shown in Fig. 22, are important as they have many practical applications. Certain of their aspects have already been investigated by many writers. Considered separately, $C_1$ and $C_2$ represent, as before, transformers, substations, or static condensers; $Z_1$ and $Z_2$ lines, cables, or rotating machines; and $L$ inductance coils or current-limiting reactors. $R_0$, $R_2$, and $r$, are the usual resistances, and remarks which have been made regarding them in previous sections of the paper also apply here.

(a) *Inductance, having Series Resistance, and Concentrated Earth Capacitance at the Junction of Two or More Lines.*

This circuit, shown in Fig. 22(a), represents:—

(i) A choking coil or reactor protecting a rotating machine and its switchgear.

(ii) A choking coil or reactor protecting a transformer or substation to which other lines or cables are connected.

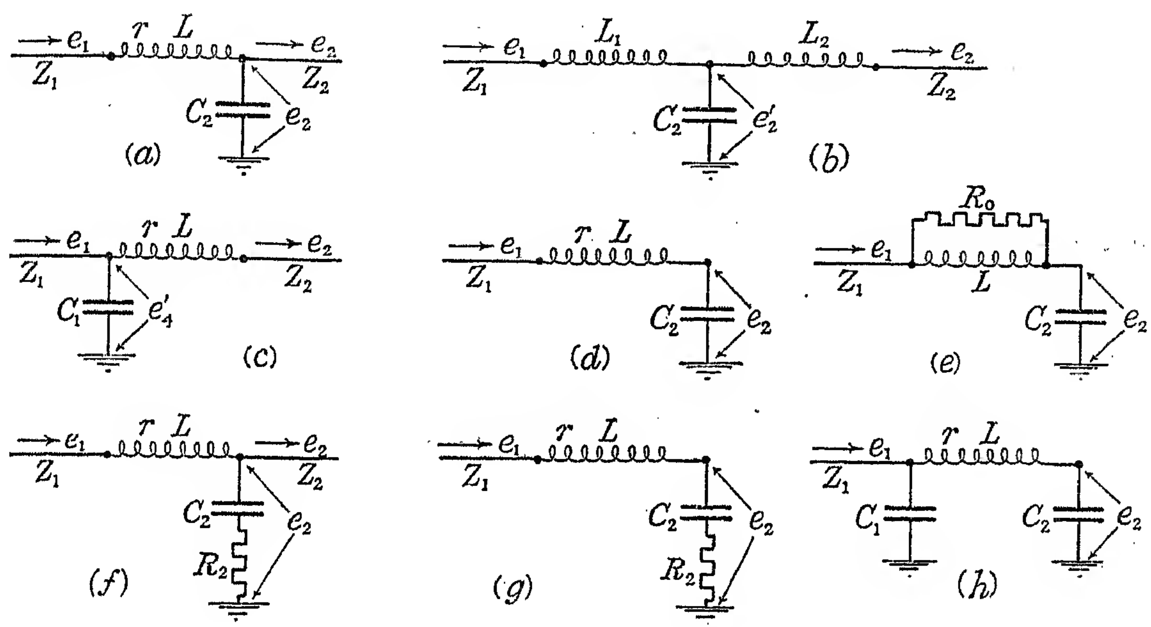



FIG. 22.

Regarding (i), as before, reflections in the machine windings are neglected. Regarding (ii), it is seen that inductance is assumed in $Z_1$ only. This assumption is frequently justifiable; for instance, when $Z_2$ represents cables or when $Z_1$ alone is susceptible to surges. On the other hand, in many cases coils would also be inserted in the outgoing feeders $Z_2$; such considerations are dealt with later.

The expressions for the voltage across the condenser $C_2$ or the transmitted voltage-wave along $Z_2$ will now be given. The symbolic equation is obtained from (2) by substituting $Z_a(p) = \infty$, $Z_b(p) = r + pL$, $Z_c(p) = 1/(C_2p)$, so that

$$e_2 = \frac{2Z_2}{p^2C_2LZ_2 + p(L + C_2Z_2r + C_2Z_1Z_2) + (Z_1 + Z_2 + r)}e_1 \quad (22)$$

The actual solutions for the aperiodic and oscillatory cases are respectively:—

$$e_2 = 2Z_2E\left[\frac{\epsilon^{-at}}{K_1} - \frac{\epsilon^{-bt}}{K_2} + \frac{b-a}{\phi(K_3-a)(K_3-b)}\epsilon^{-K_3t} - \frac{b-a}{\phi(K_4-a)(K_4-b)}\epsilon^{-K_4t}\right] \quad . \quad . \quad . \quad . \quad (23)$$

and

$$e_2 = 2Z_2E\left[\frac{\epsilon^{-at}}{K_1} - \frac{\epsilon^{-bt}}{K_2} - \frac{2}{\psi}\sqrt{\left(\frac{C_2LZ_2}{K_1}\right)}\epsilon^{-K_5t}\cos\left(\frac{\psi t}{2C_2LZ_2} - \theta_1\right) + \frac{2}{\psi}\sqrt{\left(\frac{C_2LZ_2}{K_2}\right)}\epsilon^{-K_5t}\cos\left(\frac{\psi t}{2C_2LZ_2} - \theta_2\right)\right] \quad (24)$$

where

$K_1 = Z_1 + Z_2 + r + a^2C_2LZ_2 - a(L + C_2Z_2r + C_2Z_1Z_2),$
$K_2 = Z_1 + Z_2 + r + b^2C_2LZ_2 - b(L + C_2Z_2r + C_2Z_1Z_2),$
$K_3 = (L + C_2Z_2r + C_2Z_1Z_2 - \phi)/(2C_2LZ_2),$
$K_4 = (L + C_2Z_2r + C_2Z_1Z_2 + \phi)/(2C_2LZ_2),$
$K_5 = (L + C_2Z_2r + C_2Z_1Z_2)/(2C_2LZ_2),$
$\theta_1 = \text{arc tan}\,(L + C_2Z_2r + C_2Z_1Z_2 - 2aC_2LZ_2)/\psi,$
$\theta_2 = \text{arc tan}\,(L + C_2Z_2r + C_2Z_1Z_2 - 2bC_2LZ_2)/\psi,$
$\phi = \sqrt{[(L + C_2Z_2r + C_2Z_1Z_2)^2 - 4C_2LZ_2(Z_1+Z_2+r)]},$

and

$\psi^2 = -\phi^2.$

with those circuits in which inductance alone joins $Z_1$ and $Z_2$, ohmic resistance tends to decrease the amount of flattening, this tendency is less marked when earth capacitance is present. As the number of outgoing lines is increased, so that the value of $Z_2$ decreases, there is, of course, the usual reduction in amplitude of the transmitted waves.

As was pointed out at the commencement of this section, in those cases where the circuit represents a substation with several feeders, a choking coil in one line will probably be duplicated by similar choking coils in the other lines. It has been stated* that a fair approximation to the condenser voltage is obtained by the

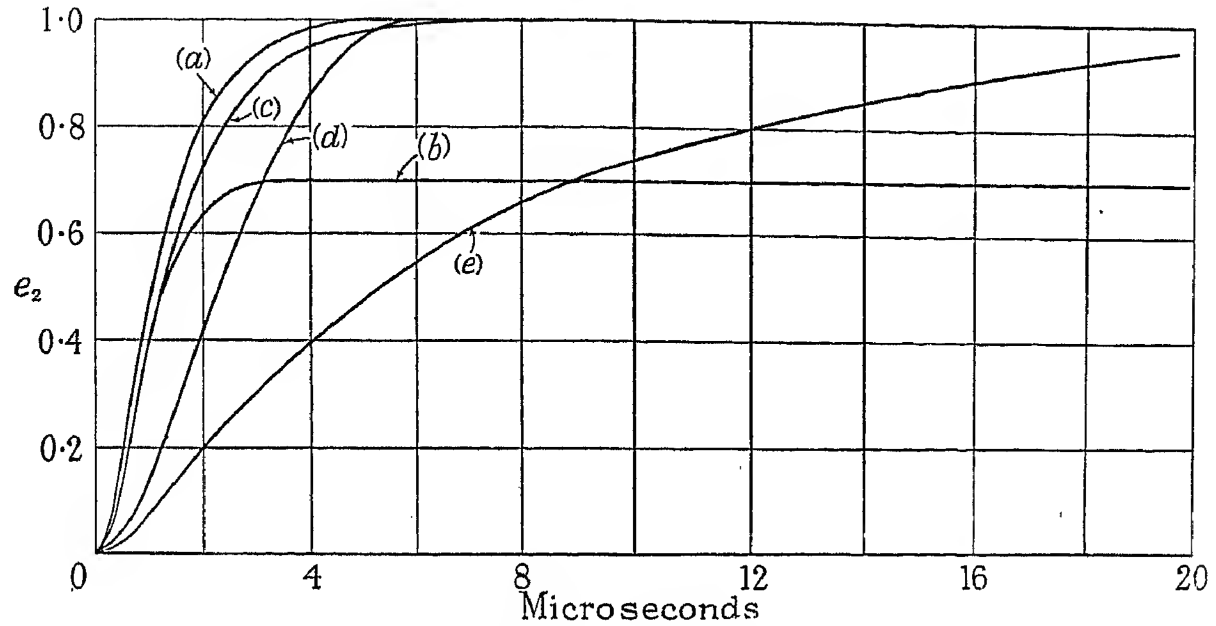



FIG. 23.—Transmitted voltage-waves when two lines are joined by an inductance; with capacitance to earth at the junction as in Fig. 22(*a*). Infinite rectangular incident wave.

$Z_1 = Z_2 = 350\ \Omega.$
(*a*) $L = 800\ \mu$H, $C_2 = 1\,000\ \mu\mu$F, $r = 0$.
(*b*) $L = 800\ \mu$H, $C_2 = 1\,000\ \mu\mu$F, $r = 300\ \Omega$.
(*c*) $L = 1\,000\ \mu$H, $C_2 = 1\,000\ \mu\mu$F, $r = 0$.
(*d*) $L = 1\,000\ \mu$H, $C_2 = 5\,000\ \mu\mu$F, $r = 0$.
(*e*) $L = 5\,000\ \mu$H, $C_2 = 1\,000\ \mu\mu$F, $r = 0$.

Using waves (i), (ii), and (iii), (23) and (24) are plotted in Figs. 23, 24, and 25, for various values of the circuit constants. In many cases values are used such that the circuits are oscillatory, but the extent of oscillation on the transmitted voltage-waves in actual practice is negligible; none of these waves for the cases where $Z_1 = Z_2$ rise above the incident voltage-wave in amplitude. Thus, the presence of one or more outgoing lines definitely prevents the voltage rising to the high values which can obtain when $Z_2$ is absent. It is interesting to note, however, that in many cases where the values of the inductance and capacitance are such that the circuit is aperiodic, increase in series resistance $r$ changes it into a strictly oscillatory one.

Regarding the curves which are plotted in Figs. 23, 24, and 25, and referring to those which have been drawn previously, it is seen that these inductance-capacitance combinations give rise to transmitted voltage-waves which have, in general, both smaller initial gradients and longer times to maxima than those circuits where either inductance or capacitance is acting alone. Also, whereas

use of one line $Z_2$ without a series choking coil as equivalent in impedance to several outgoing lines, each of surge impedance $Z_2$, but containing series coils. The smaller value of the effective parallel impedance of these several outgoing circuits brings the voltage across the condenser considerably below the limit imposed when there is no coil in a single outgoing line $Z_2$ and when $Z_1 = Z_2$. Thus the approximation is a safe one, although calculation based on it will not show the oscillations which actually occur. When there is only one outgoing line with a series coil, however, these oscillations raise the condenser voltage above the aforesaid limit.

The circuit under consideration is shown in Fig. 22(*b*). The symbolic equation for the voltage $e_2'$ across the condenser is obtained from (2) by making $Z_a(p) = \infty$, $Z_b(p) = pL_1$, $Z_c(p) = 1/(pC_2)$, and replacing $Z_2$ by $(Z_2 + pL_2)$. When $L_1 = L_2 = L$,

$$e_2' = \frac{2(pL + Z_2)}{p^3L^2C_2 + p^2LC_2(Z_1 + Z_2) + p(2L + Z_1Z_2C_2) + (Z_1 + Z_2)}e_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (24a)$$

As the denominator of this symbolic equation is a cubic in $p$, the formulæ given in (*c*) (iv) and (*c*) (v) of the Appendix must be employed to obtain actual solutions.

* See Bibliography, (38).

Two specific cases have been worked out using the infinite rectangular wave and they are plotted in Fig. 26, together with two curves obtained from a consideration of the circuit in Fig. 22($d$), which is dealt with later, and one curve obtained from a consideration of the circuit in Fig. 22($a$).

In the case of Fig. 22($b$), the choking coil in the outgoing line approximates to an open circuit initially, so that the circuit tends to respond in a manner similar to that shown in Fig. 22($d$). This accounts for the oscillatory nature of the voltage $e_2$, though, owing to the fact that the outgoing line exercises ever-increasing damping after the first instant of time, the voltage never reaches the high values which frequently occur in the case of Fig. 22($d$). Further, the rate of rise of the voltage is, in general, less than that obtained with $Z_2$ absent (as in Fig. 22$d$), but considerably greater than that which occurs when $Z_2$ is present (as in Fig. 22$a$). Thus the behaviour of this circuit is a sort of average between those of the circuits shown in Figs. 22($a$) and 22($d$). When considering the condenser voltage $e_2'$, it is not permissible, therefore, to neglect the effect of a series choking coil

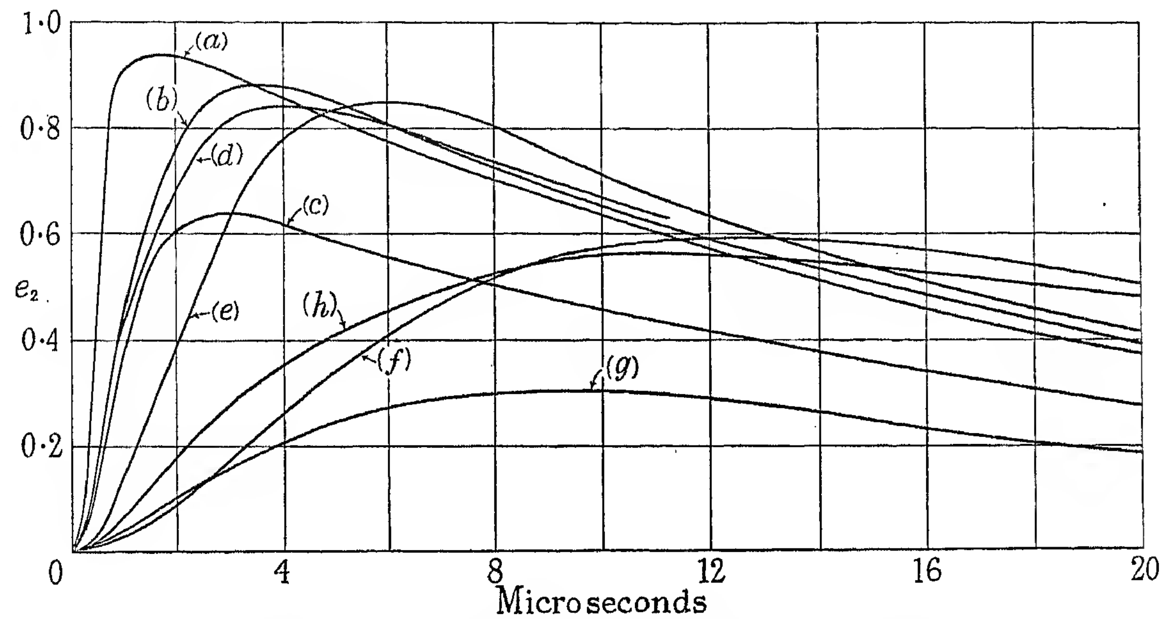



Fig. 24.—Transmitted voltage-waves when two lines are joined by an inductance; with capacitance to earth at the junction as in Fig. 22($a$). 0·4–14 microsecond incident wave.

$Z_1 = Z_2 = 350\ \Omega$.
($a$) $L = 200\ \mu H$, $C_2 = 1\,000\ \mu\mu F$, $r = 0$.
($b$) $L = 800\ \mu H$, $C_2 = 1\,000\ \mu\mu F$, $r = 0$.
($c$) $L = 800\ \mu H$, $C_2 = 1\,000\ \mu\mu F$, $r = 300\ \Omega$.
($d$) $L = 1\,000\ \mu H$, $C_2 = 1\,000\ \mu\mu F$, $r = 0$.
($e$) $L = 1\,000\ \mu H$, $C_2 = 5\,000\ \mu\mu F$, $r = 0$.
($f$) $L = 5\,000\ \mu H$, $C_2 = 5\,000\ \mu\mu F$, $r = 0$.
($g$) $L = 5\,000\ \mu H$, $C_2 = 5\,000\ \mu\mu F$, $r = 1\,000\ \Omega$.
($h$) $L = 5\,000\ \mu H$, $C_2 = 1\,000\ \mu\mu F$, $r = 0$.

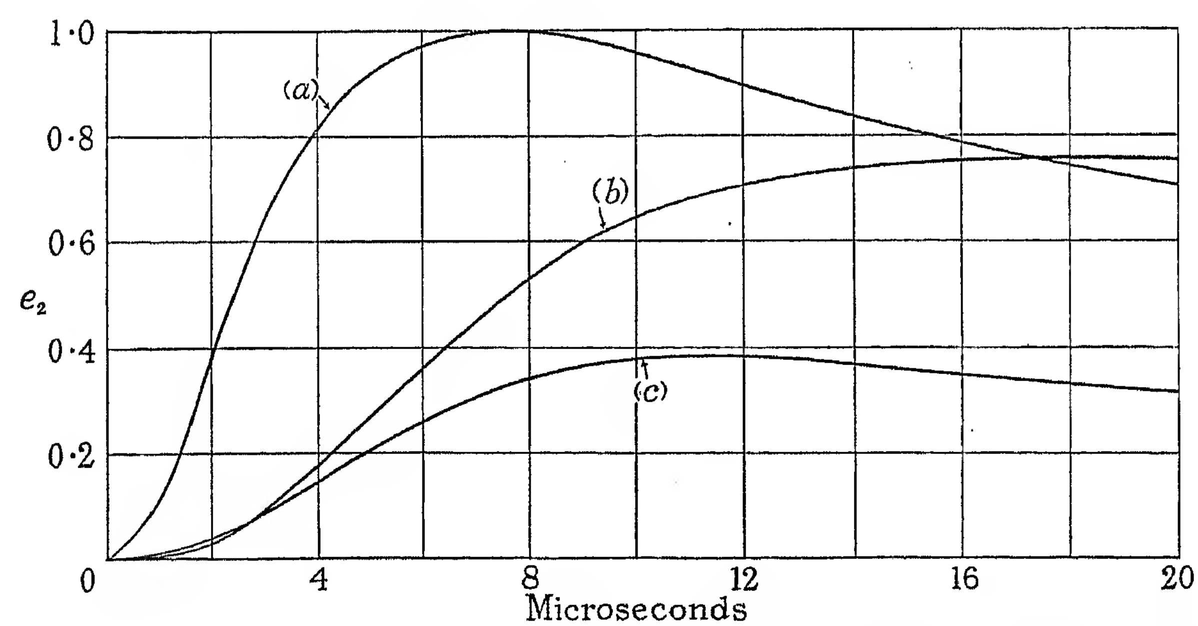



Fig. 25.—Transmitted voltage-waves when two lines are joined by an inductance; with capacitance to earth at the junction as in Fig. 22($a$). 5–30 microsecond incident wave.

$Z_1 = Z_2 = 350\ \Omega$.
($a$) $L = 1\,000\ \mu H$, $C_2 = 1\,000\ \mu\mu F$, $r = 0$.
($b$) $L = 5\,000\ \mu H$, $C_2 = 5\,000\ \mu\mu F$, $r = 0$.
($c$) $L = 5\,000\ \mu H$, $C_2 = 5\,000\ \mu\mu F$, $r = 1\,000\ \Omega$.

in a single outgoing line.* The transmitted voltage-wave along $Z_2$, however, is practically identical in Fig. 22(*b*) with that in Fig. 22(*a*) so long as the total inductance in the former is equal to the inductance in the latter.

So far, in connection with the circuit in Fig. 22(*a*), the incident wave has been regarded as travelling on $Z_1$ and reaching the choking coil before the condenser. If it is travelling on $Z_2$, so that the condenser is reached first, a different state of affairs exists. To conform to the direction of travel and notation used for all the previous cases, the circuit has been redrawn and is shown in Fig. 22(*c*).

The symbolic equation for this voltage (called $e_4'$) is obtained by substituting

$$Z_a(p) = \infty, \quad Z_b(p) = 0, \quad Z_c(p) = \frac{1}{pC_1},$$

and replacing $Z_2$ by $(r + pL + Z_2)$ in (2).*

Thus

$$e_4' = \frac{2(Z_2+r+pL)}{p^2LC_1Z_1+p(L+Z_1Z_2C_1+rZ_1C_1)+Z_1+Z_2+r}e_1$$

The actual solutions for the aperiodic and oscillatory cases are respectively:—

$$e_4' = 2E\left[\frac{Z_2+r-aL}{K_1}\epsilon^{-at} - \frac{Z_2+r-bL}{K_2}\epsilon^{-bt} + \frac{(b-a)(Z_2+r-LK_3)}{\phi(K_3-a)(K_3-b)}\epsilon^{-K_3t} - \frac{(b-a)(Z_2+r-LK_4)}{\phi(K_4-a)(K_4-b)}\epsilon^{-K_4t}\right] \quad (26)$$

$$e_4' = 2E\left[\frac{Z_2+r-aL}{K_1}\epsilon^{-at} - \frac{Z_2+r-bL}{K_2}\epsilon^{-bt} - \frac{2L}{\psi}\sqrt{(Z_1)}\epsilon^{-\left(\frac{L+C_1Z_1Z_2+C_1Z_1r}{2LC_1Z_1}\right)t}\left\{\frac{\cos\left(\frac{\psi t}{2LC_1Z_1}-\theta_1\right)}{\sqrt{(K_1)}} - \frac{\cos\left(\frac{\psi t}{2LC_1Z_1}-\theta_2\right)}{\sqrt{(K_2)}}\right\}\right] \quad . \quad (27)$$

where

$$\phi = \sqrt{[(L - C_1Z_1Z_2 - C_1Z_1r)^2 - 4LC_1Z_1^2]},$$
$$\psi^2 = -\phi^2,$$
$$K_1 = a^2LC_1Z_1 - a(L + C_1Z_1Z_2 + C_1Z_1r) + (Z_1 + Z_2 + r),$$
$$K_2 = b^2LC_1Z_1 - b(L + C_1Z_1Z_2 + C_1Z_1r) + (Z_1 + Z_2 + r),$$
$$K_3 = (L + C_1Z_1Z_2 + C_1Z_1r - \phi)/(2LC_1Z_1),$$
$$K_4 = (L + C_1Z_1Z_2 + C_1Z_1r + \phi)/(2LC_1Z_1),$$
$$\theta_1 = \text{arc tan}\,\frac{C_1Z_1(Z_2+r)^2 - aL(C_1Z_1Z_2 + C_1Z_1r - L) - L(2Z_1 + Z_2 + r)}{\psi(Z_2 + r - aL)},$$

and

$$\theta_2 = \text{arc tan}\,\frac{C_1Z_1(Z_2+r)^2 - bL(C_1Z_1Z_2 + C_1Z_1r - L) - L(2Z_1 + Z_2 + r)}{\psi(Z_2 + r - bL)}.$$

The symbolic equation for the transmitted wave is

$$e_2 = \frac{2Z_2}{p^2C_1LZ_1+p(L+C_1Z_1r+C_1Z_1Z_2)+(Z_1+Z_2+r)}e_1 \quad (25)$$

Comparing (25) with (22), and noting that $C_1$ and $C_2$ are synonymous, it is seen that the two equations are similar, and, for the case where $Z_1 = Z_2$, are identical. Thus differences between the transmitted voltage-waves in Figs. 22(*a*) and 22(*c*) only arise when $Z_1$ is different from $Z_2$.

The voltage to earth across the condenser is another matter, however. In the case of Fig. 22(*a*), this voltage is the same as the transmitted voltage-wave, but in Fig. 22(*c*) it is the sum of the incident and reflected voltage-waves.

* It is unfortunate that the general solutions already given do not apply in this important case and that the more cumbersome expressions given in (*c*) (iv) and (*c*) (v) of the Appendix have to be used. The author has found by experience, however, that in many cases where the circuit is oscillatory and where the incident wave is infinite rectangular, the simple expression

$$e_2' = E\left[1 - \frac{2}{\psi}\sqrt{(C_2L')}\epsilon^{-\frac{Z_1t}{L'}}\cos\left(\frac{\psi t}{C_2L'} - \theta_1\right)\right]$$

$\theta_1$ being arc tan $(C_2Z_1/\psi)$, and $\psi$ being $(4L'C_2 - C_2^2Z_1^2)^{\frac{1}{2}}$, is a fair approximation to the voltage $e_2'$ of Fig. 22(*b*), when $Z_1 = Z_2$ and when $L_1 = L_2 = \frac{1}{2}L'$.

Equations (26) and (27) are plotted in Fig. 27. It is seen that the voltage across the condenser rises above the incident-wave value since the inductance behind the condenser behaves as an open circuit initially. The values of inductance have to be considerably greater than those considered here, however, for this voltage to attain the double value which is approached when the line is closed only by a condenser. This is particularly the case with the longer-fronted waves, as with wave (iii) and an inductance of 1 000 microhenrys there is only a 28 per cent increase in voltage compared with the 68 per cent when the wave is infinite rectangular.

Regarding the rate of rise of voltage across the condenser, it is seen that it is considerably greater here than in the case where the inductance precedes the condenser; in fact, it approaches that occurring at the end of a line closed through a condenser. Thus, in the case of waves like (iii), the rate of rise is of the order of twice the gradient of the front of the incident wave itself. It is also worthy of note that when the circuit

* Thus $e_2$ of (2) becomes the transmitted voltage-wave along the circuit containing an inductance in series with a line. (As the inductance comes first, this is not the wave along the line $Z_2$ itself; there is a voltage drop in the former.) $Z_a(p)$ and $Z_b(p)$ being absent, this transmitted wave is the same thing as the voltage across $Z_c(p)$. This voltage is called $e_4'$.

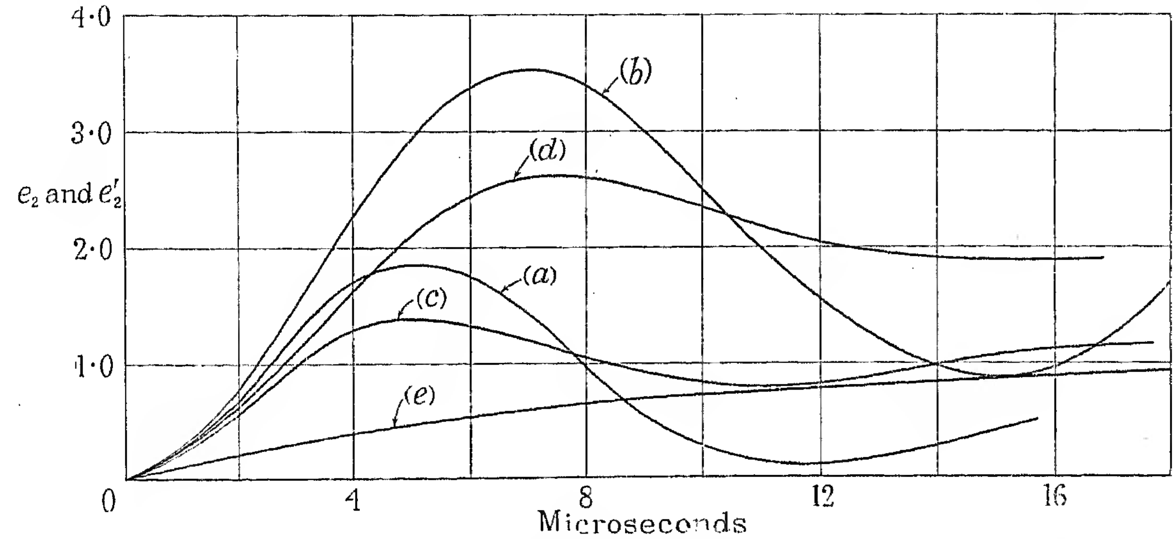



FIG. 26.—Comparison of $e_2$ in Fig. 22($d$), $e_2$ in Fig. 22($a$), and $e'_2$ in Fig. 22($b$). Infinite rectangular incident wave.

$e'_2$ in Fig. 22($b$): ($a$) $C_2 = 1\,000 \mu\mu$F, $L_1 = L_2 = 5\,000 \mu$H, $Z_1 = Z_2 = 350\ \Omega$.
$e_2$ in Fig. 22($d$): ($b$) $C_2 = 1\,000 \mu\mu$F, $L = 5\,000 \mu$H, $Z_1 = 350\ \Omega$, $Z_2 = \infty$.
$e'_2$ in Fig. 22($b$): ($c$) $C_2 = 5\,000 \mu\mu$F, $L_1 = L_2 = 1\,000 \mu$H, $Z_1 = Z_2 = 350\ \Omega$.
$e_2$ in Fig. 22($d$): ($d$) $C_2 = 5\,000 \mu\mu$F, $L = 1\,000 \mu$H, $Z_1 = 350\ \Omega$, $Z_2 = \infty$.
$e_2$ in Fig. 22($a$): ($e$) $C_2 = 1\,000 \mu\mu$F, $L = 5\,000 \mu$H, $Z_1 = Z_2 = 350\ \Omega$.

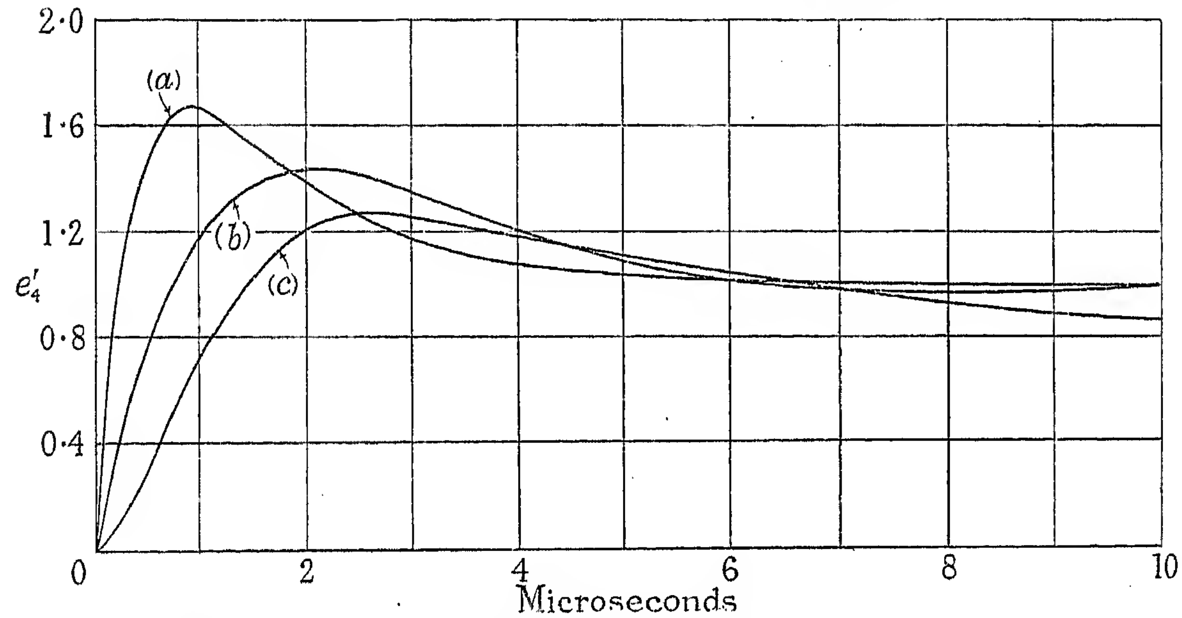



FIG. 27.—Voltage across the condenser shown in Fig. 22($c$).

$L = 1\,000 \mu$H, $Z_1 = Z_2 = 350\ \Omega$, $r = 0$.
($a$) $C = 1\,000 \mu\mu$F } Infinite rectangular incident wave.
($b$) $C = 3\,000 \mu\mu$F }
($c$) $C = 1\,000 \mu\mu$F, 5–30 microsecond incident wave.

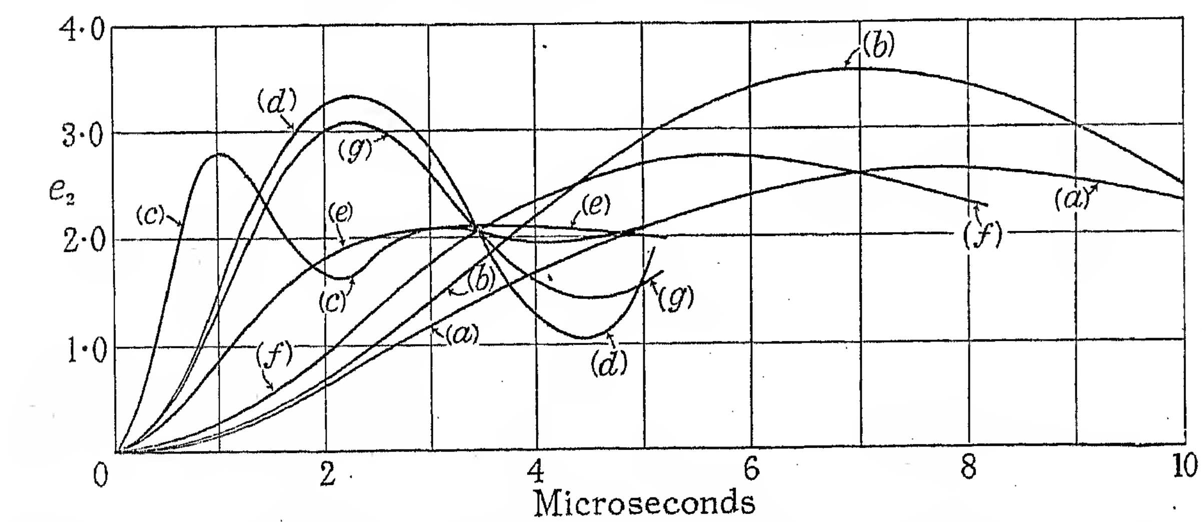



FIG. 28.—Voltages across a condenser when a line is earthed by an inductance-capacitance combination as in Fig. 22($d$). Infinite rectangular incident wave.

$Z_1 = 350\ \Omega$.
($a$) $L = 1\,000 \mu$H, $C_2 = 5\,000 \mu\mu$F, $r = 0$.
($b$) $L = 5\,000 \mu$H, $C_2 = 1\,000 \mu\mu$F, $r = 0$.
($c$) $L = 200 \mu$H, $C_2 = 500 \mu\mu$F, $r = 0$.
($d$) $L = 1\,000 \mu$H, $C_2 = 500 \mu\mu$F, $r = 0$.
($e$) $L = 200 \mu$H, $C_2 = 3\,000 \mu\mu$F, $r = 0$.
($f$) $L = 1\,000 \mu$H, $C_2 = 3\,000 \mu\mu$F, $r = 0$.
($g$) $L = 1\,000 \mu$H, $C_2 = 500 \mu\mu$F, $r = 200\ \Omega$.

is oscillatory, with the usual values of the constants no appreciable oscillation shows itself across the condenser.

(*b*) *Inductance, having Series Resistance, and Concentrated Earth Capacitance Closing the End of a Line.*

When the line terminates in an inductance with earth capacitance, as shown in Fig. 22(*d*), the symbolic equation for the voltage across the condenser is

$$e_2 = \frac{2}{p^2LC_2 + pC_2(Z_1 + r) + 1}e_1$$

The actual solutions for the aperiodic and oscillatory cases are respectively:—

$$e_2 = 2E\left[\frac{\epsilon^{-at}}{K_1} - \frac{\epsilon^{-bt}}{K_2} + \frac{b-a}{\phi(K_3 - a)(K_3 - b)}\epsilon^{-K_3t} - \frac{b-a}{\phi(K_4 - a)(K_4 - b)}\epsilon^{-K_4t}\right] \quad . \quad . \quad . \quad . \quad (28)$$

and

$$e_2 = 2E\left[\frac{\epsilon^{-at}}{K_1} - \frac{\epsilon^{-bt}}{K_2} - \frac{2}{\psi}\sqrt{(C_2L)}\epsilon^{-\left(\frac{Z_1+r}{2L}\right)t}\left\{\frac{\cos\left(\frac{\psi t}{2C_2L} - \theta_1\right)}{\sqrt{(K_1)}} - \frac{\cos\left(\frac{\psi t}{2C_2L} - \theta_2\right)}{\sqrt{(K_2)}}\right\}\right] \quad . \quad . \quad . \quad (29)$$

where $\phi = \sqrt{[C_2^2(Z_1 + r)^2 - 4LC_2]}$,

$\psi^2 = -\phi^2$,

$K_1 = C_2La^2 - a(Z_1 + r)C_2 + 1$,

$K_2 = C_2Lb^2 - b(Z_1 + r)C_2 + 1$,

$K_3 = (C_2r + C_2Z_1 - \phi)/(2C_2L)$,

$K_4 = (C_2r + C_2Z_1 + \phi)/(2C_2L)$,

$\theta_1 = \text{arc tan } (C_2Z_1 + C_2r - 2C_2La)/\psi$,

and $\theta_2 = \text{arc tan } (C_2Z_1 + C_2r - 2C_2Lb)/\psi$.

Equations (28) and (29) are plotted in Figs. 28, 29, and 30, for the usual values of the circuit constants and for waves (i), (ii), (iii), and (iv). One of the most important points to note is that under certain conditions the voltage across the condenser rises to a value between approximately three and four times the incident voltage by virtue of the oscillations that occur. These oscillations are not, in general, eliminated by small values of the series resistance $r$ of the order of 50 to 100 ohms. A suitable choice of constants may be made, however, such that the voltage is held at or near the double value, and any series resistance present then gives a further reduction in its amplitude.

It is seen that the magnitude of this voltage is greatly affected by the shape of the incident wave. In the case of the infinite rectangular wave, when $L = 1\,000 \times 10^{-12}$ henry and $C = 500 \times 10^{-12}$ farad, its maximum value is $3 \cdot 35E$. When wave (ii) is used this is reduced to $3 \cdot 15E$, and there is a further reduction to $2 \cdot 35E$ with wave (iii). When the total length of the incident wave is short, as in the case of the insulator-chopped wave, under the most violent oscillatory conditions considered ($L = 1\,000 \times 10^{-6}$ henry and $C = 500 \times 10^{-12}$ farad) the voltage never reaches even the double value; in this case the greatest voltage is negative and is only 60 per cent greater in magnitude than the incident wave. Frequently the superimposed oscillations affect adversely the initial gradient and the amount of wave-front flattening; in fact, in some cases the maximum value of the voltage occurs before the incident wave itself has reached its maximum. This effect is shown in curve (*a*) of Fig. 30, where the incident wave is wave (iii) and where $L = 1\,000 \times 10^{-6}$ henry, $C = 500 \times 10^{-12}$ farad.

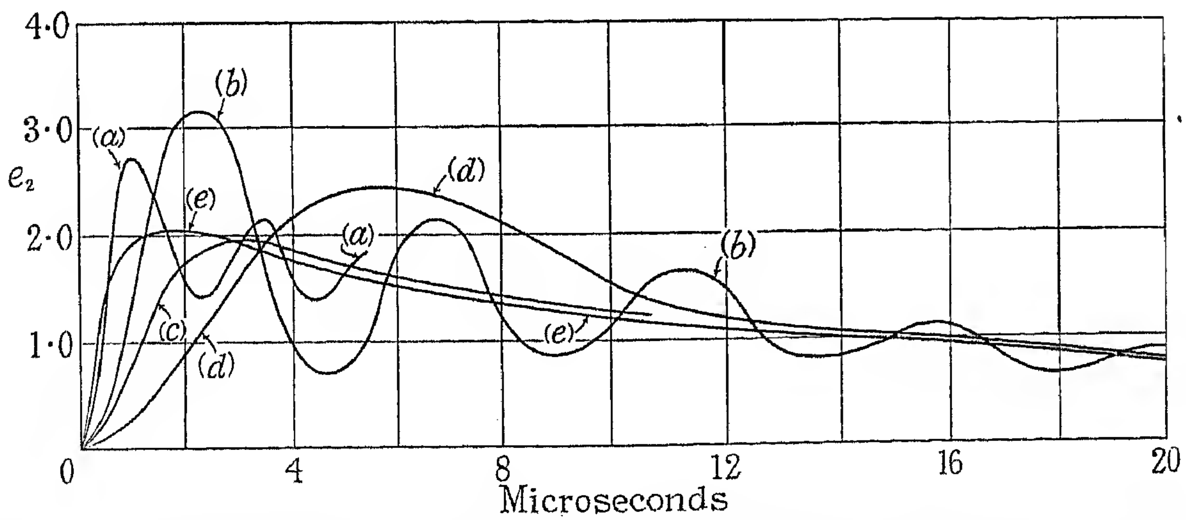



FIG. 29.—Voltage across a condenser when a line is earthed by an inductance-capacitance combination as in Fig. 22(*d*). 0·4–14 microsecond incident wave.

$r = 0, Z_1 = 350\,\Omega$.
(*a*) $L = 200\,\mu$H, $C_2 = 500\,\mu\mu$F.
(*b*) $L = 1\,000\,\mu$H, $C_2 = 500\,\mu\mu$F.
(*c*) $L = 200\,\mu$H, $C_2 = 3\,000\,\mu\mu$F.
(*d*) $L = 1\,000\,\mu$H, $C_2 = 3\,000\,\mu\mu$F.
(*e*) $L = 1\,000\,\mu$H, $C_2 = 500\,\mu\mu$F, $R_0 = 500\,\Omega$
Curve (*e*) indicates voltage when the inductance has shunt resistance as in Fig. 22(*e*).

(c) *Inductance, having Resistance in Parallel, and Concentrated Earth Capacitance Closing the End of a Line.* The oscillations discussed in Section (9b) can be prevented by the addition of resistance in parallel with the inductance. The circuit is shown in Fig. 22(e), and in this case the symbolic equation for the voltage across the condenser becomes

$$e_2 = \frac{2(R_0 + pL)}{p^2LC_2(R_0 + Z_1) + p(L + C_2R_0Z_1) + R_0} e_1$$

The actual solutions for the aperiodic and oscillatory cases are respectively:—

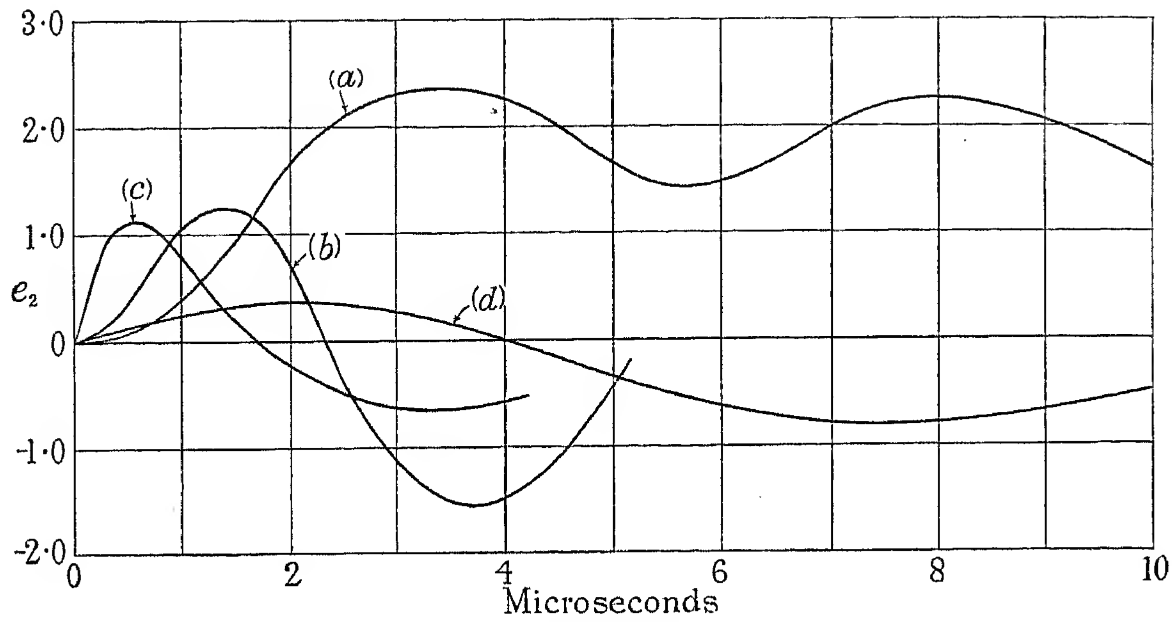



FIG. 30.—Voltages to earth when a line is earthed by an inductance-capacitance combination as in Fig. 22(*d*).

$L = 1\,000\ \mu\text{H}, r = 0, Z_1 = 350\ \Omega$.
(*a*) $C_2 = 500\ \mu\mu\text{F}$, 5–30 microsecond incident wave.
(*b*) $C_2 = 500\ \mu\mu\text{F}$, insulator-chopped incident wave.
(*c*) $C_2 = 500\ \mu\mu\text{F}$, $R_0 = 500\ \Omega$, insulator-chopped incident wave.
(*d*) $C_2 = 3\,000\ \mu\mu\text{F}$, insulator-chopped incident wave.
Curve (*c*) indicates voltage when the inductance has shunt resistance as in Fig. 22(*e*).

It is seen that for the constants used in plotting curves (b) and (e) of Fig. 29, and curves (b) and (c) of Fig. 30, a shunt resistance of 500 ohms makes the circuit aperiodic. Under certain circumstances this gives a decrease in wave-front flattening, though not necessarily an increase in maximum gradient.

(d) *Inductance, having Series Resistance, and Concentrated Earth Capacitance, the latter also having Series Resistance, at the Junction of Two Lines or Closing the End of a Line.*

When resistance occurs in series with the condenser,

$$e_2 = 2E\left[\frac{R_0 - aL}{K_1}\epsilon^{-at} - \frac{R_0 - bL}{K_2}\epsilon^{-bt} + \frac{(R_0 - K_3L)(b - a)}{\phi(K_3 - a)(K_3 - b)}\epsilon^{-K_3t} - \frac{(R_0 - K_4L)(b - a)}{\phi(K_4 - a)(K_4 - b)}\epsilon^{-K_4t}\right] \quad . \quad (30)$$

and

$$e_2 = 2E\left[\frac{R_0 - aL}{K_1}\epsilon^{-at} - \frac{R_0 - bL}{K_2}\epsilon^{-bt} - \frac{2R_0}{\psi}\sqrt{(LC_2R_0)}\epsilon^{-\left\{\frac{L+C_2R_0Z_1}{2LC_2(R_0+Z_1)}\right\}t}\left\{\frac{\cos\left\{\frac{\psi t}{2LC_2(R_0 + Z_1)} - \theta_1\right\}}{\sqrt{(K_1)}} - \frac{\cos\left\{\frac{\psi t}{2LC_2(R_0 + Z_1)} - \theta_2\right\}}{\sqrt{(K_2)}}\right\}\right] \quad . \quad (31)$$

where

$$K_1 = a^2LC_2(R_0 + Z_1) - a(L + C_2R_0Z_1) + R_0$$
$$K_2 = b^2LC_2(R_0 + Z_1) - b(L + C_2R_0Z_1) + R_0$$
$$K_3 = [L + C_2R_0Z_1 - \phi]/[2LC_2(R_0 + Z_1)]$$
$$K_4 = [L + C_2R_0Z_1 + \phi]/[2LC_2(R_0 + Z_1)]$$
$$\theta_1 = \text{arc tan}\,[R_0^2Z_1C_2 - R_0L + a(L^2 - 2R_0^2LC_2 - R_0Z_1LC_2)]/[\psi(R_0 - aL)]$$
$$\theta_2 = \text{arc tan}\,[R_0^2Z_1C_2 - R_0L + b(L^2 - 2R_0^2LC_2 - R_0Z_1LC_2)]/[\psi(R_0 - bL)]$$
$$\phi = \sqrt{[(L - C_2Z_1R_0)^2 - 4LC_2R_0^2]}$$

and

$$\psi^2 = -\phi^2.$$

as in Fig. 22($f$), the symbolic equation for the transmitted voltage-wave is:—

$$e_2 = \frac{2Z_2(1 + R_2C_2p)}{p^2LC_2(R_2 + Z_2) + p[Z_2C_2(R_2 + Z_1 + r) + R_2C_2(Z_1 + r) + L] + Z_1 + Z_2 + r}e_1 \quad . \quad . \quad . \quad (32a)$$

and for the case where $Z_2$ is absent, as in Fig. 22($g$),

$$e_2 = \frac{2(1 + R_2C_2p)}{p^2LC_2 + p(C_2r + C_2Z_1 + R_2C_2) + 1}e_1 \quad (32b)$$

Consideration of (32$a$), in conjunction with (22) where series resistance is in the inductance only, shows that the condition for oscillation is not greatly affected by the location of resistance, and consideration of (32$b$) shows that when $Z_2$ is absent it is quite immaterial whether the resistance is in the inductance or in the condenser earth-connection. Further, calculations based on (32$a$) show that, for the usual values of the constants, the voltage across the condenser in the case of Fig. 22($g$) is similar to that in Fig. 22($d$).*

(e) *Approximations to Inductance Coils having Distributed Earth Capacitance.*

An arrangement, based on an inductance coil surrounded by an earthed metal cylinder such that the coil has distributed constants and losses, has been produced for connection between lines and inductive apparatus in order to flatten the fronts of waves incident on the latter.† Such an arrangement has a considerable influence in many ways on the response of a system to transients. Only its wave-front flattening properties are dealt with here, and the discussion is therefore confined to a consideration of the transmitted voltage-wave $e_2$.

The complete mathematical treatment of such a coil is difficult, and it is therefore desirable to represent it by some simple equivalent circuit which will simulate its behaviour fairly closely. The choice of an approximation depends to a large extent on the design of the coil and the shape of the incident wave.

In general, when the front of the wave is long compared with the length of the conductor comprising the coil, the latter can be regarded as a combination of concentrated inductance and capacitance. Thus Figs. 22($a$) and 22($d$) are equivalent circuits, $C_2$ being the sum of the earth capacitances of the coil and the system in which it is placed. A better approximation is shown in Fig. 22($h$), $Z_2$ being added thereto when necessary.

When the front of the wave is short compared with the length of the winding in the coil, it is evident that the constants of the latter can only be regarded as distributed. It has been shown by Wagner,* for instance, that such coils under these conditions behave as finite transmission lines, so that the method of treatment is somewhat different from that considered hitherto.

Dealing first with the circuit shown in Fig. 22($h$), the symbolic equation for the voltage across $C_2$, when $Z_2$ is absent, is obtained from (2) by substituting $Z_a(p) = 1/(pC_1)$, $Z_b(p) = r + pL$, and $Z_c(p) = 1/(pC_2)$. It is

$$e_2 = \frac{2}{p^3LC_1C_2Z_1 + p^2C_2(L + C_1Z_1r) + pC_2\left(Z_1 + \frac{C_1}{C_2}Z_1 + r\right) + 1}e_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

$C_1$ being one half of the lumped earth capacitance of the coil and $C_2$ being the other half plus the system earth capacitance.

This circuit has recently been fully treated by Thomson† for finite waves similar to those used here. The symbolic equation (33) was solved directly by means of the Expansion Theorem for each set of constants, and no general formulæ were given. For the sake of completeness, therefore, it seems desirable to include such formulæ here, though the inconvenient necessity of solving a cubic equation will remain. If the roots $-\rho_1$, $-\rho_2$, $-\rho_3$, of the denominator of (33) are real, this equation can be written in the form:—

$$e_2 = 2K\frac{1}{(p + \rho_1)(p + \rho_2)(p + \rho_3)}e_1$$

For incident waves given by (1), as shown in the Appendix, the solution of this equation is:—

$$e_2 = 2EK\left[\frac{(b-a)\epsilon^{-\rho_1 t}}{(\rho_2 - \rho_1)(\rho_3 - \rho_1)(a - \rho_1)(b - \rho_1)} + \frac{(b-a)\epsilon^{-\rho_2 t}}{(\rho_1 - \rho_2)(\rho_3 - \rho_2)(a - \rho_2)(b - \rho_2)} + \frac{(b-a)\epsilon^{-\rho_3 t}}{(\rho_1 - \rho_3)(\rho_2 - \rho_3)(a - \rho_3)(b - \rho_3)} - \frac{\epsilon^{-at}}{(a - \rho_1)(a - \rho_2)(a - \rho_3)} + \frac{\epsilon^{-bt}}{(b - \rho_1)(b - \rho_2)(b - \rho_3)}\right] \quad . \quad (34)$$

If the roots are $-\rho$, $(-\sigma + j\xi)$, $(-\sigma - j\xi)$, then

$$e_2 = 2K\frac{1}{(p + \rho)(p + \sigma - j\xi)(p + \sigma + j\xi)}e_1$$

and the solution for incident waves given by (1) is

$$e_2 = 2EK\left[\frac{\epsilon^{-bt}}{[b - \rho][\xi^2 + (\sigma - b)^2]} - \frac{\epsilon^{-at}}{[a - \rho][\xi^2 + (\sigma - a)^2]} + \frac{(b-a)\epsilon^{-\rho t}}{[\xi^2 + (\rho - \sigma)^2][a - \rho][b - \rho]} - \frac{\epsilon^{-\sigma t}}{\xi\sqrt{[\xi^2 + (\rho - \sigma)^2]}}\left\{\frac{\cos(\xi t - \theta_1)}{\sqrt{[\xi^2 + (\sigma - a)^2]}} - \frac{\cos(\xi t - \theta_2)}{\sqrt{[\xi^2 + (\sigma - b)^2]}}\right\}\right] \quad . \quad (35)$$

* See Bibliography, (43).
† This arrangement is adopted as the basis of design in the Ferranti surge absorber.

* See Bibliography, (44).
† *Ibid.*, (45).

where $\theta_1 = \arc\tan \dfrac{\xi^2 + \rho\sigma - \sigma^2 - a(\rho - \sigma)}{\xi(a + \rho - 2\sigma)}$,

$$\theta_2 = \arc\tan \frac{\xi^2 + \rho\sigma - \sigma^2 - b(\rho - \sigma)}{\xi(b + \rho - 2\sigma)},$$

and, for both solutions, $K = 1/(LC_1C_2Z_1)$.

Equations (34) and (35) are not plotted, as the behaviour of the circuit can be fully examined by reference to Thomson's work. It should be mentioned, however, that although the equations do not take into account the variable decrease in inductance of the coil due to the short-circuiting effect of the secondary (or surrounding metal cylinder), Thomson covered this point by employing, in many cases, very low values of the inductance in his calculations. The power loss in the secondary was taken into account empirically by assuming that the coil had a constant resistance; in general, low values of the order of 10 to 100 ohms were used. Despite this very pessimistic choice of constants, it was found that there was always considerable wave-front flattening.

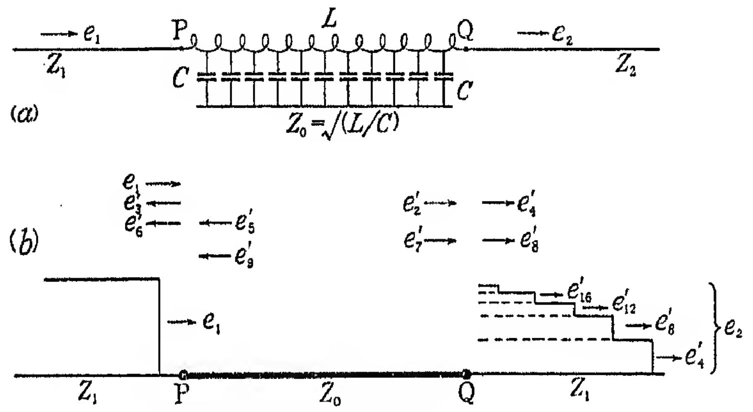


FIG. 31.

The case where the coil is behaving as a finite line of surge impedance $Z_0$ will now be examined. The arrangement is shown in Fig. 31($a$), where the coil is inserted between two lines of surge impedance $Z_1$ and $Z_2$. At once, as in Section (5), it becomes necessary to take into account reflections at the points P and Q, and these have an important bearing on the shape of the transmitted voltage-wave $e_2$. Fig. 31($b$) shows the manner in which the repeated reflections take place, the incident voltage-wave being infinite rectangular. The voltage $e'_4$ of the first transmitted wave (neglecting losses, and assuming $Z_1 = Z_2$) is

$$\frac{4e_1Z_0Z_1}{(Z_1 + Z_0)^2}$$

and $e'_8$, the voltage of the second transmitted wave, is

$$\frac{4e_1Z_0Z_1(Z_1 - Z_0)^2}{(Z_1 + Z_0)^4}.$$

If $Z_0 = 0\cdot1Z$, say, then $e'_4 = 0\cdot33e_1$, and $e'_8 = 0\cdot22e_1$. This shows that the transmitted voltage-wave is of "stepwise" form as indicated in Fig. 31($b$), though naturally, in practice, losses "round off" the steps. In the practical case of a transformer connected to Q, there is further "rounding off" due to its earth capacitance, and if $Z_2$ is absent the steps are doubled in amplitude, so that the normal final voltage of $2e_1$ is eventually attained.

Transmitted waves having stepped characteristics have been observed experimentally by Krug and others.* The effective surge impedance of any such coil can be approximately determined by measurement of the heights of the steps in an oscillogram. Also, if the time-interval between steps and the length of the coil winding is known, the velocity of propagation and hence the effective earth capacitance and surge inductance of the coil can be determined.

It is found that the value of the capacitance calculated in this manner is very nearly equal to the measured 50-cycle value, but that the surge inductance is considerably smaller than the 50-cycle value. The large decrease in inductance is due partly to incomplete counter e.m.f.'s (as was explained in Section 7), and partly to the short-circuiting effect of the secondary.

The exact consideration of the questions of the variation in inductance due to incomplete flux linkages and the mathematics of the coil from the point of view of a transmission line with losses is either difficult or impossible in the present state of the art, and is beyond the scope of the present paper. These remarks on coils behaving as finite lines are merely included to illustrate another aspect of their response to travelling waves.

Cathode-ray studies have shown that both the approximations here considered apply in their respective spheres. With incident waves having fronts of $0\cdot2$ microsecond or longer and for usual values of the conductor lengths comprising the coils, except in the case of certain designs, the circuit shown in Fig. 22($h$) is the nearest equivalent.

## (10) SOME EXPERIMENTAL RESULTS.

This section of the paper contains a series of oscillograms illustrating experimentally the behaviour of some of the circuits already mathematically examined. These oscillograms are included rather with the object of confirming that the response to transients of various apparatus can generally be calculated with good accuracy, and that the assumptions made in the calculations are justifiable, than for the purpose of discussing in detail any small discrepancies that occur between experimentally and theoretically obtained results for any particular apparatus. To this end, therefore, only records dealing with the salient points of the more important circuits are given. In order to facilitate comparison the mathematically obtained results, plotted on the appropriate exponential time-bases, are set out where necessary side by side with the oscillograms.†

The oscillograms were obtained using a high-voltage high-speed cathode-ray oscillograph connected through an electrostatic potentiometer to the various circuits, these being coupled to a high-voltage surge generator through a length of overhead line having a surge impedance of 370 ohms. The surge generator was in every case (except for Figs. 41$a$ and 41$b$) adjusted to send out the same incident wave, which rose to its maximum value in $0\cdot3$ microsecond and fell to half value in $11\cdot5$ microseconds. An oscillogram of this

* See Bibliography, (46).

† As the oscillograms given here were obtained at various times and not originally for the specific purpose of illustrating this paper, both the time and the voltage scales are frequently different for different records.

wave is shown in Fig. 32($a$), and a wave similar in shape and represented by $1 \cdot 02\,E(\epsilon^{-0\cdot06\times10^6t} - \epsilon^{-15\times10^6t})$ is drawn in Fig. 32($b$). The latter is used for all the calculations in this section of the paper.

*Discussion of Oscillograms.*

*Fig.* 33.—The voltage across one phase of a 500-kVA 11 000-volt 3-phase transformer is shown in Fig. 33($a$). The transformer was connected to the end of the transmission line and the incident wave was that shown in Fig. 32($a$).

Minor high-frequency oscillations are present, probably caused by interaction between leads connecting the transformer to the line and the condenser terminal. It should be noted, however, that the transformer internal oscillations do not manifest themselves on the voltage wave to any marked extent* and that there is some wave-front flattening. The transformer, as far as external circuits are concerned, is thus behaving as a condenser.

Fig. 33($c$) shows the calculated voltage across a condenser closing the end of a line as in Fig. 4($c$), the incident wave being that shown in Fig. 32($b$) and $C_1$ being $0 \cdot 001$ microfarad. The agreement between Figs. 33($a$) and 33($c$) is sufficiently close to suggest that the effective earth capacitance of the transformer is approximately of this value.

In Fig. 33($b$) is shown the voltage across the same transformer with a $0 \cdot 0033$-microfarad condenser inserted in front of it. The total earth capacitance was thus $0 \cdot 0043$ microfarad, and Fig. 33($d$) shows the calculated voltage using a pure capacitance of this value.

Regarding Fig. 33($b$), it is worthy of note that, for the time durations considered here, there is no evidence of high-frequency oscillation between the inductance of the transformer and the external capacitance. The hump at the beginning of this record is due to the inductance of the leads connecting the various units of the external capacitance.

*Fig.* 34.—In Fig. 34($a$) is shown the voltage across a condenser and a resistance in series at the end of the line, as in Fig. 4($d$). The value of the former was $0 \cdot 0017$ microfarad and the latter 250 ohms. Fig. 33($b$) shows the calculated voltage for the same values of the circuit constants.

*Fig.* 35.—Fig. 35($a$) shows the transmitted voltage-wave when an air-core inductance of 1 000 microhenrys is inserted between two lines of equal surge impedance, this condition having been simulated by closing the line

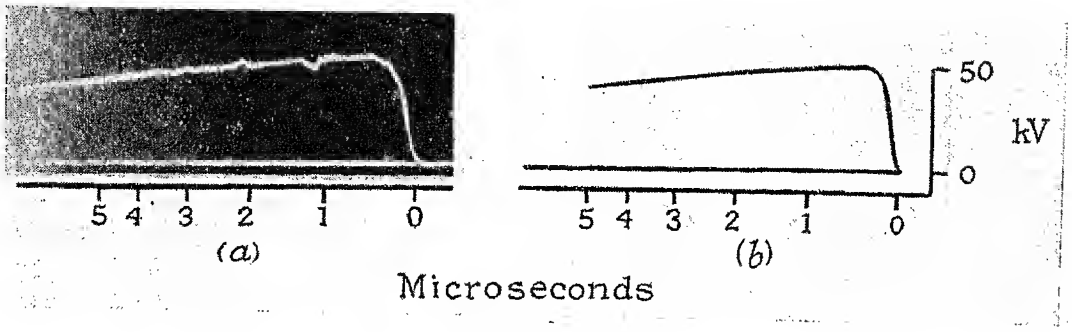



FIG. 32.

($a$) Incident wave used in obtaining the oscillograms shown in Figs. 33–40.
($b$) Empirical wave used in obtaining the calculated voltages in Figs. 33–39.

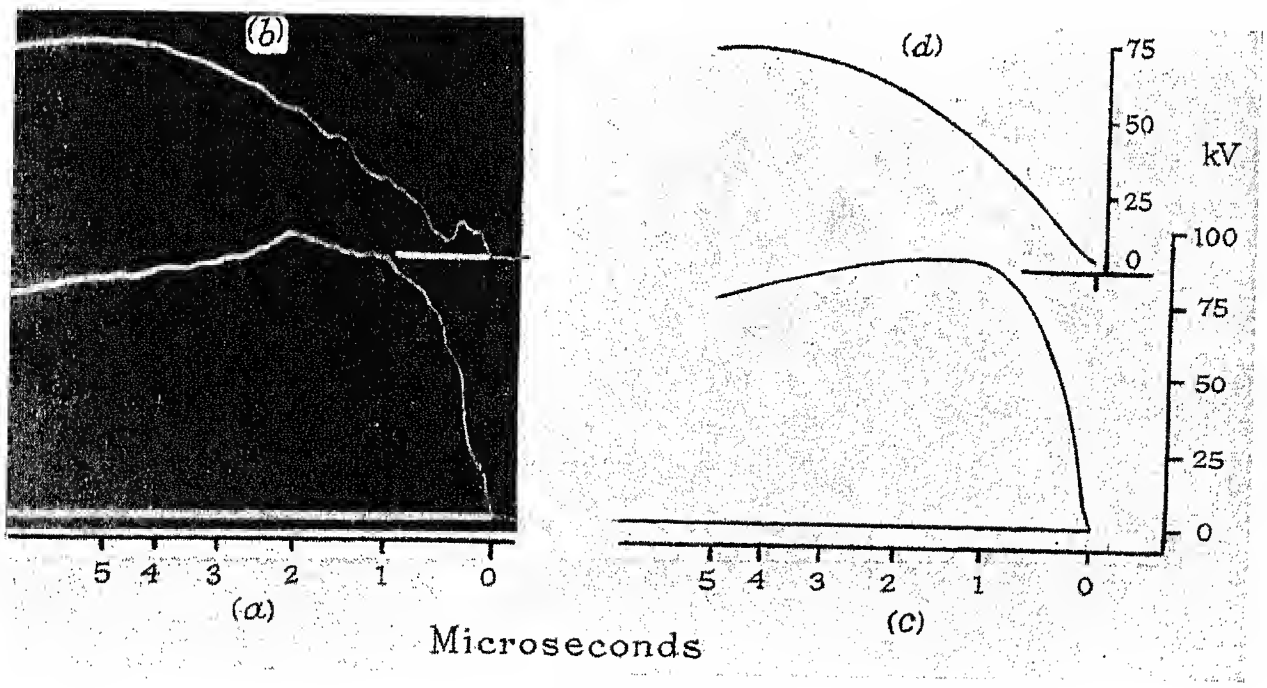



FIG. 33.

($a$) Voltage across one phase of a 500-kVA 11 000-volt transformer at the end of the line.
($b$) Voltage across the same transformer when protected by a capacitance of $0 \cdot 0033$ $\mu$F.
($c$) Calculated voltage across a $0 \cdot 001$-$\mu$F capacitance at the end of a line as in Fig. 4($c$). ($Z_1$ = 370 Ω.)
($d$) Calculated voltage across a $0 \cdot 0043$-$\mu$F capacitance at the end of a line as in Fig. 4($c$). ($Z_1$ = 370 Ω.)

* For the relatively short, though practical, wave shown in Fig. 33, only evidence of the higher harmonics of the transformer internal oscillations could be expected.

beyond the inductance by a resistance of 370 ohms, the oscillograph deflection-plates being appropriately connected across this resistance. The calculated transmitted voltage-wave using the circuit shown in Fig. 11($a$) is given in Fig. 35($b$), and it is seen that good agreement is obtained.

core inductance closing the end of the line, and Fig. 36($b$) shows the corresponding calculated voltage, the value of the inductance being 1 000 microhenrys. The characteristic fall of voltage typical of this circuit is well shown.

*Fig.* 37.—In Fig. 37($a$) is shown the transmitted

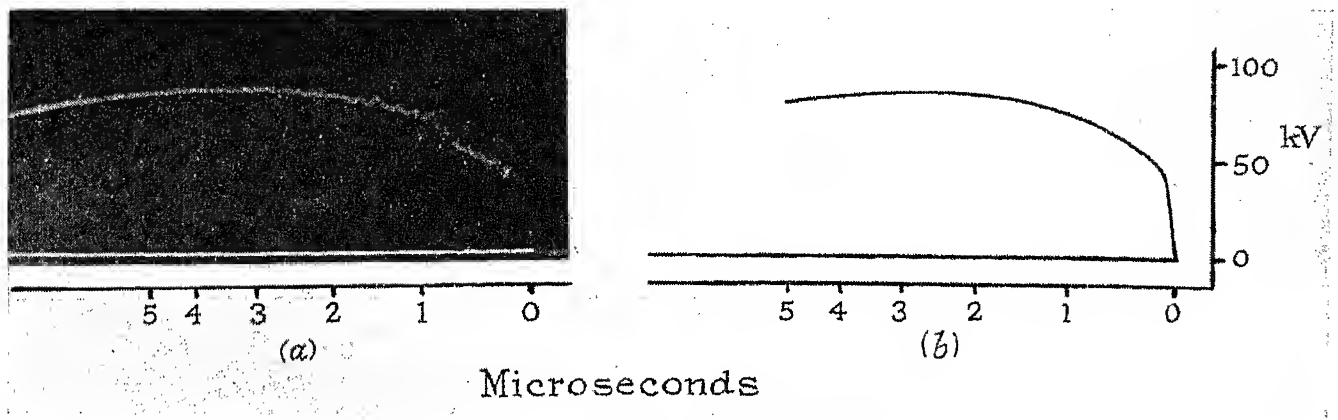



Fig. 34.

($a$) Voltage across a condenser and resistance in series at the end of the line, as in Fig. 4($d$).
($b$) Calculated voltage. ($C_1 = 0\cdot0017\ \mu F$; $R_1 = 250\ \Omega$; $Z_1 = 370\ \Omega$.)

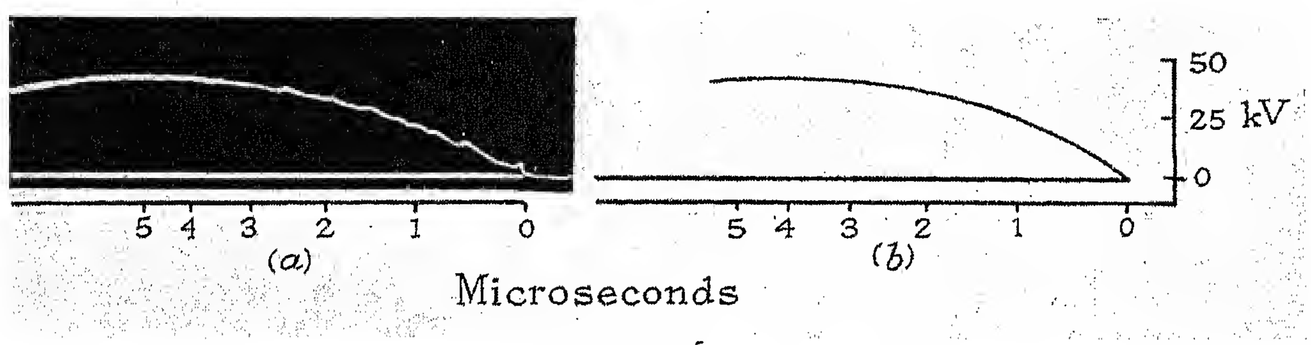



Fig. 35.

($a$) Transmitted voltage-wave when an air-core inductance coil joins two lines of equal surge impedance, as in Fig. 11($a$).
($b$) Calculated voltage. ($L = 0\cdot001$ H, $r = 0$, $Z_1 = Z_2 = 370\ \Omega$.)

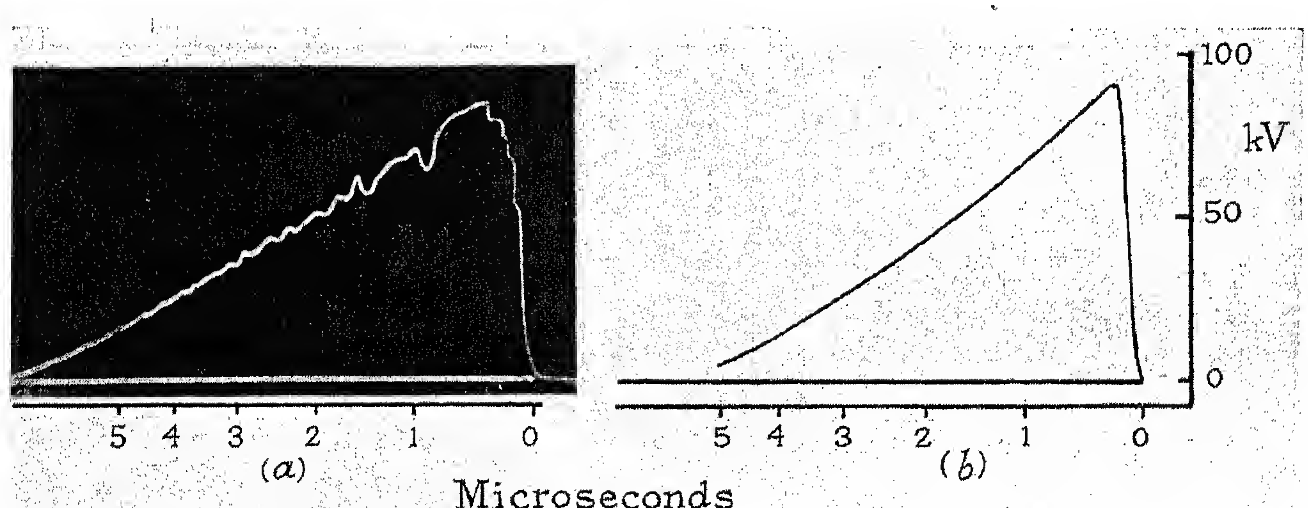



Fig. 36.

($a$) Voltage across an air-core inductance closing the end of the line, as in Fig. 11($c$).
($b$) Calculated voltage. ($L = 0\cdot001$ H, $r = 0$, $Z_1 = 370\ \Omega$.)

The inductance coil used in the experiment was a single-layer one with the turns well separated, but even so the shunt capacitance was sufficiently great to transmit practically unchanged a small proportion of the front of the incident wave. This is readily seen at the beginning of Fig. 35($a$).

*Fig.* 36.—Fig. 36($a$) shows the voltage across an air-

voltage-wave when an air-core single-layer inductance coil of 1 000 microhenrys, having a condenser of $0\cdot00025$ microfarad connected across its terminals, joins two lines of equal surge impedance, as in Fig. 18($a$). Fig. 37($b$) shows the calculated transmitted voltage-wave for the same values of the constants.

Fig. 37($c$) shows the transmitted voltage-wave when

the two lines are joined by an air-core 5-layer close-wound inductance coil of 1 000 microhenrys. This coil had considerable distributed shunt capacitance (no external lumped capacitance being connected across it), and from the oscillogram it is seen that this is behaving in a manner similar to that of a lumped shunt capacitance over the periods of time considered here. Comparison between Figs. 37($a$) and 37($c$) suggests that in this case this distributed self-capacitance is equivalent to a lumped capacitance of 0·00025 microfarad. The measured self-capacitance at 200 kilocycles per sec. was 0·00023 microfarad.

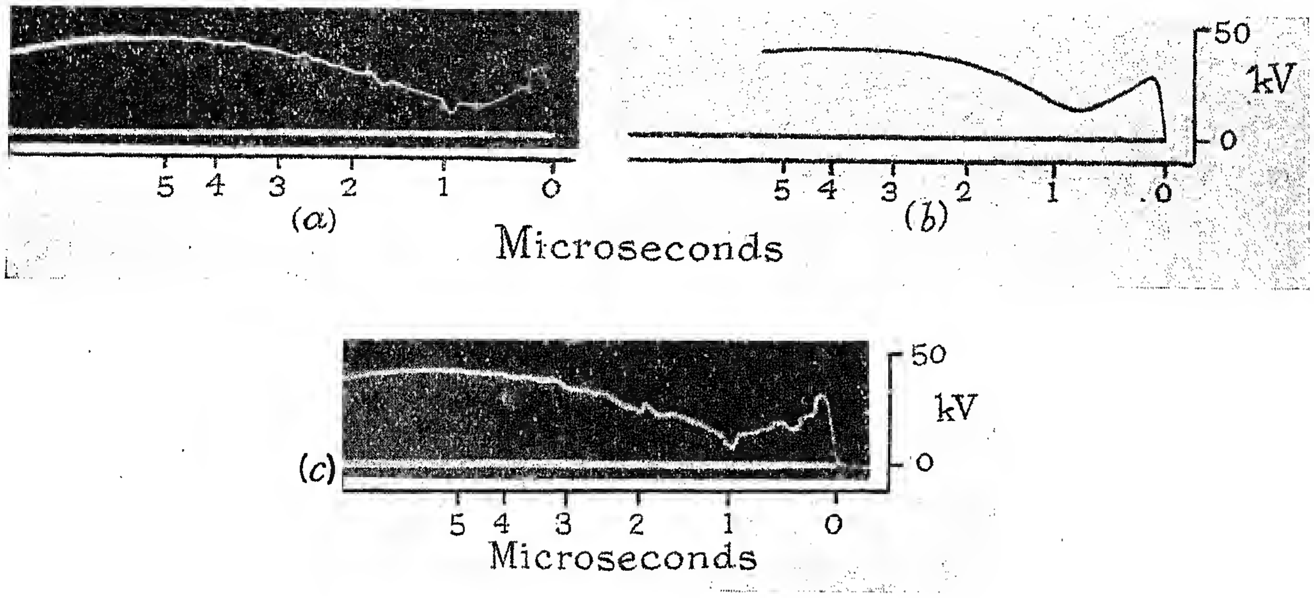



FIG. 37.

($a$) Transmitted voltage-wave when an air-core single-layer inductance coil, having a condenser connected in parallel with it, joins two lines of equal surge impedance, as in Fig. 18($a$).
($b$) Calculated voltage. ($L = 0 \cdot 001$ H, $C_0 = 0 \cdot 00025\ \mu$F, $r = 0$, $Z_1 = Z_2 = 370\ \Omega$.)
($c$) Transmitted voltage-wave when an air-core multi-layer inductance coil having considerable self-capacitance joins two lines of equal surge impedance. ($L = 0 \cdot 001$ H, $Z_1 = Z_2 = 370\ \Omega$, self-capacitance at 200 kilocycles per sec. $= 0 \cdot 00023\ \mu$F.)

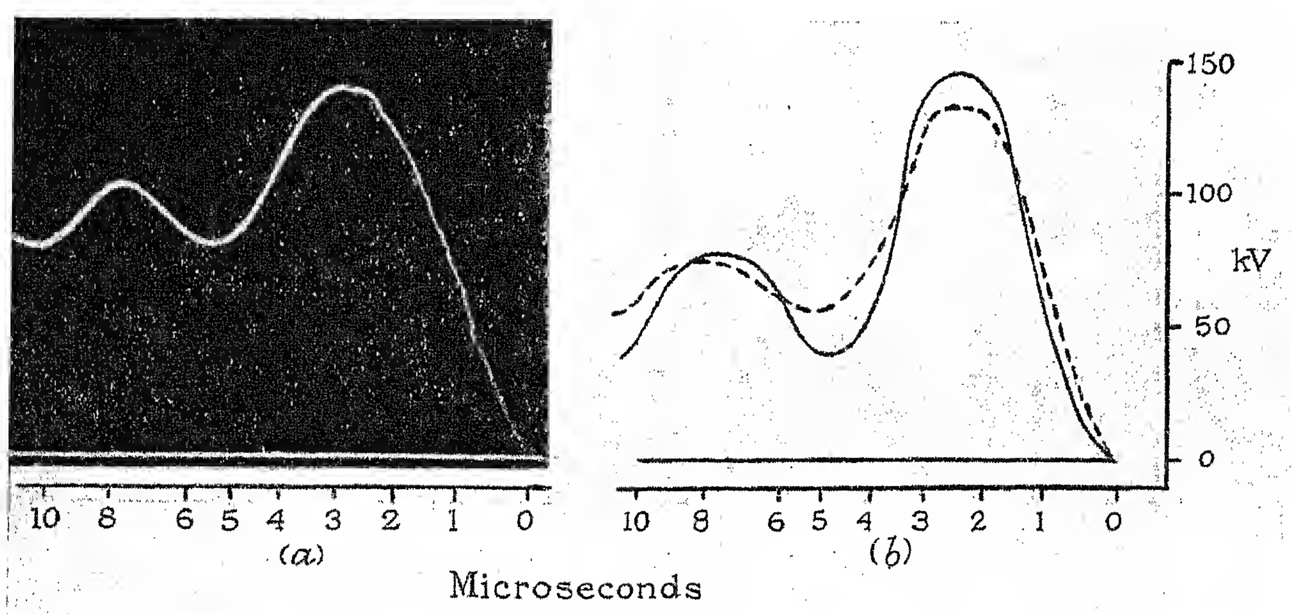



FIG. 38.

($a$) Voltage across a capacitance at the end of the line and preceded by an air-core inductance as in Fig. 22($d$).
($b$) Full curve: Calculated voltage for Fig. 22($d$). ($L = 0 \cdot 001$ H, $C_2 = 0 \cdot 0006\ \mu$F, $r = 50\ \Omega$, $Z_1 = 370\ \Omega$.) Dotted curve: Calculated voltage for Fig. 22($e$). ($L = 0 \cdot 001$ H, $C_2 = 0 \cdot 0006\ \mu$F, $r = 0$, $R_0 = 5\,000\ \Omega$, $Z_1 = 370\ \Omega$.)

*Fig.* 38.—Fig. 38($a$) shows the voltage across a condenser of 0·0006 microfarad at the end of the line and preceded by an air-core single-layer inductance coil of 1 000 microhenrys, as in Fig. 22($d$). The oscillogram confirms the excessive voltages that can be attained with this circuit. The full line in Fig. 38($b$) shows the calculated voltage for the same values of the constants, and it is seen that there is some difference between the amplitudes of the superimposed oscillations in the oscillogram and in the calculated voltage-wave.

This difference is due to the fact that the calculation neglected the considerable damping caused by the presence of corona leakage between turns of the coil, which had the effect of a shunt resistance. It is interesting to note, therefore, that reasonable agreement between theory and practice can be obtained under such adverse conditions. In order to take into account empirically the effect of corona, a calculation was made

based on Fig. 22(*e*). With $R_0$ equal to 5 000 ohms, it was found that agreement was much better, and the voltage across $C_2$ under these conditions is shown dotted in Fig. 38(*b*).

transformer at the end of the line when preceded by a multi-layer air-core inductance coil. The transformer is behaving as a lumped capacitance, so that the excessive voltages obtained in Fig. 38(*a*) are also in evidence here.

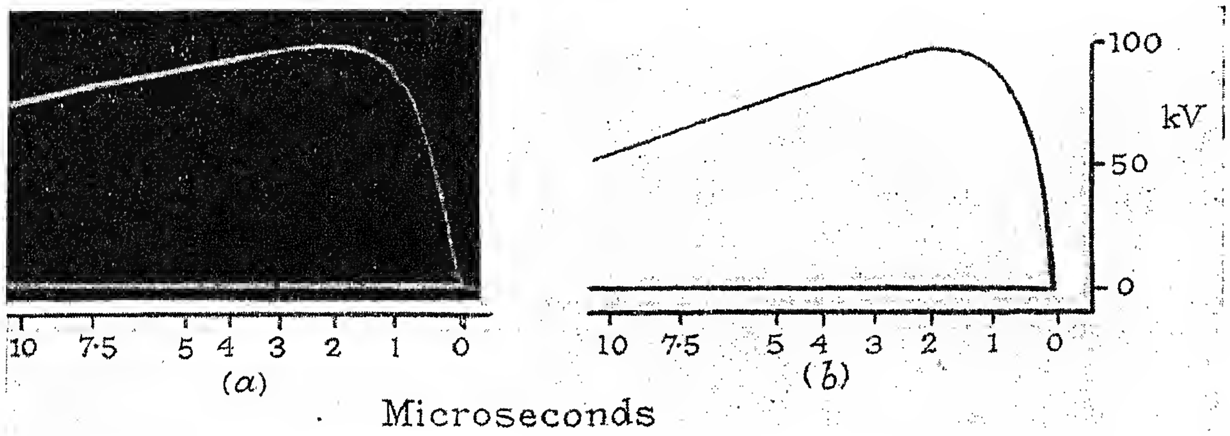



FIG. 39.

(*a*) Voltage across a capacitance at the end of the line and preceded by an air-core inductance having resistance in parallel as in Fig. 22(*e*).
(*b*) Calculated voltage. ($L = 0{\cdot}001$ H, $C_2 = 0{\cdot}0006\ \mu$F, $r = 0$, $R_0 = 500\ \Omega$, $Z_1 = 370\ \Omega$.)

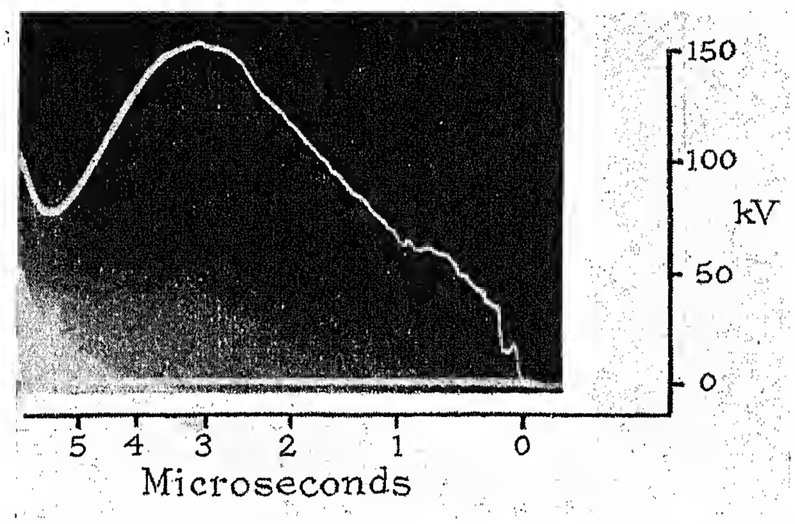
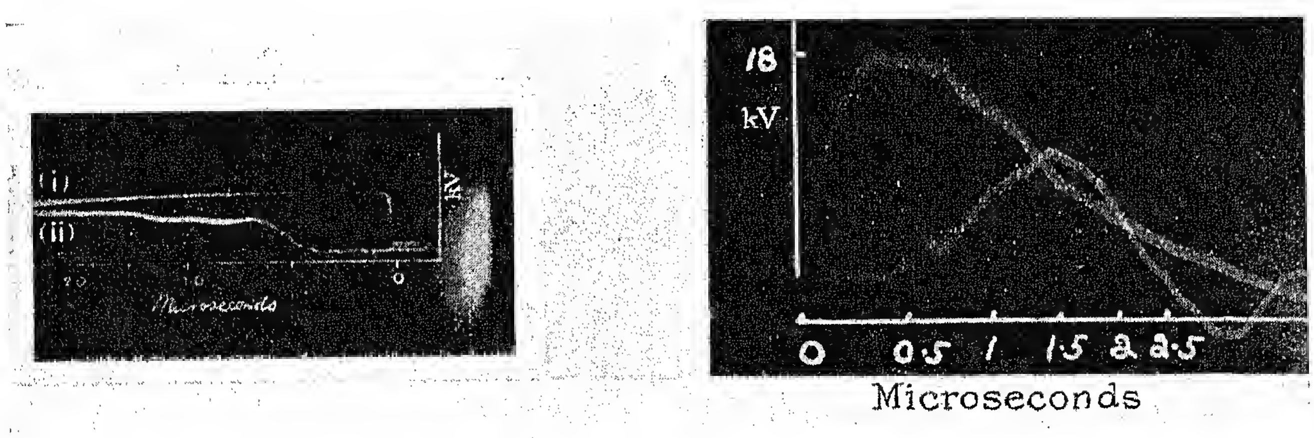



FIG. 40.

Voltage across a transformer at the end of the line and preceded by a multi-layer inductance coil.




(*a*) (*b*)

FIG. 41.

Oscillograms illustrating the two types of wave transmission discussed in Section 9(*e*). In both cases the upper oscillogram shows the incident wave and the lower the wave after passing through a surge absorber.

*Fig.* 39.—Fig. 39(*a*) shows the effect of connecting a resistance of 500 ohms across the inductance of the circuit used to obtain the record given in Fig. 38(*a*), and it is seen that the voltage across the condenser is no longer oscillatory. The calculated voltage, for the circuit in Fig. 22(*e*), is shown in Fig. 39(*b*).

*Fig.* 40.—This oscillogram shows the voltage across a transformer at the end of the line and preceded by a multi-layer air-core inductance coil. Also, the effect of the distributed shunt capacitance of this coil is shown at the beginning of the time period considered.

*Fig.* 41.—Figs. 41(*a*)* and 41(*b*), obtained when using Ferranti surge absorbers, show respectively the two types of wave transmission discussed in Section 9(*e*). In

* This oscillogram was taken by Prof. Binder in the High-Tension Laboratory of the Technische Hochschule, Dresden.

the first case the front of the incident wave was steep and the absorber behaved as a distributed circuit. In the second case the front of the incident wave was less steep and the absorber (which was of different type from the other) responded as a concentrated circuit.

### (11) Conclusion.

The paper has been written with the object of stimulating interest in travelling-wave phenomena among engineers engaged in the design and operation of transmission lines and connected apparatus. It is hoped that the material presented may serve two useful purposes. For those interested in the more academic side of the question, there is a demonstration of the method of ascertaining the manner in which travelling waves are affected by transmission-line associated apparatus; and the ease of extending the treatment to other cases will be evident. On the other hand, for those more interested in the immediate practical side of the question, there are actual values of the magnitudes and durations of the waves, the equivalent values of the various equipments connected to lines, and the curves showing the effect of these equipments on the waves. The paper could, of course, have been shortened considerably by making it a purely mathematical one, but it was felt that much of its value would then have been lost.

It was the author's intention in the first instance to deal with many other questions, including the effect of adjacent phases or earth wires on the travelling waves, the surges appearing on the secondaries of transformers, and the voltage stresses produced between the turns of the winding of a transformer. The inclusion of such considerations would have made the paper unduly long and considerably more mathematical, and it was therefore decided to defer the discussion of such aspects of the surge problem.

In view of the trouble and expense incurred both in this country and in many others by interruptions of supply and breakdowns due to lightning transients, some remarks on this danger and methods of preventing it would no doubt have been interesting. As this branch of the subject is so vast and opinion is so divided, it was felt that such matter would be best left to a separate paper.

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### BIBLIOGRAPHY.

(1) J. L. Miller: "Lightning Transients," *Electrician*, 1932, vol. 109, p. 805.

(2) P. Sporn and W. L. Lloyd: "Lightning Investigations," *Transactions of the American I.E.E.*, 1931, vol. 50, p. 1111.

(3) P. Sporn and W. L. Lloyd: "Lightning Investigations on the Ohio Power Co.'s 132-kV System," *ibid.*, 1930, vol. 49, p. 905.

(4) N. N. Smeloff and A. L. Price: "Lightning Investigations on the 200-kV System of the Pennsylvania Power and Light Co.," *ibid.*, 1930, vol. 49, p. 895.

(5) W. W. Lewis: "Summary of Five Years' Work in the Study of Lightning Effects on Transmission Lines," Paper No. 45, International E.H.T. Conference, 1931.

(6) E. T. Norris: "Protection of Electrical Apparatus from High-Voltage Surges," Paper No. 27, International E.H.T. Conference, 1931.

(7) R. R. Pittman and J. J. Torok: "Lightning Investigations on a Wood Pole Transmission Line," *Transactions of the American I.E.E.*, 1931, vol. 50, p. 568.

(8) J. R. Eaton, J. K. Peck, and J. M. Dunham: "Experimental Studies of Arcing Faults," *ibid.*, 1931, vol. 50, p. 1469.

(9) J. F. Peters and J. Slepian: "Voltages Induced by Arcing Grounds," *ibid.*, 1923, vol. 42, p. 478.

(10) L. V. Bewley: "Travelling Waves on Transmission Lines," *ibid.*, 1931, vol. 50, p. 532.

(11) F. W. Peek: "Lightning," *ibid.*, 1931, vol. 50, p. 1077.

(12) L. V. Bewley: "Travelling Waves due to Lightning," *ibid.*, 1929, vol. 48, p. 1050.

(13) E. T. Norris: "Lightning Surges," *Electrician*, 1932, vol. 108, p. 632.

(14) G. C. Simpson: "Lightning," *Proceedings of the Royal Society*, A, 1926, vol. 111, p. 56.

(15) G. C. Simpson: "The Mechanism of the Thunderstorm," *ibid.*, 1927, vol. 114, p. 376.

(16) E. Bell and A. L. Price: "Lightning Investigation," *Transactions of the American I.E.E.*, 1931, vol. 50, p. 1101.

(17) S. Tezner and L. Barbillion: "The Use of Condensers and Derived Arrangements for the Protection of Electric Systems against Voltage Surges," Paper No. 50, International E.H.T. Conference, 1929.

(18) C. Ledoux: "The Protection of Networks and the Damping Down of Surges," Paper No. 71, International E.H.T. Conference, 1931.

(19) J. L. Miller: "Condensers and Travelling Waves," *Elektrotechnik und Maschinenbau*, 1932, vol. 50, p. 281.

(20) W. W. Lewis and C. M. Foust: "Lightning Investigations on Transmission Lines," *Transactions of the American I.E.E.*, 1930, vol. 49, p. 917.

(21) J. C. Dowell: "Attenuation and Successive Reflections of Travelling Waves," *ibid.*, 1931, vol. 50, p. 551.

(22) R. N. Conwell and C. L. Fortescue: "Lightning Laboratory at Stillwater," *ibid.*, 1930, vol. 49, p. 872.

(23) L. V. Bewley: "Critique of Ground Wire Theory," *ibid.*, 1931, vol. 50, p. 1.

(24) K. W. Wagner: "Inductive Effects of Surges in Neighbouring Conductors," *Elektrotechnische Zeitschrift*, 1914, vol. 35, p. 639.

(25) C. W. Boehne: "Voltage Oscillations in Armature Windings under Lightning Impulses," *Transactions of the American I.E.E.*, 1930, vol. 49, p. 1587.

(26) F. D. FIELDER and C. BECK: "The Effects of Lightning Voltages on Rotating Machines and Methods of Protecting against them," *ibid.*, 1930, vol. 49, p. 1577.

(27) R. M. SOUTHGATE, A. E. BROOKES, E. R. WHITEHEAD, and W. G. ROMAN: "Impulse Tests on Substations," *ibid.*, 1931, vol. 50, p. 562.

(28) L. F. BLUME and A. BOYAJIAN: "Abnormal Voltages within Transformers," *ibid.*, 1919, vol. 38, p. 577.

(29) K. K. PALUEFF: "The Effect of Transient Voltages on Power Transformer Design," *ibid.*, 1929, vol. 48, p. 681; 1930, vol. 49, p. 1179; 1931, vol. 50, p. 802; 1932, vol. 51, p. 601.

(30) E. T. NORRIS: "Distribution Transformers," *Electrician*, 1931, vol. 106, p. 193.

(31) J. K. HODNETTE: "The Effect of Surges on Transformer Windings," *Transactions of the American I.E.E.*, 1930, vol. 49, p. 68.

(32) L. V. BEWLEY: "Transient Oscillations in Distributed Circuits," *ibid.*, 1931, vol. 50, p. 1215.

(33) E. T. NORRIS: "The Surge Dielectric Strength of Transformers," *Electrical Review*, 1931, vol. 108, pp. 140 and 182.

(34) H. V. PUTMAN: "Surge Proof Transformers," *Transactions of the American I.E.E.*, 1932, vol. 51, p. 579.

(35) L. V. BEWLEY: "Transient Oscillations of Mutually Coupled Windings," *ibid.*, 1932, vol. 51, p. 299.

(36) F. J. VOGEL and J. K. HODNETTE: "Grounding Banks of Transformers with Neutral Impedances," *ibid.*, 1931, vol. 50, p. 61.

(37) K. B. MCEACHRON: "Cathode-Ray Oscillograph Study of the Operation of Choke Coils on Transmission Lines," *General Electric Review*, 1929, vol. 32, p. 668.

(38) F. H. KIERSTEAD, H. L. RORDEN, and L. V. BEWLEY: "Shunt Resistors for Reactors," *Transactions of the American I.E.E.*, 1930, vol. 49, p. 1161.

(39) H. M. TOWNE: "Lightning Arrester Grounds," *General Electric Review*, 1932, vol. 35, pp. 173, 215 and 280.

(40) P. D. MORGAN and H. G. TAYLOR: "The Resistance of Earth Electrodes," *Journal I.E.E.*, 1933, vol. 72, p. 515.

(41) R. KLEIN: "The Inter-Turn Capacitance of Choke Coils," *Elektrotechnik und Maschinenbau*, 1930, vol. 48, p. 337.

(42) R. BAUCH: "Travelling Waves and Coils, with particular reference to the Effect of Mutual Inductance and Capacitance between Turns and to Earth," *ibid.*, 1932, vol. 50, p. 181.

(43) R. MAYER and A. SEGALL: "The Flattening of Steep-Fronted Surges by Arrangements of Capacitances, Choking Coils, and Resistances," *ibid.*, 1931, vol. 49, p. 397.

(44) K. W. WAGNER: *ibid.*, 1915, vol. 33, p. 89.

(45) J. M. THOMSON: "Surge Absorbers," *World Power*, 1932, vol. 17, p. 325.

(46) W. KRUG: "The Flattening of Travelling Waves by Surge Absorbers," *Elektrotechnische Zeitschrift*, 1932, vol. 53, p. 627.

(47) J. R. CARSON: "Electric Circuit Theory," McGraw-Hill, 1926.

(48) T. J. I'A. BROMWICH: "Normal Co-ordinates in Dynamical Systems," *Proceedings of the London Mathematical Society*, 1916, series 2, vol. 15, p. 401.

(49) OLIVER HEAVISIDE: "Electromagnetic Theory," vols. 1, 2, and 3.

(50) OLIVER HEAVISIDE: "Operators in Physical Mathematics," *Proceedings of the Royal Society*, A, 1893, vol. 52, p. 504; and 1894, vol. 54, p. 105.

(51) L. COHEN: "Heaviside's Electric Circuit Theory," McGraw-Hill, 1928.

(52) H. JEFFREYS: "Operational Methods in Mathematical Physics," Cambridge Tract in Mathematics and Mathematical Physics, No. 23.

(53) V. BUSH: "Operational Circuit Analysis," pub. J. Wiley, 1929.

## APPENDIX.

### MATHEMATICAL DEVELOPMENT OF EQUATIONS.

#### (a) *The General Operational Impedance Equation.*

The general operational impedance equation, giving the transmitted voltage-wave $e_2$ for the circuit shown in Fig. 2, will here be derived.

The symbols used are:—

$Z_b(p)$ = operational series impedance.

$Z_a(p), Z_c(p)$ = operational shunt impedance.

$Z_1, Z_2$ = surge impedances of incoming and outgoing lines respectively.

$e_1$ and $i_1$ = incident voltage-wave and incident current-wave respectively.

$e_3$ and $i_3$ = reflected voltage-wave and reflected current-wave respectively.

$e_2$ and $i_2$ = transmitted voltage-wave and transmitted current-wave respectively.

$i_2, i_4, i_5$, and $i_6$ = currents in the various parts of the circuit, as shown in Fig. 2.

$e_4$ = total voltage to earth at the point P.

Since the network is a concentrated one, only reflection of the incident voltage-wave and the incident current-wave at the point P are taken into account.

Now at any transition point the conditions imposed are that the current must be continuous, and that the potentials on either side must be identical.

Thus
$$e_1 + e_3 = e_4$$
$$i_1 + i_3 = i_4 + i_6$$
$$i_6 = i_2 + i_5$$
Further,
$$e_4 - i_6 Z_b(p) = e_2$$
$$i_4 = \frac{e_4}{Z_a(p)}$$
$$i_5 = \frac{e_2}{Z_c(p)}$$

In addition, for waves moving from left to right,
$$e_1 = i_1 Z_1; \; e_2 = i_2 Z_2$$
and for waves moving from right to left,
$$e_3 = -i_3 Z_1$$

Eliminating $i_1$, $i_2$, $i_3$, $i_4$, $i_5$, $i_6$, $e_3$, and $e_4$, from these nine simultaneous equations, the transmitted voltage-wave $e_2$ in operational form is given by

$$e_2 = \frac{2Z_2 Z_a(p) Z_c(p)}{[Z_1 + Z_2]Z_a(p)Z_c(p) + Z_1 Z_2[Z_a(p) + Z_b(p) + Z_c(p)] + Z_b(p)[Z_1 Z_c(p) + Z_2 Z_a(p)] + Z_a(p)Z_b(p)Z_c(p)} e_1 \quad (1\text{A})$$

which as equation (2), page 477, is used throughout this paper.

It should be remarked that although $Z_2$ has been defined as a surge impedance, there is no reason why the symbol, as it is used here, should not represent some other network having an operational equivalent $Z_2(p)$, so that a further range of circuits is covered by the arrangement in Fig. 2.

(b) *Remarks on the Operational Equation and the Method of Solution.*

Equation (1A) can be expressed as the ratio of two integral rational functions of $p$. Thus

$$e_2 = 2K\frac{f(p)}{F(p)}e_1 \quad . \quad . \quad . \quad . \quad . \quad (2\text{A})$$

where $e_1$ and $e_2$ are functions of the time $t$, $p$ being the Heaviside operator $d/dt$, and $K$ a constant.

When $e_1 = E1$, where $E$ is a constant and $1$ is the Heaviside unit function (a discontinuous function whose value is zero for $t < 0$, and unity for $t > 0$), then (2A) becomes

$$e_2 = 2KE\frac{f(p)}{F(p)}1 \quad . \quad . \quad . \quad . \quad (3\text{A})$$

Now the actual solution of the operational equation $\frac{f(p)}{F(p)}1$ is given at once by the Heaviside Expansion Theorem,* namely

$$\frac{f(p)}{F(p)}1 = \frac{f(0)}{F(0)} + \sum_{p_r = p_1, p_2 \ldots p_n} \frac{f(p_r)}{p_r \dfrac{dF(p_r)}{dp}} \epsilon^{p_r t} \quad . \quad . \quad (4\text{A})$$

where $p_1$, $p_2$, $p_3$, $\ldots p_n$, are the roots of the equation $F(p) = 0$.

When $e_1 = E\epsilon^{-mt}$, $e_2$ can be determined by means of (4A) in conjunction with the Heaviside Shifting Theorem,† which states

$$h(p)(\epsilon^{-mt}u) = \epsilon^{-mt} . h(p - m)u \quad . \quad . \quad (5\text{A})$$

* For a full discussion of Heaviside's Expansion Theorem (and also other branches of the operational calculus) reference should be made to the many excellent works on the subject (see Bibliography, 47 to 52 inclusive).

It is necessary to mention here, however, that the Expansion Theorem does not apply when the powers of $p$ in $f(p)$ exceed those in $F(p)$, but in such a case it is only necessary to divide out. It also fails if one root in the equation $F(p) = 0$ is zero, or if two roots are equal; in these cases, however, the difficulty can be overcome by a slight readjustment of the circuit constants or by the inclusion of the critical conditions in the equation before solution. For circuits having many degrees of freedom there is the difficulty of determining the roots of $F(p) = 0$, and for distributed circuits the use of the Expansion Theorem is not always the best method of attack.

† The Boltzmann-Hopkinson superposition theorem, namely

$$h(p)g(t) = g(t)q(0) + \int_0^t g(u)\frac{d}{d(t-u)}q(t-u)du \quad . \quad . \quad . \quad . \quad (a)$$

can also be employed for obtaining, from the unit-function solution, the solution for waves of any form. If the process of calculation has led to an operational expression $h(p)g(t)$, where $g(t)$ is the impressed wave, and if by means of the Expansion Theorem $h(p)1 = q(t)$ has been determined, the right-hand side of $(a)$ contains nothing but known quantities and therefore, if the integral can be evaluated, $h(p)g(t)$ can be found at once as a function of time. Thus the actual solution for a wave of any form can be determined (see Bibliography, 53).

where $h(p)$ is a complete operational function and $u$ can be any continuous function of $t$; in all the cases under consideration here $u$ is the "unit" function.

Equations (4A) and (5A) enable all the present problems to be solved.

The most complicated case which occurs in the paper is that where $F(p)$ is a cubic in $p$ and $f(p)$ is linear. For the sake of generality it would be desirable to obtain actual solutions when $f(p)$ is also cubic, but since such operational equations are immediately reducible by division so that the numerator becomes quadratic, solutions for the latter only will be given. Thus (3A) can be written

$$e_2 = 2KE\left[\frac{\lambda p^2 + \mu p + \nu}{\delta p^3 + \alpha p^2 + \beta p + \gamma}\right]1 \quad . \quad (6\text{A})$$

where $\delta$, $\alpha$, $\beta$, $\gamma$, $\lambda$, $\mu$, and $\nu$, are functions of resistance, inductance, capacitance, and surge impedance, depending on the circuit conditions. When $e_1 = E\epsilon^{-at}$, by means of the Shifting Theorem (6A) becomes

$$e_2 = 2KE\epsilon^{-at}\left[\frac{\lambda(p-a)^2+\mu(p-a)+\nu}{\delta(p-a)^3+\alpha(p-a)^2+\beta(p-a)+\gamma}\right]1 \quad (7\text{A})$$

When $Z(p)$ is quadratic or linear, the two operational equations to be solved are

$$e_2 = 2KE\epsilon^{-at}\left[\frac{\lambda(p-a)^2 + \mu(p-a) + \nu}{\alpha(p-a)^2 + \beta(p-a) + \gamma}\right]1 \quad (8\text{A})$$

and

$$e_2 = 2KE\epsilon^{-at}\left[\frac{\mu(p-a) + \nu}{\beta(p-a) + \gamma}\right]1 \quad . \quad . \quad (9\text{A})$$

Three separate equations are given because solutions for a certain order of $p$ in $Z(p)$ cannot be obtained from those for a higher order by equating the appropriate coefficient to zero.

The equations will now be solved by means of the Expansion Theorem. Of course, absence of numerator coefficients does not invalidate the solutions.

(c) *Actual Solutions of the General Symbolic Equations.*

Five solutions have to be considered, one for (9A), two (corresponding to the aperiodic and oscillatory conditions) for (8A), and two for (7A). Equation (9A) will be solved first.

(i) *Linear Case.*—The actual solution of (9A) is given by

$$p\frac{dF(p)}{dp} = a\beta - \gamma; \quad f(p) = \frac{\nu\beta - \mu\gamma}{\beta}$$

from which

$$e_2 = \frac{2KE}{\beta(\gamma - a\beta)}\left[\beta(\nu - a\mu)\epsilon^{-at} - (\nu\beta - \mu\gamma)\epsilon^{-\frac{\gamma t}{\beta}}\right] \quad (10\text{A})$$

The subtraction of a term due to the presence of $- E\epsilon^{-bt}$ enables this solution to be put in the form given in equation (5), page 477.

(ii) *Quadratic Case: Aperiodic Condition.*—The actual solution of (8A) for this case is given by

$$p_1\frac{dF(p_1)}{dp} = \frac{\phi}{2a}(2aa - \beta + \phi); \quad p_2\frac{dF(p_2)}{dp} = -\frac{\phi}{2a}(2aa - \beta - \phi)$$

$$f(p_1) = \frac{\lambda(-\beta + \phi)^2 + 2a\mu(-\beta + \phi) + 4a^2\nu}{4a^2}; \quad f(p_2) = \frac{\lambda(-\beta - \phi)^2 + 2a\mu(-\beta - \phi) + 4a^2\nu}{4a^2}$$

where $\phi^2 = \beta^2 - 4a\gamma$.

From which

$$e_2 = 2KE\left[\frac{a^2\lambda - a\mu + \nu}{a^2a - a\beta + \gamma}\epsilon^{-at} + \frac{\lambda(-\beta \pm \phi)^2 + 2a\mu(-\beta \pm \phi) + 4a^2\nu}{\pm 2a\phi(2aa - \beta \pm \phi)}\epsilon^{\frac{-\beta\pm\phi}{2a}t}\right] \quad . \quad . \quad . \quad . \quad (11\text{A})$$

(iii) *Quadratic Case: Oscillatory Condition.*—The actual solution of (8A) for this case is obtained as follows:—

For the given conditions, $(\beta^2 - 4a\gamma)^{\frac{1}{2}} = j\psi$. Thus (11A) can be written

$$e_2 = 2KE\left[\frac{\lambda a^2 - \mu a + \nu}{aa^2 - \beta a + \gamma}\epsilon^{-at} + \frac{\lambda\beta^2 - a\beta\mu - 2a\gamma\lambda + 2a^2\nu \pm (a\mu - \lambda\beta)j\psi}{\pm j\psi a(2aa - \beta \pm j\psi)}\epsilon^{\left(-\frac{\beta}{2a}\pm\frac{j\psi}{2a}\right)t}\right] \quad . \quad . \quad (12\text{A})$$

By rationalizing the numerators of the coefficients of the second and third exponential terms, the latter are obtained in the form

$$\frac{2[(a\mu - \beta\lambda)(\gamma\mu - \beta\nu) + (a\nu - \gamma\lambda)^2]}{\psi^2[a(a\mu - \beta\lambda) + (\gamma\lambda - a\nu)] \pm j\psi[(\gamma - a\beta)(a\mu - \beta\lambda) + a(\gamma\mu - \beta\nu) + 2aa(a\nu - \gamma\lambda)]}\epsilon^{\left(-\frac{\beta}{2a}\pm\frac{j\psi}{2a}\right)t} \quad . \quad (13\text{A})$$

The modulus of the denominator of the coefficient of (13A) is

$$\psi\{4a[aa^2 - \beta a + \gamma][(a\mu - \beta\lambda)(\gamma\mu - \beta\nu) + (a\nu - \gamma\lambda)^2]\}^{\frac{1}{2}}$$

so that (13A) reduces to

$$+\frac{2}{\psi}\sqrt{\left[\frac{(a\nu - \gamma\lambda)^2 + (a\mu - \beta\lambda)(\gamma\mu - \beta\nu)}{a(aa^2 - \beta a + \gamma)}\right]}\epsilon^{-\frac{\beta}{2a}t}$$
$$\times \cos\left\{\frac{\psi t}{2a} - \text{arc tan}\,\frac{(\gamma - a\beta)(a\mu - \beta\lambda) + a(\gamma\mu - \beta\nu) + 2aa(a\nu - \gamma\lambda)}{\psi[a(a\mu - \beta\lambda) + (\gamma\lambda - a\nu)]}\right\} \quad . \quad (14\text{A})$$

On transforming (14A) into the more useful form with a negative coefficient, the complete solution becomes

$$e_2 = 2KE\left\{\frac{\lambda a^2 - \mu a + \nu}{aa^2 - \beta a + \gamma}\epsilon^{-at} - \frac{2}{a^{\frac{1}{2}}\psi}\sqrt{\left[\frac{(a\nu - \gamma\lambda)^2 + (a\mu - \beta\lambda)(\gamma\mu - \beta\nu)}{aa^2 - \beta a + \gamma}\right]}\epsilon^{-\frac{\beta}{2a}t}\cos\left(\frac{\psi t}{2a} - \theta_1\right)\right\} \quad . \quad (15\text{A})$$

where $$\theta_1 = \text{arc tan}\,\frac{(\gamma - a\beta)(\beta\lambda - a\mu) - a(\gamma\mu - \beta\nu) - 2aa(a\nu - \gamma\lambda)}{\psi[a(\beta\lambda - a\mu) + (a\nu - \gamma\lambda)]}$$

The subtraction of terms due to the presence of $-E\epsilon^{-bt}$ enables (11A) and (15A) to be put in the forms given in equations (7) and (8), page 478.

(iv) *Cubic Case: Aperiodic Condition.*—The actual solutions of (7A) in this and the next case are cumbersome, but they become simpler and more usable when some numerator coefficients are absent, as frequently occurs in practice. The treatment here is somewhat different from that used previously, as it is necessary to derive the actual solution in terms of the roots of $Z(p)$.

Let these be $-\rho_1$, $-\rho_2$, $-\rho_3$. Then (7A) can be written as follows:—

$$e_2 = 2KE\epsilon^{-at}\left[\frac{\lambda(p - a)^2 + \mu(p - a) + \nu}{(p - a + \rho_1)(p - a + \rho_2)(p - a + \rho_3)}\right]1$$

which can be put in the form $$e_2 = 2KE\epsilon^{-at}\left[\frac{L}{p-a+\rho_1} + \frac{M}{p-a+\rho_2} + \frac{N}{p-a+\rho_3}\right]1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16\text{A})$$

where $$L = \frac{\lambda\rho_1^2 - \mu\rho_1 + \nu}{(\rho_2 - \rho_1)(\rho_3 - \rho_1)}; \quad M = \frac{\lambda\rho_2^2 - \mu\rho_2 + \nu}{(\rho_1 - \rho_2)(\rho_3 - \rho_2)}; \quad N = \frac{\lambda\rho_3^2 - \mu\rho_3 + \nu}{(\rho_1 - \rho_3)(\rho_2 - \rho_3)}$$

The three terms of (16A) can be solved directly by the Expansion Theorem.

Taking the first term, $2LKE\epsilon^{-at}\left[\dfrac{1}{p-a+\rho_1}\right]1$, its actual solution is $2LKE\epsilon^{-at}\left[-\dfrac{1}{a-\rho_1}+\dfrac{\epsilon^{(a-\rho_1)t}}{a-\rho_1}\right]$.

By solving the other two terms, inserting the values of the constants $L$, $M$, and $N$, and rearranging, the complete solution of (16A) is obtained in the form

$$e_2 = 2KE\left[\frac{\lambda a^2-\mu a+\nu}{(\rho_1-a)(\rho_2-a)(\rho_3-a)}\epsilon^{-at} - \frac{\lambda\rho_1^2-\mu\rho_1+\nu}{(\rho_1-a)(\rho_2-\rho_1)(\rho_3-\rho_1)}\epsilon^{-\rho_1 t}\right.$$
$$\left. - \frac{\lambda\rho_2^2-\mu\rho_2+\nu}{(\rho_2-a)(\rho_1-\rho_2)(\rho_3-\rho_2)}\epsilon^{-\rho_2 t} - \frac{\lambda\rho_3^2-\mu\rho_3+\nu}{(\rho_3-a)(\rho_1-\rho_3)(\rho_2-\rho_3)}\epsilon^{-\rho_3 t}\right] \quad . \quad (17\text{A})$$

(v) *Cubic Case: Oscillatory Condition.*—In this case the equation which has to be solved in order to arrive at the actual solution of (7A) is

$$e_2 = 2KE\epsilon^{-at}\left[\frac{\lambda(p-a)^2+\mu(p-a)+\nu}{(p-a+\rho)(p-a+\sigma-j\xi)(p-a+\sigma+j\xi)}\right]1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (18\text{A})$$

Its solution can be obtained directly by the substitution of the appropriate roots in (17A),

$$\text{e.g.} \qquad \rho_1=\rho, \quad \rho_2=\sigma-j\xi, \quad \rho_3=\sigma+j\xi.$$

The first and second terms of (17A) then become respectively

$$\frac{\lambda a^2-\mu a+\nu}{[\rho-a]\left[\xi^2+(\sigma-a)^2\right]}\epsilon^{-at} \quad \text{and} \quad -\frac{\lambda\rho^2-\mu\rho+\nu}{[\rho-a]\left[\xi^2+(\sigma-\rho)^2\right]}\epsilon^{-\rho t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19\text{A})$$

and the last two terms become

$$-\frac{(\lambda\sigma^2-\lambda\xi^2-\mu\sigma+\nu)\pm j(\mu\xi-2\lambda\sigma\xi)}{2\xi\left\{\xi(a+\rho-2\sigma)\pm j\left[\xi^2+\sigma\rho-\sigma^2-a(\rho-\sigma)\right]\right\}}\epsilon^{(-\sigma\pm j\xi)t}$$

On rationalizing the numerators of these coefficients, transforming according to the usual rule, and collecting terms, one obtains the expression

$$-\frac{1}{\xi}\left\{\frac{\left[\lambda(\sigma^2+\xi^2)-\mu\sigma\right]^2+[\mu\xi-\nu]^2+2\nu[\sigma-\xi]\left[\lambda(\sigma+\xi)-\mu\right]}{\left[\xi^2+(\rho-\sigma)^2\right]\left[\xi^2+(\sigma-a)^2\right]}\right\}^{\frac{1}{2}}\epsilon^{-\sigma t}\cos(\xi t-\theta_1) \quad . \quad . \quad . \quad (20\text{A})$$

where $\theta_1$ is

$$\text{arc tan}\ \frac{\left\{(\sigma^2+\xi^2)\left[\lambda(\sigma\rho-\xi^2-\sigma^2)-\mu(\rho-\sigma)\right]+\nu(\rho\sigma+\xi^2-\sigma^2)\right\}+a\left\{[\xi^2+\sigma^2][\lambda\sigma-\mu]+\lambda\rho[\xi^2-\sigma^2]+\mu\sigma\rho+\nu(\sigma-\rho)\right\}}{\xi\left\{\nu(\rho-2\sigma)-\lambda\rho(\xi^2+\sigma^2)+\mu(\xi^2+\sigma^2)+a\left[\nu-\lambda(\xi^2+\sigma^2)+2\lambda\rho\sigma-\mu\rho\right]\right\}}$$

Equations (19A) and (20A) jointly give the actual solution of (18A).

When $\lambda=\mu=0$, and when similar terms incorporating $b$ instead of $a$ are subtracted, this solution and (17A) respectively enable equations (35) and (34) of the paper to be written down. Similar solutions can be obtained for equation (24$a$).

[The discussion on this paper will be found on page 519.]

# THE DESIGN AND OPERATION OF A HIGH-SPEED CATHODE-RAY OSCILLOGRAPH.

By J. L. Miller, Ph.D., B.Eng., Associate Member, and J. E. L. Robinson, M.Sc., Graduate.

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Summary.

Following an Introduction reviewing the essential requirements of a high-speed cathode-ray oscillograph for the recording of short-time transients, consideration is devoted to the design and construction of an instrument of this description adapted to both internal electron-blackening and to external light-blackening photography. The problems discussed include the construction of a cold-cathode discharge tube, the disposition of deflection plates, the operation of the time-sweep and beam-trap circuits, and the methods of synchronizing these with the transient under examination; this last leading to questions of voltage-division and delay necessary for synchronization.

## (1) Introduction.

Experimental lightning research on transmission-system associated apparatus, involving the application of short-time transients, necessitates the use of a high-voltage impulse generator together with suitable measuring equipment. In its essentials the action of an impulse generator is well known; it consists only of the abrupt discharge of a condenser so arranged as to impress on the test object a transient of the required nature. For the measurement of the transients three instruments —the spark-gap, the klydonograph, and the cathode-ray oscillograph—are available. While the first measures maximum values and the second gives an approximate idea of the general characteristics and wave-shape of the surges, the cathode-ray oscillograph is the only instrument capable of completely delineating the voltage/time relationships.

Since lightning transients are not repeatable in the same sense as is a sustained oscillation, they must be recorded in a single traverse of the cathode ray, and since also their duration is of the order of microseconds only, they cannot be measured by the well-known low-voltage sealed-off type of oscillograph, although this is eminently suitable for repeated phenomena up to the highest frequencies. This failure is a consequence of the fact that at the high speeds of traverse of the spot involved (of the order of hundreds of thousands of feet per second) the trace on the fluorescent screen is insufficiently intense to allow either of visual inspection or of photography with a camera; and in turn this lack of visual and actinic sensitivity arises from the low velocity of the electron beam generated by a voltage only of the order of 1 000 volts between cathode and anode.

The photographic sensitivity may be increased both by employing higher-velocity electrons and by placing photographic plates inside the vacuum so that a record of the phenomena under investigation is obtained by direct action of the electrons on the sensitive emulsion. High-velocity electrons are obtained by using voltages of the order of 50 000 volts between cathode and anode, and the use of this high voltage constitutes the main difference between the sealed-off glass-type and the high-speed oscillograph.

In general, these instruments employ a cold cathode requiring a low vacuum of the order of a few millionths of an atmosphere in the discharge tube. The insertion of photographic materials inside the vacuum demands continuous evacuation, and accordingly a controllable air leak is necessary in order to obtain the requisite working gas-pressure in the discharge tube.

Two pairs of deflection plates at right angles, serving to delineate the transient in Cartesian co-ordinates, are connected respectively to the test object and a capacitive circuit, the latter providing a uniformly varying voltage which sweeps the beam across the recorder to give the time or abscissa motion.

When the cathode is kept energized for immediate recording, some means must be adopted to prevent the beam lingering on the photographic plate and causing fogging. This is frequently achieved by means of a diaphragm and a third deflection-plate system in the upper part of the oscillograph. These plates, when charged, deflect the beam to one side, so preventing its passing through an aperture in the diaphragm into the lower part of the oscillograph. On the arrival of the transient to be measured, the potential on the plates is made to collapse in a very short time-interval, with the result that the beam is available almost immediately for recording, and at the same instant time-sweeping is initiated. The record completed, the plates automatically recharge and retrap the beam with the minimum of delay.

When the transient is controlled (that is, when it is generated at will in the laboratory) some method must be adopted whereby spark-over of the impulse generator is synchronized with the release of the beam and the commencement of time-sweeping. If the transient is uncontrolled (as in the case of lightning surges in a transmission line) it must be made to initiate release of the beam and time-sweeping. In both cases some delaying device is necessary to ensure that its arrival at the deflection plates is delayed by a fraction of a microsecond until time-sweeping is in full operation. Finally, since the oscillograph deflection plates will only withstand relatively low voltages, potentiometers are required capable of giving faithful division of the fastest transients.

The succeeding sections of the paper deal with these problems in detail.

(2) THE CONSTRUCTION OF THE OSCILLOGRAPH.

The oscillograph, which is built up from sections of copper-alloy tubing, is shown diagrammatically in Fig. 1, whilst Fig. 2 gives a general idea of the sturdy and transportable construction of the instrument and its auxiliary equipment.

At the lower end of the oscillograph is the photographic chamber, which is suitable for either internal electron-blackening photography or, alternatively, outside light-blackening photography.* In the first method the plates are contained in a holder (made integral with the cover plate) divided into two compartments, and are transferred as they are used from one to the other by turning a rotatable vacuum joint. Another rotatable joint controls the shutter of the holder, on which the traces can be examined through an inspection window before recording. In the second method a separate cover plate incorporating a thin glass window is used, against which a photographic film in an external holder is pressed in close contact. The window is supported on a fine-mesh grid built up of slotted and hard-soldered steel strips set into the cover plate (see Fig. 3). The whole is of very rigid construction and, the surface being ground to a very fine finish, readily supports against atmospheric pressure a 5-inch diameter glass disc of 0·12 mm thickness. The disc extends outside the grid section on to the cover plate, where the vacuum seal is effected by tap grease between the glass and plate surfaces. The glass is coated on the inside with calcium tungstate, which fluoresces under the action of the beam, and in this way a record is obtained on the external photographic film by secondary light emission. Fig. 4 shows oscillograms taken by means of this apparatus.

Evacuation, which is carried out by means of a two-stage mercury diffusion pump backed by the usual oil pump, takes place from the photographic chamber.

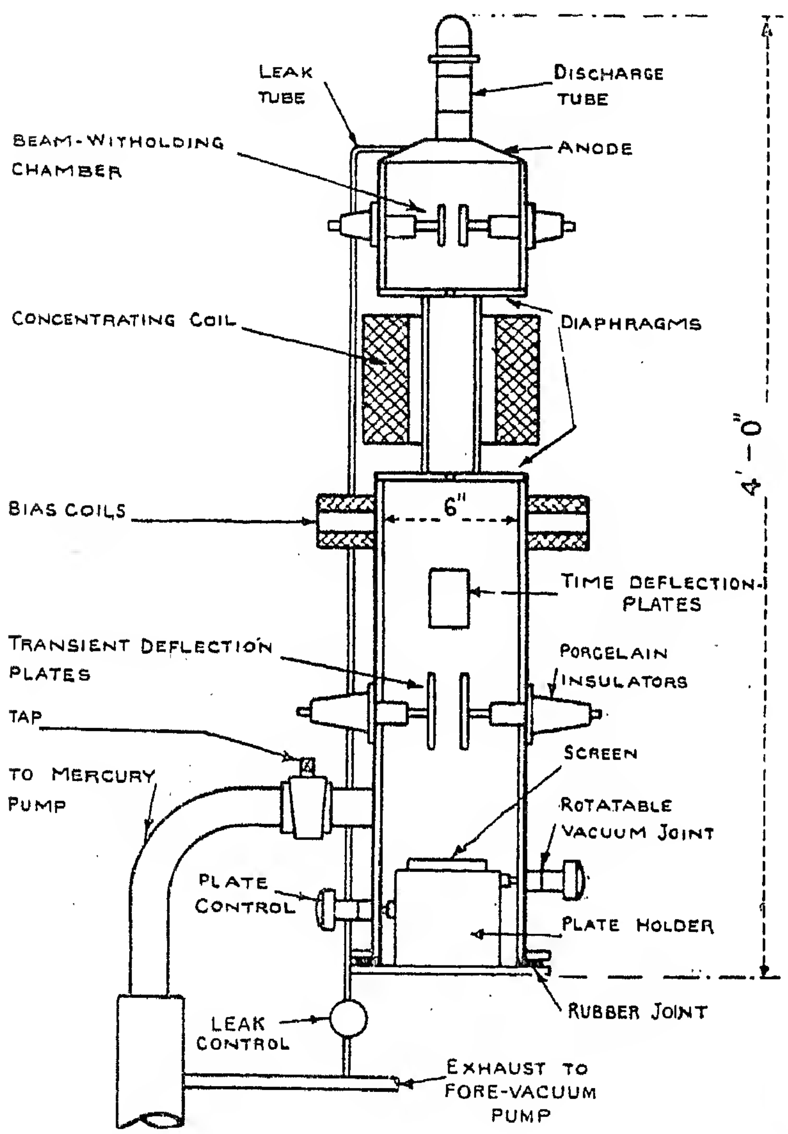



FIG. 1.—General disposition of the oscillograph.

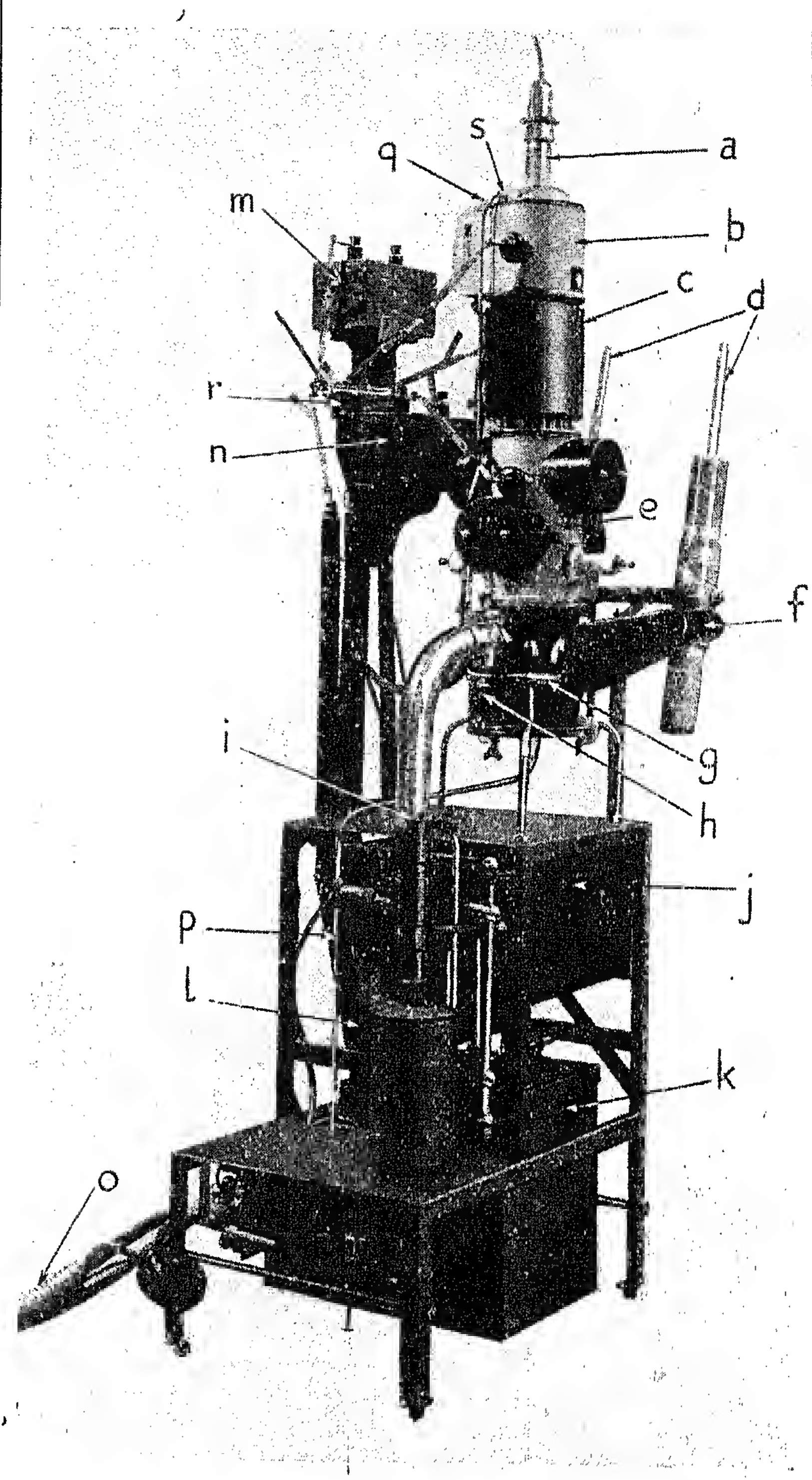



FIG. 2.—General view of the oscillograph and its auxiliaries mounted in one transportable unit.

*a*. Discharge tube.
*b*. Beam-withholding chamber.
*c*. Concentrating coil.
*d*. Potentiometers.
*e*. Deflection chamber.
*f*. Viewing window.
*g*. Air-leak control.
*h*. Rotatable plane-face joint for photographic-plate control.
*i*. Mercury diffusion pump.
*j*. Time calibration oscillator.
*k*. Supply transformer for time-sweep and beam-withholding circuits.
*l*. Rectifying valve for time-sweep and beam-withholding circuits.
*m*. Tripping-circuit condenser.
*n*. Time-sweep condenser.
*o*. Cooling water, fore-vacuum, and 230-volt a.c. supplies.
*p*. Fore-vacuum connection.
*q*. Air leak to discharge tube.
*r*. Initiating spark-gap.
*s*. Water cooling for anode.

* See Bibliography, (1), (2), (3), and (4).

The pump connection is made rigid by means of sealed screwed joints and includes a large-bore metal tap which enables the oscillograph vacuum to be broken immedi-

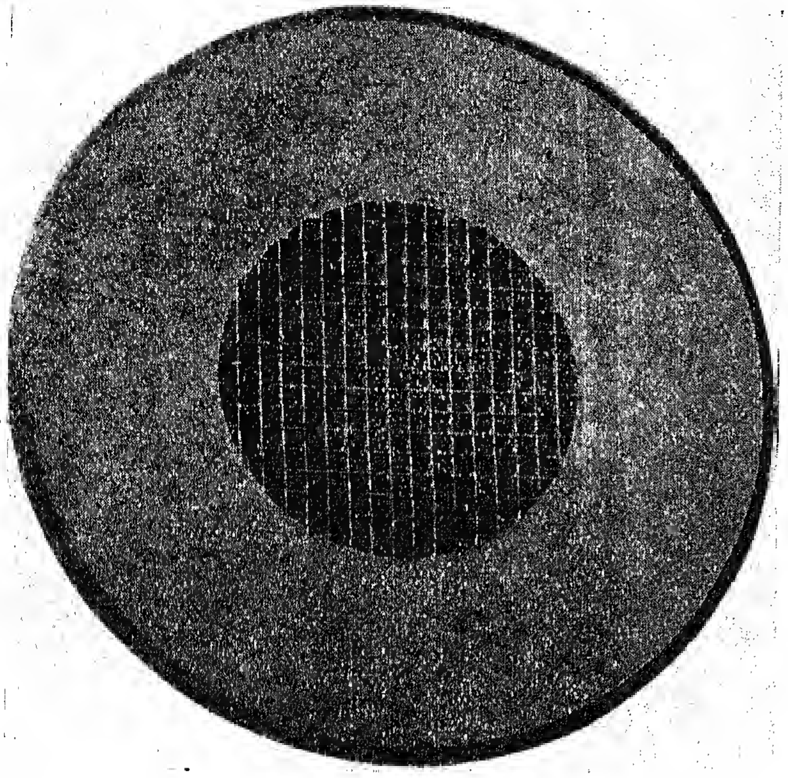


FIG. 3.—Grid support for thin glass window employed in external light-blackening photography.

ately in order to change the photographic plates, thus avoiding the customary delay during cooling-down of the pump. The speed of pumping is such that the apparatus

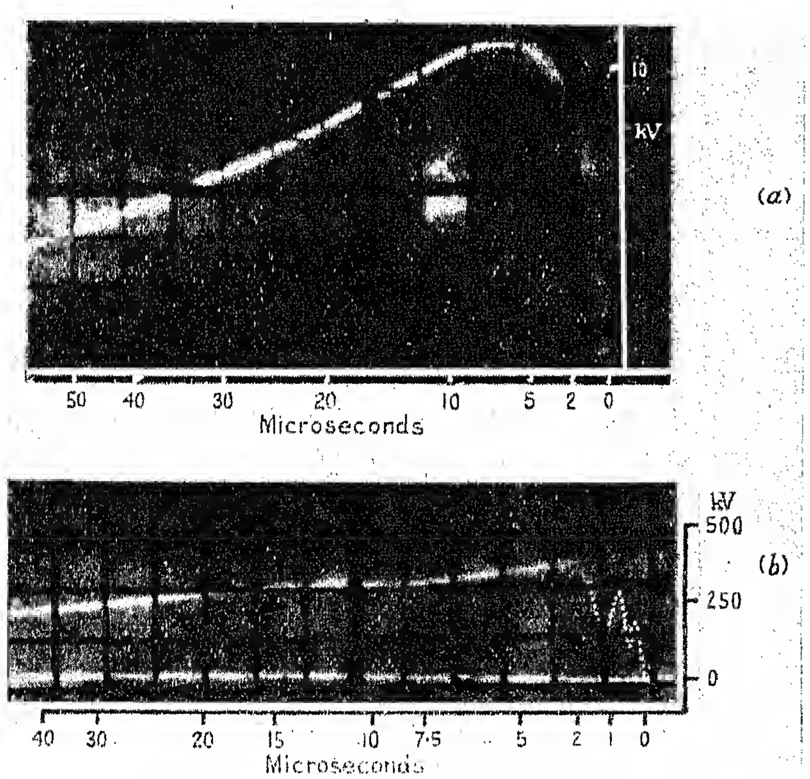



FIG. 4.—Examples of external photography.

(a) Slow-fronted wave. Writing current 100 per cent greater than that used for internal photography.
(b) Fast-fronted wave. Writing current the same as that used for internal photography. The high-frequency oscillations on the front of the wave, shown dotted in the reproduction, are readily discernible on the negative.

is ready for operation within a few minutes from renewal of the plates.

On top of the photographic chamber is the deflection chamber equipped with the transient and time-sweep deflection plates, which are mounted on porcelain insulators. Here also are fixed the electromagnetic bias coils for varying the mean position of the beam relative to the photographic plate. This control is necessary in the ordinate or transient direction, since several traces may be required on one plate without superimposing, and again in the abscissa or time direction, because, where the simpler time-sweep circuits are in use, the initial or final position of the beam would otherwise occur in the middle of the plate. An example of the use of this control will be seen in the oscillogram of Fig. 5.

Situated above the deflection chamber is the concentrating coil, followed immediately by the beam-trap diaphragm and the withholding plates.

The anode, disposed above the withholding chamber, is constructed of steel. To provide for the use of varying anode apertures the anode core is detachable, while to ensure good cooling it is made of steel and is a push-fit

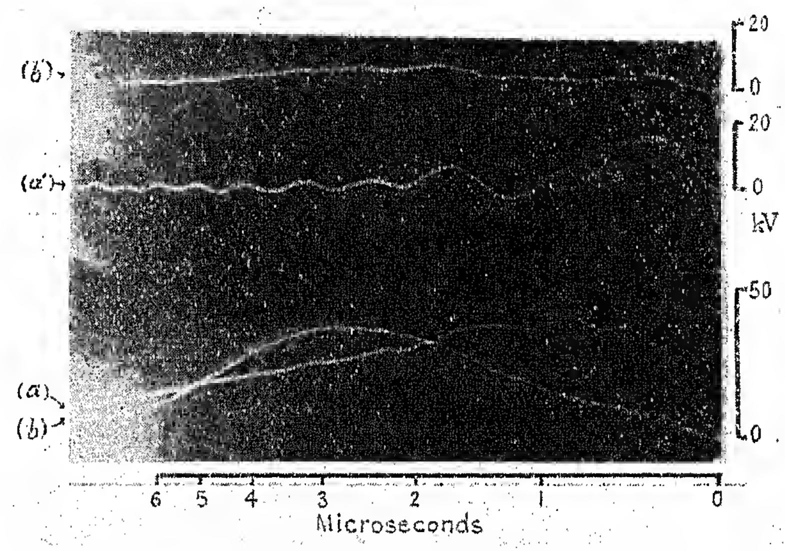



FIG. 5.—Illustration of the use of ordinate bias control.

(a′) and (b′) are the voltages appearing across 5 per cent of the line end-turns of a transformer winding when the waves incident on it are respectively those shown in (a) and (b).

in the main housing. An internal water duct closely surrounding the centre core is provided for cooling purposes.

The discharge tube, fed from a single-valve rectifier through a 1-MΩ resistance,* is of the cold-cathode type and is mounted on the anode. The cathode, which is adjustable in a vertical direction, is of aluminium. Air leakage takes place directly into the discharge tube from the fore-vacuum system through an adjustable needle valve, which provides a very accurate control of pressure. The tube is evacuated through an internal channel in the anode body, which includes an externally controlled cut-off valve for the purpose of maintaining a lower air pressure in the main body than in the discharge tube during operation.

### (3) DISCHARGE TUBES.

At the time of first consideration several designs of discharge tubes were available, the most advanced being

* Finch has successfully employed a saturated diode as a feeding and regulating resistance in the cathode circuit. See Bibliography, (5).

probably the metal tube described by Knoll,* while the more recent constructions due to Burch and Whelpton† in this country and Binder‡ in Germany, both of which are very satisfactory, had not then been described. In view, however, of the generally critical nature of the operation of discharge tubes and the lack of operating data in the various publications, it was decided to develop a design from first-hand experience.

It is self-evident that to be satisfactory a discharge tube should be electrostatically and thermally stable and capable of generating a beam of good intensity. While the first two of these requirements are attained by minimizing stress on the solid dielectrics by shielding them from stray charges and by providing for adequate heat dissipation, they are closely inter-related with the third, which requires some comment.

Experimentally it has been noted that for a given post-anode beam current the size of the focused spot is controlled by the nature of the cathode assembly, the discharge current, and the anode aperture. The dependence on cathode assembly is well known, and arises, as shown by Busch,§ from the focused-spot diameter being a function of the minimum useful section of the discharge; this latter is probably close to or at the emitting area on the cathode and is determined by the field shape about the cathode face. The dependence on discharge current is probably explained by a corresponding increase in the cathode emitting area with increased current. The observed increase in focused-spot diameter with increased anode aperture may result from loss of efficacy of concentration with the larger attendant beam-section.

As a result of these observations it is realized that in order to obtain a small concentrated writing spot both a small cathode emitting area and a small anode aperture are desirable and, in view of the latter requirement, to obtain adequate post-anode energy the pre-anode discharge density must be correspondingly high.

With these requirements in view, a whisky bottle and a specially constructed bottle-like tube of heat-resistant glass were successively employed, in both cases the cathode being situated in the neck on an adjustable mounting.‖ While both operated very quietly, the former gave an insufficiently dense beam and the latter, which had a very narrow neck to overcome this defect, was not capable of running continuously for more than 10 minutes.¶

A subsequent design of tube capable of continuous operation was therefore evolved; it is shown in Fig. 6. The introduction of a metal shield round the cathode provides a high degree of electrostatic focusing and at the same time overcomes the difficulty of glass stressing and heating. The focusing is such that the cathode emitting area is small, and the discharge section is so concentrated that only a narrow anode aperture is necessary to give the required post-anode current. Actually, with an aperture of 0·35 mm and a discharge current of 1 milliamp., a current of as much as 25 microamps. may be obtained in the post-anode beam, although this is considerably greater than is required for internal writing. A further, though totally unexpected, advantage of this tube is the long useful life of its cathode.

The shield takes the form of an inverted tulip-shaped cup, the cathode being disposed therein and provided with a control affording vertical adjustment in and about the neck of the tulip. The shield, free to assume any potential, is insulated from the cathode, and the dielectric (in the form of a flat glass disc) is outside the dynamic field. The focusing of the discharge is influenced by the disposition of the cathode relative to the shield, the best position being determined experimentally. There is also a maximum clearance between the two for stable running,

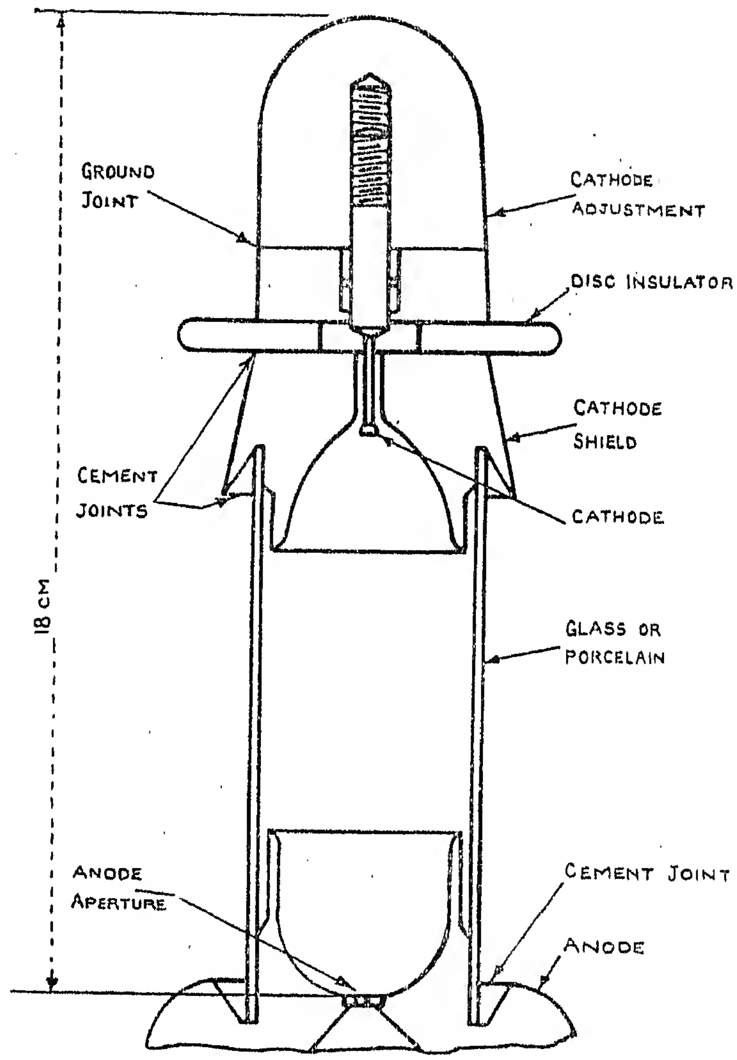



Fig. 6.—Discharge tube.

and in general the larger the clearance the higher the cathode position demanded.

Stability of operation depends on the inter-electrode length and the disposition of the anode. With long tubes stability is easily obtained, but for a given size of anode aperture there may not be the requisite post-anode beam energy and in addition there is considerable heating of the glass wall above the anode. As the tube is shortened the post-anode energy increases, until finally a minimum length is reached where instability occurs. When the length is decreased below this minimum the heating eventually disappears, but the tube is very unstable. By maintaining the cathode-anode separation at its minimum for stable running and by bringing the circumference of the anode up to within the distance from the cathode assembly necessary for elimination of heating, the two rather contradictory requirements are met and a satisfactory tube is obtained. This anode extension does not prevent heating by merely shielding the glass from

* See Bibliography, (6).
† Ibid., (7).
‡ Ibid., (8).
§ Ibid., (9).
‖ Ibid., (10).
¶ One of the authors has discussed previously some of the difficulties associated with glass discharge tubes. See Bibliography, (11) and (12).

stray bombardment, but rather by altering the disposition of the electric field and hence the characteristics of the tube. Further, since the distance between the cathode and the anode aperture is greater than that between the lower extremity of the hood and the anode extension, there is no tendency for discharge to take place down the walls of the tube unless this path is made very short.

### (4) Deflection-Chamber Design.

The design and disposition of the deflection system are determined by the contending requisites of sensitivity and small overall length, together with the degree of distortion which can be tolerated. In oscillographs used for high-voltage transient measurements actual deflection sensitivity is seldom of major importance and it may accordingly be sacrificed, in the interests of decreasing the oscillograph length, by minimizing the deflection length employed. For a required size of record there is, however, a lower limit to deflection length imposed by the exaggeration of deflection as the beam diverges from the normal. While this may be corrected for by calibration, it is preferable to minimize it by limiting the maximum angular divergence or, where $D_1$ is the maximum displacement on the recorder and $L$ is the deflection length, the ratio $D_1/L$. The differential error introduced by this divergence is $D_1^2/L^2$, and hence for a maximum permitted percentage error $p$ and a required displacement $D_1$ the minimum deflection length which may be employed is $10D_1\sqrt{p}$. The adoption of electromagnets or other devices for biasing* the beam permits sweeping to be provided equally about the axis, the total deflection obtained being then double this divergence. This fact is conveniently made use of in connection with the abscissa motion.

For a given deflection length it is usually most logical to provide for maximum sensitivity, and for this condition geometrically the ratio of the plate length $l$ to the separation $d$ should be equal to that of the deflection length $L$ to the total desired deflection $D$, although in practice the considerable section of the beam as it passes through the deflection chamber, and the necessity of keeping it clear of the electrodes by imposing a minimum on the separation which may be employed, require some deviation from strict equality.

The deflection sensitivity is determined by the four dimensions $d$, $D$, $l$, and $L$, and for a given beam velocity† $v$ (cm per sec.) the necessary deflection voltage $E$‡ is given by

$$E = 0\cdot565 \times 10^{-15}\frac{v^2}{[1-(v^2/c^2)]^{\frac{1}{2}}}\cdot\frac{dD}{lL}$$

where $c$ is the velocity of light (cm per sec.). Accordingly the sensitivity $D/E$ is proportional to $lL/d$ for a given oscillograph and, when the relation $d/l = D/L$ is maintained, for a required deflection the maximum sensitivity is nearly proportional to the square of the deflection length.

The sequence of the time-sweep and the transient-deflection plates also requires some consideration. At the upper deflection plates the deflecting field requires only to be uniform over the small width occupied by the beam, whereas at the lower plates the inter-electrode field must be uniform throughout the whole locus of movement of the beam caused by the deflection of the upper pair. Consequently the upper plates are narrower than the lower, and they therefore have less capacitance, which may be advantageous for their connection to the transient voltage. This arrangement is also advantageous when it is desired to secure the greater deflection sensitivity for the transient-deflection plates. On the other hand the time axis is conventionally the longer and accordingly requires the greater minimum deflection length. If then it is desired to minimize the overall length of the apparatus, it may be preferable to mount the time-sweep plates the farther from the recorder. Actually, in the design of the present instrument, the latter sequence has been adopted.

In regard to the necessary width of the plates, this must be sufficient only to provide an undistorted field throughout the locus of motion of the beam, although it must be remembered that at this point the section of the latter is relatively great. This question is closely related to that of the annular clearance to the metal casing, in that narrow electrodes require a large clearance and have small earth capacitance, while broader electrodes, permitting a reduced casing diameter, have a greater capacitance. Considerable reduction of this annular clearance is permissible when employing a symmetrical field, and this is generally obtainable in the time-sweep circuit.

### (5) Time-Sweeping.

A single-sweep time motion is employed, the time deflection plates being connected across the condenser of a condenser-resistance circuit, which on being discharged

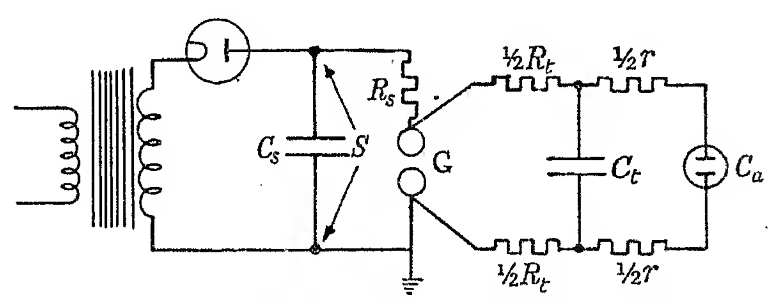



Fig. 7.—Schematic diagram of time-sweep circuit.

sweeps the beam uniformly across the recorder according to the simple exponential law.

The circuit is shown diagrammatically in Fig. 7, the connections of the d.c. voltage supply $S$ (which in the present case is 15 kV) being also shown. $R_t$ and $C_t$ are respectively the resistance and capacitance of the time-sweep circuit, and $C_a$ represents the capacitance of the time-sweep deflection plates, the resistances $\frac{1}{2}r$ being included to prevent any local oscillation in this part of the circuit. G is a sphere-gap whose setting is just greater than that required for spark-over of the supply voltage. $C_s$ is the capacitance of the supply reservoir condenser,

* In view of the symmetrical disposition and saving of length afforded, the practice of dividing time-sweeping equally about both sides of the oscillograph axis is superior to off-setting the recorder. In the abscissa circuit, where stray earth capacitance is not of primary importance, electrostatic bias may be introduced by the inclusion of condensers suitably charged from a very high-impedance source. Since the preparation of the paper one arrangement on these lines, providing also a symmetrical deflection field, has been developed by the authors.

† The velocity of high-voltage rays may be obtained from the relationship $v = c\sqrt{[V(V+1\cdot02\times10^6)]}/(V+0\cdot51\times10^6)$, where $V$ is the total accelerating potential (volts). See Bibliography, (13).

‡ See Bibliography, (14).

which is several times $C_t$; while $R_s$ is the supply resistance, which must be so large as to allow extinction of the gap arc in the time determined by $R_tC_t$, so that this arc has no tendency to persist and discharge $C_s$. When this condition is obtained the time of recharge of $C_t$ is of the order of a fraction of a millisecond. Normally, of course, such recharge would involve the slow retracing of the beam across the recorder, but this is prevented by withdrawal of the beam when $C_t$ is discharged.

When initiation of discharge is effected by reducing the gap separation or by increasing the supply voltage, on recharge of $C_t$ spark-over of the gap tends to occur repeatedly with a periodicity determined by $R_sC_t$. Since each spark-over releases the beam, an unwanted series of traces along the time axis will then be drawn. It is undesirable to avoid this by adequately increasing $R_s$, and the difficulty is overcome by the use of a rotating switch which is arranged to sweep within sparking distance of the gap electrodes for a time less than the periodicity of spark-over. Thus, release of this switch, which is spring-loaded, ensures that only one time-sweep is made. When the gap is tripped electrically the question of time of recharge of $C_t$ has no significance.

In order to prevent loss of focus of the beam it is desirable that during discharge of $C_t$ the potential of the time-sweep plates should be equally divided about earth. This is achieved by dividing symmetrically the discharge resistance $R_t$, as shown in Fig. 7. With the simple arrangement shown the condenser voltage under static conditions is not symmetrical, but symmetry does obtain on spark-over of the gap.

### (6) Beam-Withholding.

By reason of the fogging which would otherwise result some means must be provided to prevent the beam lingering on or near the recorder before and after time-sweeping. To this end a large number of arrangements have been developed, and the devices of Norinder,* Krug,† Rogowski,‡ Boekels,§ and the American workers,|| all have their respective advantages. The present authors preferred, however, to use a beam-trap system such as that described in Section (1), since this arrangement adds little complication to the equipment, is sufficiently rapid in action to allow of very fast phenomena being recorded, is capable of very complete trapping, and requires only a relatively simple operating circuit which is readily synchronized with the timing circuit.

The withholding chamber is designed to maintain the capacitance of the deflection plates at the lowest possible value, which minimizes the time-constant of the connected circuit and ensures rapid release of the beam. In connection with the arrangement of this electrode system a practice which is much favoured is the use of two pairs of oppositely-connected deflection plates.¶ In favour of this system are the facts that the beam may be arranged to emerge through the aperture when the withholding voltage has fallen to any desired value and that any small oscillations in the withholding circuit are not evidenced on the record. In the present instance, however, only one pair of deflection plates is employed, and with careful design of the associated circuits the want of the above advantages has never been apparent.

A circuit similar to that of Fig. 7, but of much smaller time-constant, is suited to controlling this type of withholding arrangement, and by connecting it across the spark-gap in parallel with the time-sweep circuit synchronization of action is automatically obtained. This connection, together with the time-sweeping circuit, is shown diagrammatically in Fig. 8, where $C_0$ and $R_0$ represent the capacitance of the withholding deflection plates and the damping resistance respectively, the other symbols being as in Fig. 7. In practice it is unnecessary to include any withholding capacitance other than that inherent in the withholding plates. The value of $R_0$ is just sufficient to prevent local oscillation, so that the time-constant $R_0C_0$ is only of the order of $10^{-8}$ sec. On this account the speed of release of the beam is controlled by the rate of development of the spark arc, and thus, where extremely rapid release is required, it has been found beneficial in obtaining the appropriate duration of time-sweep to employ a relatively

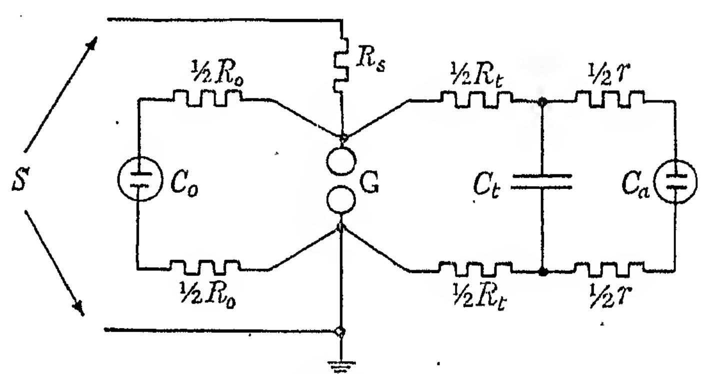



Fig. 8.—Schematic diagram of time-sweep and withholding circuits.

large value of $C_t$ and a low value of $R_t$ so as to give a hot spark. This procedure also has the advantage of minimizing coupling between circuits, and in the absence of any oscillatory component in the spark current the use of a separate heating condenser has been found unnecessary.

After completion of time-sweeping, when the spark is extinguished, it is necessary to effect withdrawal as rapidly as possible in order to prevent retracing of the beam across the recorder while $C_t$ slowly recharges through the high feeding-resistance $R_s$. The time required for this withdrawal may be investigated by considering the recovery voltage on the capacitance $C_0$. Neglecting the time-constant $rC_a$, the equation for this recovery voltage $e_0$ is:—

$$e_0 = S\left[1+\frac{1}{\phi}\left(\mu-\frac{2a}{\beta-\phi}\right)\epsilon^{-\frac{\beta-\phi}{2a}t}+\frac{1}{\phi}\left(-\mu+\frac{2a}{\beta+\phi}\right)\epsilon^{-\frac{\beta+\phi}{2a}t}\right]$$

where $a = (R_0R_t + R_0R_s + R_sR_t)C_0C_t$;

$\beta = C_0(R_s + R_0) + C_t(R_s + R_t)$;

$\mu = R_tC_t$;

$\phi = (\beta^2 - 4a)^{\frac{1}{2}}$;

and $S$ is the supply voltage.

Consequently the voltage recovers in two steps, this being shown by the presence of the two exponentials.

* See Bibliography, (15).
† Ibid., (16).
‡ Ibid., (17).
§ Ibid., (18).
|| See, for instance, Bibliography, (19) to (23).
¶ Ibid., (7).

As has already been mentioned, $R_s$ is large and so is generally much greater than $R_t$ and $R_0$, as also is $C_t$ than $C_0$, and hence the approximate recovery equation is

$$e_0 = S\left[1 - \left(1 - \frac{R_t}{R_s}\right)\epsilon^{-\frac{t}{R_s C_t}} - \frac{R_t}{R_s}\epsilon^{-\frac{t}{R_t C_0}}\right]$$

Thus the amplitude of the rapid-recovery term (the last) is so small that the effective withdrawal speed of the beam is determined by the time-constant $R_s C_t$. Accordingly, recovery of the voltage to 10 per cent occupies about $0 \cdot 1 R_s C_t$ second, which, by suitable choice of constants, withholding-plate deflection sensitivity, and size of aperture in the diaphragm, may be arranged to effect withdrawal in a time of the order of $10^{-4}$ second.*

As regards the disposition of these circuits, it is desirable that the connections should be short and direct. As shown in Fig. 2, the equipment is separated only a few inches from the oscillograph body and so the use of short leads follows naturally. Further, by careful arrangement of the connections, despite the close proximity of the various units, interaction is obviated.

## (7) The Measurement of Controlled Transients.

As has already been mentioned in Section (1), the measurement of voltage transients produced by impulse generators† introduces two distinct problems. The first is the synchronization of the oscillograph auxiliary circuits (i.e. the time-sweep and beam-withholding controls) with the impulse generator, and the second is the arrangement of suitable potentiometers as intermediaries between the test object and the deflection plates.

### (a) *Synchronization of the Impulse Generator with the Auxiliary Circuits.*

Essentially the first problem reduces to a consideration of whether the transient should initiate spark-over of the gap controlling the withholding and time-sweep circuits, or whether arbitrarily-controlled spark-over of this gap, besides commencing time-sweeping, should also trip the impulse-generator spark-gap and so initiate the transient.

* Several authorities have found it necessary to increase the speed of withdrawal by the use of auxiliary circuits. For instance, Rogowski (see Bibliography, 18) has employed a system of Faraday cages whereby the beam retraps itself. It would appear, however, that even with beam currents of the order of 10 microamps. withdrawal still takes $10^{-4}$ sec. Boekels (see Bibliography, 18) has also suggested the use of an auxiliary condenser circuit containing a triode whose grid voltage is neutralized when the condenser voltage has fallen to a predetermined value; the passage of anode current can then effect withdrawal either electromagnetically or mechanically by actuating a solenoid-operated shutter. The present authors once used an adaptation of Boekels's suggestion; they placed the grid of the triode across a resistance in the time-sweep circuit, and when its current fell to a predetermined value withdrawal was effected electrostatically. The need for such a device, however, does not arise when time-sweeping is arranged to carry the beam right off the recorder.

† It is desirable to draw attention to certain important factors regarding the various arrangements of impulse generators for transient research. For the purpose of impressing a predetermined transient on a test object, such as a bushing, a transformer, or insulation, its direct connection across the discharge resistance of the generator is usual, although in the case of transformers this arrangement probably does not impose such severe voltage stresses as does the discharge of the generator capacitance directly into the transformer. With some test objects, however, such as rotating machines, cables, and certain types of surge absorbers, it is important to investigate internal to-and-fro reflections caused by incident travelling waves; and on occasion it is important to examine external reflections. In such cases the impulse generator should simulate the surge-impedance characteristics of a transmission line. For some purposes sufficient approximation to this has been obtained by including a resistance equal to the desired surge impedance between the surge generator and the test object. Another arrangement, and one which very closely simulates a transmission line, is the use of a generator with constants adjusted to the critical state so that its response is resistive to any equivalent frequency (see Bibliography, 24). Since these arrangements do not set up travelling-wave conditions, for reasons apart altogether from the question of delay it is desirable, on occasion, to employ a length of actual transmission line.

In the former system it is necessary to delay the transient before it reaches the deflection plates, and this can be readily achieved by interposing a cable of suitable length between these and the potentiometer across the test object. In the latter system the necessary delay can also be obtained in this manner, or alternatively by delaying the arrival of the tripping pulse at the surge-generator spark-gap,* when only direct connection of the deflection plates across the potentiometer is necessary.

For the measurement of uncontrolled transients, other than the use of an oscillatory time motion, there is no practical alternative to the adoption of the first system, and it is widely employed with satisfactory results. On the other hand, when the transients are under control, as in laboratory work, both systems are applicable.

It is apparent, however, that the joint use of cable and capacitive or resistive dividers has some disadvantages not wholly shared by the corresponding simple dividers. For instance, the potentiometer-and-cable arrangement has characteristically† a lower impedance than the simple potentiometer, and under certain circumstances imposes excessive loading on the test object with consequent distortion. This is chiefly of importance in laboratory work for such purposes as the measurement of transient voltages between turns or coils of a transformer, where the local inherent capacitances may be very small. In such a case there is also the complication of duplicating the cable delay, while in the transmission of very abrupt impulses even short lengths of cable always introduce a small percentage of distortion. In view of the fact that the present equipment is designed for laboratory work, it is an obvious choice to employ the second system and arrange for the impulse generator to be tripped from the oscillograph auxiliary circuits, delay being obtained in the tripping circuit.‡

This circuit is shown in Fig. 9. The normal 2-electrode gap of the impulse generator (or the first inter-stage gap of a Marx generator) is replaced by a 3-electrode one with insulated middle sphere. In operation, with the generator supply voltage adjusted just below spark-over of the gap, the middle electrode takes up a mean potential and application to it of a small tripping voltage serves to initiate spark-over.

This tripping voltage is obtained from an additional capacitance discharge circuit $C_b R_b$ connected across the common spark-gap G of the time-sweeping and withholding circuits, and operation of the rotary switch, besides initiating these circuits, generates across $R_b$ the impulse fed to the third electrode of the impulse generator. Spark-over of the latter thus lags on the operation of the gap G with a delay determined by the circuit constants. The low value of the coupling condenser $C_k$ in conjunction with a high value of $R_2$ limits the back-voltage injected into the cable at breakdown of the 3-electrode gap to a few volts only, and this is readily absorbed at the sending end.

For some investigations a short double overhead line is employed. The time of a single traverse of a wave along

* See Bibliography, (25) and (26).

† Burch has recently demonstrated methods of considerably reducing the capacitance, and therefore the loading, of a capacitive-cable device without increasing the distortion (see Bibliography, 27).

‡ Some interesting discussions on cable delays and potentiometers have been published since the preparation of this paper (see *Transactions of the American I.E.E.*, 1933, vol. 52, pp. 555–567).

this line is 0·8 microsecond, and thus the arrival of the transient at the oscillograph is delayed by this amount on spark-over of the impulse generator. This is disadvantageous in very short-time investigations, when the total duration of time-sweep may only be of the same order, with the result that the transient appears towards the end of the trace on the record. This difficulty is overcome by arranging for the impulse generator to initiate the auxiliary circuits, the normal gap G being replaced by a 3-electrode one whose middle sphere is suitably fed through a coupling condenser or antenna from the generator terminal. (The arrangement, of course, is suitable for amphipolar tripping of uncontrolled transients.) This ensures that arrival of the transient at the oscillograph only lags behind the initiation of time-sweeping by the difference of the delays in the line and the tripping circuit, and as this latter is under control the difference may be made as small as desired.

Distortion which may arise in the overhead line is not of importance, since the transient measurements are made at the terminal and are therefore only in respect of the received wave-shape. Actually, however, the length of the line is often artificially made very great by the use of a generator tuned to give pure resistive response (equal to the line surge-impedance) to reflections returned from the terminal, and on this account the line distortion-factor is minimized in order that absorption of reflection may be as complete as possible.

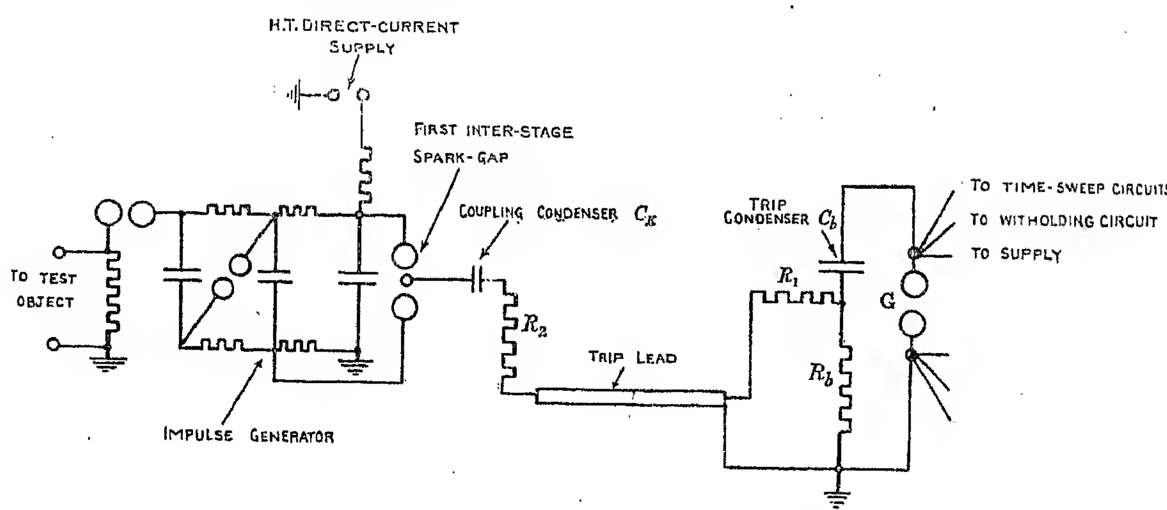



FIG. 9.—Schematic diagram of trip circuit.

### (b) *Potentiometers.*

In the case of resistance potentiometers the capacitive loading of the oscillograph deflection plates and leads imposes a minimum on the value of the resistance if distortion is to be avoided, and the consequent considerable and maintained loading of these potentiometers greatly restricts their application. Except, therefore, for such conditions as the measurement of impulse-generator wave-form, where voltage division is sometimes conveniently obtained by tapping the discharge resistor, resistance dividers are not employed. Further, at high voltages, where the dividers are of necessity long, inductive and earth-capacitance effects give rise to distortion with the faster transients even when the dividers take the form of liquid tube resistances.

The use of condenser potentiometers is therefore favoured, since they need only introduce a very small capacitive loading and are suitable for work involving measurement of transient voltages superimposed on power frequency. These dividers where convenient take the form of concentric cylinders surrounding the high-tension electrode, and are so proportioned that the outer earth-connected cylinder overlaps the middle electrode and effectively shields it from stray electric fields.

For measurement of inter-turn voltages on transformers and similar windings, two carefully-matched condenser potentiometers are employed, the voltage appearing across the deflection plates being the difference between turns reduced in the appropriate ratio. In such cases since the local earth and inter-turn capacitances are often very small, these may be very sensitive to additional loading, so that efforts are made to reduce as far as possible the potentiometer capacitances.

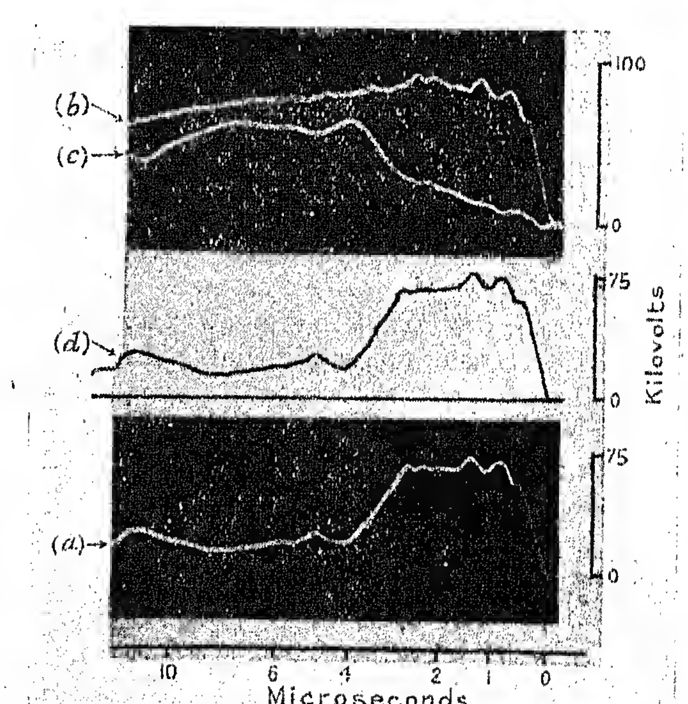



FIG. 10.

(a) Voltage appearing across 10 per cent of the line end-turns of a transformer winding.
(b) Voltage to earth at the transformer terminal.
(c) Voltage to earth at a point 10 per cent down the winding.
(d) Arithmetical difference of (b) and (c).

(a) taken using two condenser potentiometers,
(b) and (c) taken using one condenser potentiometer.

Various tests can be devised for estimating the error introduced. Thus measurements can be made with and

without an additional identical pair of potentiometers in circuit to estimate the extent of the effect of such added capacitances. Another test is made by comparing the arithmetical difference of separately-made oscillograms of the voltages at the two points with the record of the difference obtained electrically. In this connection Fig. 10 is an interesting oscillogram; (*a*) shows the voltage across 10 per cent of the line end-turns of a transformer using two potentiometers, while (*b*) and (*c*) respectively show the voltage to earth at the terminal and the voltage to earth at a point 10 per cent down the leg, using one potentiometer in each case. The arithmetical difference of (*b*) and (*c*) is drawn in (*d*), and superimposes satisfactorily on (*a*).

(8) ACKNOWLEDGMENTS.

In conclusion, the authors wish to accord thanks to Messrs. Ferranti, Ltd., for permission to publish this paper, and to Mr. M. Taylor of the High-Voltage Laboratory for assistance with many of the details of the mechanical construction. They desire to express their gratitude to Mr. L. G. Carpenter, of University College, Southampton, for co-operation in connection with certain of the technical problems involved.

## BIBLIOGRAPHY.

(1) M. KNOLL: *Zeitschrift für Technische Physik*, 1930, vol. 11, p. 491.
(2) M. KNOLL and B. VON BORRIES: *ibid.*, 1930, vol. 11, p. 493.
(3) W. ROGOWSKI, E. FLEGLER, and P. ROSENLÖCHER: *Archiv für Elektrotechnik*, 1929, vol. 23, p. 149.
(4) J. L. MILLER: *Electrician*, 1932, vol. 109, p. 805.
(5) G. I. FINCH and A. G. QUARRELL: *Proceedings of the Royal Society*, A, 1933, vol. 141, p. 398.
(6) M. KNOLL, H. KNOBLAUCH, and B. VON BORRIES: *Elektrotechnische Zeitschrift*, 1930, vol. 51, p. 966.
(7) F. P. BURCH and R. V. WHELPTON: *Journal I.E.E.*, 1932, vol. 71, p. 380.
(8) L. BINDER, FÖRSTER, and FRÜHAUF: *Zeitschrift für Technische Physik*, 1930, vol. 11, p. 379.
(9) H. BUSCH: *Archiv für Elektrotechnik*, 1927, vol. 18, p. 583.
(10) J. L. MILLER and J. E. L. ROBINSON: *Journal of Scientific Instruments*, 1932, vol. 9, p. 264.
(11) *Journal I.E.E.*, 1932, vol. 71, p. 78.
(12) *Ibid.*, 1932, vol. 71, p. 832.
(13) F. B. PIDDUCK: "Treatise on Electricity," 2nd ed., 1925, p. 587.
(14) J. T. MACGREGOR-MORRIS and R. MINES: *Journal I.E.E.*, 1925, vol. 63, p. 1056.
(15) H. NORINDER: *Zeitschrift für Physik*, 1930, vol. 63, p. 672.
(16) W. KRUG: *Elektrotechnische Zeitschrift*, 1930, vol. 51, p. 605.
(17) W. ROGOWSKI, E. FLEGLER, and R. TAMM: *Archiv für Elektrotechnik*, 1927, vol. 18, p. 513.
(18) H. BOEKELS: *ibid.*, 1931, vol. 25, p. 497.
(19) G. F. HARRINGTON and A. M. OPSAHL: *Electric Journal*, 1927, vol. 24, p. 384.
(20) J. J. TOROK and F. D. FIELDER: *ibid.*, 1929, vol. 26, p. 320.
(21) C. M. FOUST: *General Electric Review*, 1931, vol. 34, p. 235; also L. B. SNODDY and J. C. STREET, *ibid.*, 1931, vol. 34, p. 258.
(22) R. B. MCEACHRON and V. E. GOODWIN: *Transactions of the American I.E.E.*, 1929, vol. 48, p. 953.
(23) C. L. FORTESCUE, A. L. ATHERTON, and J. H. COX: *ibid.*, 1929, vol. 48, p. 449.
(24) E. W. BOEHNE: *ibid.*, 1930, vol. 49, p. 1587.
(25) K. GIROD: *Archiv für Elektrotechnik*, 1931, vol. 25, p. 695.
(26) H. BAATZ, M. FREUNDLICH, and W. HOLZER: *Elektrotechnische Zeitschrift*, 1932, vol. 53, p. 696.
(27) F. P. BURCH: *Philosophical Magazine*, 1932, vol. 13, p. 760.

# DISCUSSION BEFORE THE INSTITUTION, 4TH JANUARY, 1934, ON THE PAPERS BY DR. MILLER (SEE PAGE 473), AND DR. MILLER AND MR. ROBINSON (SEE PAGE 511) RESPECTIVELY.

**Mr. B. L. Goodlet:** Regarded from the purely mathematical point of view the first paper has several commendable features. It is in the first place thoroughly orderly. The author considers a general type of line transition point and deduces a general equation for the voltage he requires. All the various combinations which he considers are treated as special cases of the general equation. This general method of approach has in my opinion many advantages. Although special solutions can often be obtained by special tricks, the general method of approach is easier to remember and also more educative. The second good point of the paper is the very large number of numerical examples it contains. Personally when I have worked out a solution sufficiently far to see what happens in a general kind of way I publish it, and hope that someone else will do all the tedious arithmetical computations. The author on the contrary, has worked out some 30 sheets of curves with truly Germanic thoroughness. These curves will undoubtedly be very useful to less industrious writers. On the other hand I am bound to say that apart from the clarity imparted by these numerical examples the paper adds little to our knowledge of travelling waves. The paper is essentially an orderly presentation of existing knowledge, rather than a piece of pioneer work. No new problems are solved and no new mathematical methods are employed. Bewley in his book "Travelling Waves on Transmission Systems" covers the same ground as the author, and much more besides. Considering the paper from the practical point of view, I am inclined to think that the author has allowed his mathematics to run away with him. The function of mathematics in this kind of engineering research is not the prediction of what *will* occur but the quantitative explanation of what *does* occur. In other words, observation should precede or accompany calculation. Predictions and assumptions not checked by observation are frequently found to be incorrect. For this reason many of the author's conclusions regarding the practical applications of his results must be accepted with reserve pending

their confirmation by experiment. It is, for example, hard to believe that the complicated equipment of a substation reacts on the line like a simple condenser. I should anticipate internal reflections and oscillations to be apparent, and I do not think the American tests quoted on this point are conclusive.

The second paper is essentially the record of a piece of experimental research carried to a successful conclusion. The authors are not the first investigators to build a high-speed cathode-ray oscillograph in England—this, I think, was done by Burch and Whelpton—but their design nevertheless incorporates many features of interest. I congratulate them on their success with the non-metal discharge tube and with external photography. The authors appear to have worked only with controlled transients, and express a preference for this method in the laboratory while recognizing its limitations in the field. We, on the contrary, treat all transients as uncontrolled. The authors' method as indicated in Fig. 9 has one disadvantage, which in the case of a generator with many gaps might be serious. Their timing motion is initiated before, not after, the impulse has been propagated through the impulse generator itself. Their delay therefore includes this propagation time, which is variable with the settings of the impulse-generator spark-gaps. This uncertainty is avoided in our method, and the resulting accuracy of timing—even on a multi-gap generator—is very high indeed. The authors also seem to prefer condenser-type potentiometers. For voltages of the order of 100 kV, as employed in their experiments, these may be satisfactory. For higher voltages, of the order of 1 000 kV, any arrangement of concentric cylinders would be exceedingly large and awkward. The capacitance errors in resistance potentiometers are not serious in practice. I congratulate the authors on the admirable agreement obtained in Fig. 10, and should be glad if they would incorporate in their reply a diagram of the connections of the matched potentiometers to which they refer.

**Mr. R. A. Watson Watt:** The authors of the second paper are unduly kind to the glass-type cathode-ray oscillograph in saying that it "is eminently suitable for repeated phenomena up to the highest frequencies." The well-known sealed-off oscillograph in general use in this country contains gas, and that gas content is such that two very serious troubles arise in its application to high-frequency working. There is the familiar phenomenon of "origin distortion," and there is the much more serious problem of the defocusing of the spot, owing to the failure of the ionization mechanism which is depended on to maintain the focus in the gasfilled tube. In connection with focusing problems, I should like to ask the authors whether, when using the apparatus described in the paper, they do not find difficulty in focusing by the comparatively simple solenoid type of electron lens. This method has the inherent disadvantage of converting the original Cartesian co-ordinates into curved co-ordinates, and I should have thought that the distortion of the diagram might be sufficiently serious to impair the usefulness of the apparatus. I was very much surprised to hear Mr. Goodlet suggest that the mathematician should be a historian rather than a forecaster. Every engineering designer, whether he likes it or not, is compelled to predict the performance of his apparatus in conditions about which he may not know very much. My main text in this discussion is just this point of how far the difficulties faced by the authors and those who are dealing with the same kind of problem are due to the inability of the methods which have been used up to now to provide the data for forecasting the performance of new apparatus and equipment, in the conditions imposed by the incidence of lightning. In Section 3 of the first paper it is stated that the characteristic shapes of true lightning waves are known, and, although this is qualified on the previous page by the statement that "the analysis given later deals with the waves when they reach the circuit under examination," I feel that we shall soon be in difficulties because we do not know the characteristic wave of lightning. What we do know is the time-curve of variation of voltage at a particular pair of terminals after a wave released by the lightning has reached these terminals. Much of the data obtained from the American observations is robbed of its value because it has been filtered through the behaviour of the transmission-line system before reaching the measuring terminals. In my work on the effects of lightning-flashes on radio receivers I meet with the same essential difficulties as those faced by the authors and other workers, namely that the atmospherics reach me not directly from the lightning flash but after propagation through the "transmission network" of the ionosphere. There are obvious difficulties and discomforts about examining a lightning flash at its place of origin, but the trouble in the breakdown conditions with which the engineer has to deal is that the most damage is caused when the place of origin of the lightning flash happens to be a generating station or a substation. I suggest we must get to the place of origin of the lightning flash before we can submit the problem to the analysis which it requires. The observations will be a good deal easier to make now that we have a "pocket" oscillograph of the variety designed by the authors, which may be taken to places where we may reasonably hope to find a good supply of lightning. It is a matter of regret to many, including myself, that those responsible for the design of important distributing systems should appear to have worked on the basic assumption that there is no lightning in this country! I think that pressure of experience may lead to a modification of this assumption, and the work of the authors will facilitate the resulting fresh attack on the lightning problem.

**Prof. J. T. MacGregor-Morris:** In setting out the results obtained mathematically, and also experimentally, in these cathode-ray oscillograms, I would put forward a strong plea that in all cases the time scale should increase from left to right—as is the common practice in almost all graphs. I agree with Mr. Watson Watt that in the second paper the authors are too kind to the sealed-off oscillograph, and they omit to mention that the focusing fails. On a sinusoidal wave the focusing fails at a frequency of something like 200 000 to 500 000 cycles per sec., depending on the gas used in the tube. I should be glad to know the dimensions of the focusing coil shown in Fig. 1, and the ampere-turns that are required for a given voltage on the tube.

What is the relation between the ampere-turns in the coil and the exciting voltage to give correct focusing? I should like to congratulate the authors on the excellent results of their external photography. This is the first time, so far as I know, that such photography has been carried out in England. If their time scale were a uniform one, the grid arrangement shown in Fig. 3 might be used as the co-ordinates for making measurements, and it would be even more helpful in that way. Have the authors determined the relation between the photographic action and the current in the beam? Some years ago Dr. A. B. Wood* predicted a certain relation between chemical activity, photo-actinic activity, and beam intensity; perhaps the authors have figures by which they could check this theory. With regard to the last paragraph of Section (2), I should like to ask whether the authors have measurements of the pressure in the upper part of the tube, where the air leak is, and also in the lower part. If so, I think it would be well to give them, because there is a considerable amount of vagueness about this point even now in connection with the relative degrees of exhaustion in sealed-off tubes and in tubes working on pumps. I do not agree with the statement in the third paragraph of Section (3), to the effect that "The dependence on cathode assembly is well known." If the authors have some knowledge on this subject which they feel they can give, I should be glad if they would do so. I notice from a German publication that Malsch† has recently done a good deal of work which bears on this problem, and I think it would be well to add his name to the Bibliography given. I have tried to compare the efficiency of the tube of which the authors give details with that of the one referred to by Malsch. From what is stated in Section (3) of the paper it will be seen that there are 50 watts in the discharge and $1\frac{1}{4}$ watts in the beam; in other words, only $2\frac{1}{2}$ per cent of the energy which is being delivered to the tube gets down the beam and goes on to the photographic plate. It seems to me that if we understood the electro-optics of the tube thoroughly we should be able to get a higher efficiency than this. Working out the current density, I find that in the beam it is 260 microamperes per mm$^2$, whereas Malsch gets 20 000 microamperes per mm$^2$—about 80 times as much. Malsch's value was obtained at 40 kV; probably the authors' results were obtained at 50 kV, and, if so, the density of the beam might be higher. The last point to which I wish to refer is in connection with Fig. 6. A metal cathode shield is shown in the assembly, and it is stated in the paper that the shield is free to assume any potential. If the potential of that shield is important, it seems rather haphazard to allow it to vary. Will the authors state the potential of that shield, and also draw in Fig. 6 a few of the lines of electrostatic force and of the equipotential lines? This would materially assist in giving a true idea of the method of working of the shield.

**Mr. V. Z. de Ferranti:** It seems to me that the authors have before them a great opportunity to make electricity both cheaper and more reliable. I say cheaper, because the grid is based on the idea of making it possible to shut down unwanted plant; unless the grid is reliable we shall not be able to do this, and therefore the expected advantages will not be realized. It seems to me that if the engineers responsible for electricity supply will co-operate with manufacturers of apparatus and others who have studied this subject, we shall make very rapid progress towards a cheaper and more reliable supply.

**Prof. G. W. O. Howe:** About six months ago the Vienna journal *Elektrotechnik und Maschinenbau* published a jubilee number, and asked various authorities to contribute articles on special subjects. One of these was a review of recent progress in cathode-ray oscillograph work, by Prof. Rogowski.* I suggest that this article be added to the Bibliography of the second paper. It describes the great progress which is being made in oscillographs for high voltages and high-speed work, and in external photography. Dealing with the amount of light available for external photography, it is stated that if the spot of light be unfocused a 100 000-volt oscillograph tube will give a candle-power of over 100. With this instrument a photographic record of a transient can be obtained in which the spot moves across the screen with a speed exceeding one-third of the velocity of light. There is one point which I should like the authors of the second paper to explain. In the Introduction they say: "When the transient is controlled (that is, when it is generated at will in the laboratory) some method must be adopted whereby spark-over of the impulse generator is synchronized with the release of the beam and the commencement of time-sweeping. If the transient is uncontrolled (as in the case of lightning surges in a transmission line) it must be made to initiate release of the beam and time-sweeping." Surely, if the apparatus will work automatically when it is uncontrolled by a lightning flash, it would work equally well uncontrolled in the laboratory with an impulse generator? It would, of course, be very convenient to have an alternative to the automatic release when dealing with controlled transients, but I cannot see that it is essential.

**Lieut.-Col. A. G. Lee:** Mr. Watson Watt has put his finger on a weak spot in connection with the mathematics, namely that the mathematics and the oscillographs merely indicate the effect of the current in the line, given certain artificial attack conditions which are not related to lightning. The line conditions determine the current and voltage obtained, and these in turn depend on the form of the attack. I should be doubtful about the results of an artificial spark system of attack as compared with those obtainable from real lightning. When I was at Perth, in Scotland, a few months ago, I was informed that, in the summer, lightning occurs very frequently in the country between Perth and Inverness. I imagine, therefore, that the Grampian Power Co., which operates in that area, will have ample evidence regarding the effect of lightning, and could afford the authors every opportunity of trying out their oscillographs on real lightning. I have known of only one case of attack by lightning on wireless aerials and masts. This occurred at the Lyons station, France, where the mast was brought down by a 90-m.p.h. gale. I do not know of any case of lightning having attacked our large wireless masts in this country. My observa-

* *Journal I.E.E.*, 1932, vol. 71, p. 41.
† "Electron Current Density in Cathode-Ray Tubes," *Archiv für Elektrotechnik*, 1933, vol. 27, p. 642.

* *Elektrotechnik und Maschinenbau*, 1933, vol. 50, p. 249.

tions have led me to believe that the high masts, which are provided with good earthing systems, drain the atmosphere above of electricity, so that the chances of lightning occurring in their vicinity become smaller. When the black thunder clouds approach the station the lightning ceases, and after they have passed beyond it the lightning comes on again. It seems to me, therefore, that the efficient earthing of power-line masts is a very important factor which might be studied in this connection. In wireless work, copper bonding of the masts is quite usual practice, and especially across the concrete base to the earth plate. With regard to the effect of line plant in determining the form of current attack by lightning, I suggest that to reduce the effect of lightning at a power station or substation it might be desirable to convert the line into a low-pass filter. If air-core chokes or coils, such as are used in wireless work, were inserted at intervals along the line, these coils, coupled with the capacitance of the line, could form a low-pass filter and tend to cut off any steep wave-fronts which might result from the attack of lightning. Earthing arrangements would be necessary at each coil, so that if the line were struck by lightning a discharge could take place to earth at once. Isolating the attack in this way would ensure that a relatively small proportion of the line capacitance was concerned in the attack. The amount of energy to be subsequently dissipated in the resulting oscillations would then be reduced as compared with the case where the capacitance of the whole line is allowed to absorb energy from the lightning flash.

**Dr. E. H. Rayner:** The modern cathode-ray oscillograph, and especially the high-voltage oscillograph, has the valuable property that in an ordinary laboratory the incident to be photographed can be seen at a writing speed of the order of 200 km per sec. It can be repeated several times for visual inspection before the photograph is taken. The fact that the hit-or-miss methods in use some years ago need no longer be employed removes one of the serious disadvantages of internal photography, namely that of having to break and raise the vacuum every time a plate is exposed. With modern pumps, of course, this process is not nearly so troublesome as it used to be. At one time $1\frac{1}{2}$ hours had to elapse before a second photograph could be taken; now the time is 5 to 10 minutes, and with films one can take 6 exposures in a minute if necessary. I notice that the authors use only one vacuum pump, placed at the bottom of the container. A leak has to be arranged at the top to give the poor vacuum (of the order of $10^{-3}$ or $10^{-4}$ mm) for the discharge tube, whereas in the bottom part, where the beam is deflected, the best vacuum available is desirable. Certain brands of cathode-ray oscillographs have two exhausts, one to pump away the leak gas and the other to keep the vacuum in the bottom part as complete as possible. The authors, however, deliberately let the leaking gas pass into the main chamber; it would be interesting to know whether this course has any disadvantages in practice. I do not think they say anything about the relative merits of the use of plates and of films. The little experience which we have had shows that the slowest plates are the best; they are not so liable to fog, and they give blacker lines than ordinary commercial film. The authors mention the use of the saturated diode; Prof. Finch has made use of this very valuable arrangement for steadying the voltage applied to the top of the cathode-ray oscillograph, this being essential when recording quantitative values of the type he has measured recently. Nevertheless, the method has its disadvantages. It cuts down the voltage a good deal, and a resistance has been found preferable in some circumstances. When an a.c. rectified supply is used for generating the cathode beam the ripple in the supply has some effect on the quantitative value of the result, but for impulse work the accelerating voltage may be regarded as constant during the exposure. With regard to the focusing coil, one of the latest fashions is to encase this completely in a sheet-iron box, except for a small slit; I should like to know whether the authors have done this. Their focusing coil appears to be well above the half-way position, but in other instruments the coil is placed as far down as possible. I should like to mention the necessity for shielding all the circuits, and indeed the whole instrument; some information as to the necessity or desirability of this would be very useful. It is an important matter, because at high voltages a direct inductive action of the impulse generator on the whole system is met with. Do the authors use complete metallic shielding, is expanded metal adequate, and can non-magnetic metals be used for this purpose? They mention loss of focus due to using a sweeping system which is not evenly balanced as regards voltage to earth; we find that the voltage to earth of the sweeping system can be appreciably different on both sides without loss of focus. For their time-sweeping arrangement the authors have adopted, probably on account of their technical method, a scheme necessarily involving a spark-gap. Other methods have been developed which do not require spark-gaps, and they are distinctly promising. It would be of interest to know whether the authors have any experience of such methods.

**Mr. E. T. Norris:** Apart from its use in checking calculations, the high-speed cathode-ray oscillograph has a very important application in making those calculations possible in the first place, and that is in the determination of the values of the constants of the circuit. Although a mathematical study of the ordinary inductance coil at the end of a line will enable one to predict the effect of a coil of given inductance and resistance on a certain type of travelling wave, there are no means of knowing whether any particular choke coil has that particular inductance or that particular resistance under surge conditions. One can measure quite easily the 50-cycle inductance or the d.c. resistance, and one can measure fairly readily the resistance, inductance, and capacitance, under sustained high-frequency conditions; but I think it is impossible, or at any rate so difficult as to be impracticable, to calculate the values of these constants under surge conditions, because they all depend on the distribution of electrostatic or magnetic fields. As no mathematics has been invented so far which is well adapted to the study of field distribution, calculation is practically impossible. The cathode-ray oscillograph, however, enables one to measure those constants and to formulate a general relation showing how they vary from the simple d.c. or 50-cycle values, which can be experimentally determined for circuits of different kinds.

**Mr. P. F. Stritzl** (*communicated*): I should like to put forward some comments concerning the practice of excess-voltage protection, dealt with in the first paper. The author refers to one kind of protective apparatus, namely a choking coil placed in the run of a line. Fig. 41 illustrates the flattening of the wave-front produced by this apparatus, and shows that the peak voltage is only slightly reduced. Dr. R. Willheim* has proved beyond doubt that the essential factor endangering transformer windings is not the steepness of the wave-front, but the peak value of the voltage. Hence any device

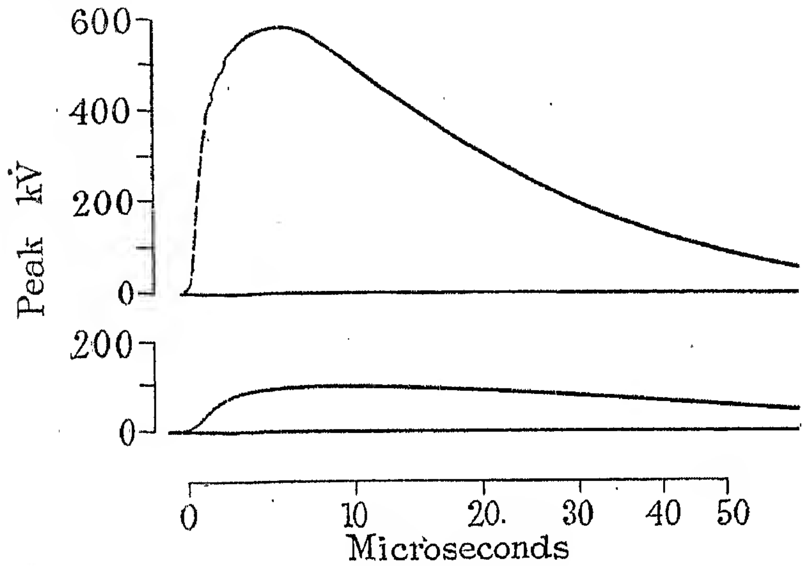



FIG. A.

merely aiming at the flattening of the front can necessarily only offer a very limited, if any, protective value. Effective protection from excess-voltage surges can only be achieved by a multi-spark-gap, in series with a suitable resistance, and connected between each phase and earth. Such devices have been known for many years, but experience with them has as a rule been most unsatisfactory owing to the fact that they permitted a number of half-cycles of operating current to follow the discharge of a surge to earth, thus often creating, instead of preventing, trouble and interruptions of service. In Germany and in the U.S.A. a type of arrestor was developed a few years ago which has proved very satisfactory in practice and may be considered to be a real solution of the surge problem. This arrestor consists of a resistance (whose ohmic value decreases inversely with about the third root of the voltage) in series with a multi-spark-gap so designed that uniform distribution of potential is maintained under any condition, and so perfectly enclosed that no moisture can enter the gaps. Fig. A shows a transient with a peak value of 600 kV, and also the same wave as reduced by an A.E.G. surge arrestor.

* *Elektrotechnik und Maschinenbau*, 1932, vol. 50, pp. 16 and 28.

**Mr. J. M. Thomson** (Canada) (*communicated*): The first paper shows the value of the Heaviside operator in reducing the work necessary to obtain a solution of transient conditions in a network. This is especially true for travelling waves, as it enables the worker to develop general solutions for circuits having up to three degrees of freedom. A general form can be developed for circuits of more than three degrees of freedom by using the Expansion Theorem given in equation (4A), page 508. In this case it is necessary to use the known circuit constants in order to obtain the solution. It is interesting to note the good agreement between the calculated and the measured values in Figs. 34, 35, 36, etc. These comparisons show that if the correct assumptions are made it is possible to obtain calculated results which are in close agreement with the measured results. I should like to point out that the formulæ presented can be used to determine the voltages developed in a short line with associated apparatus at each end. In this case the reflected waves will affect the results. It is only necessary to determine the time $t$ for the wave to travel to the end B of the line, be reflected, and travel back to end A. The reflected voltage $e_3$ at B is calculated by the methods outlined in the paper: it is then treated as an input wave at A, and its $e_2$ component obtained. The total transmitted wave at A is due to the $e_2$ component of the original wave plus the $e_2$ component of the reflected wave, which is $t$ microseconds out of phase with the original wave. With care, any reasonable number of reflections can be used in the calculations. It is only necessary to use the correct formulæ as developed in the paper and to shift the waves to allow for the time of travel from one end of the line to the other.

[The authors' reply to this discussion will be found on page 528.]

## MERSEY AND NORTH WALES (LIVERPOOL) CENTRE, AT LIVERPOOL, 8TH JANUARY, 1934.

**Prof. E. W. Marchant:** The first paper turns to practical use the work of Oliver Heaviside, who was the first to deal with the surge problem. It was he who originally defined the quantity we now call "surge impedance," a quantity which is beginning to be treated in a very practical and simple way. It is not unlikely that in a few years' time we shall be treating problems of surge impedance as we now treat problems involving Ohm's law. It is only in comparatively recent times that surge impedance has been fully worked out, and now that this has been done it is realized that the problem is not so difficult as it had been thought to be. This is clear from the simple nature of the equations given on page 479. One of the most valuable parts of the first paper is that in which the values of the effective capacitances and inductances of various kinds of machinery are determined. Take, for example, the transformer; it is quite a new idea to regard a transformer as an effective capacitance or a miniature transmission line. The figures for the effective capacitances of transformers of various sizes, given on page 483, will be of very considerable value. It is evident from these figures that the effective capacitance of small transformers is relatively greater than that of large transformers, and therefore that the effective capacitance of a group of small transformers will be greater than that of a single large transformer. A more advantageous result, as far as surge smoothing is concerned, is therefore obtained with a number of small transformers instead of a single large one. With regard to Fig. 22, it is evident from this

that when a limiting reactor is placed in front of a transformer a much greater surge voltage is produced than when the limiting reactor is absent. It is therefore desirable to make certain that the inductance of the limiting reactor is of such a value as to avoid trouble. Can the author give any information as to the approximate values of the limiting reactance which should be avoided, with transformers of various sizes? I should like to refer to one other point, namely the surge voltage that can be produced at the neutral end of a 3-phase transformer. The author mentions that the surge voltage at the neutral point is 2 to 3 times that entering from the line. Some years ago Mr. Paton, of the North Wales Power Co., told me that he had found it necessary to have the end turns on the earth side of his transformers reinforced, and this reinforcement of the insulation is now included in the appropriate British Standard Specification.

With regard to the second paper, there are two points to which I should like to refer. The authors employ a capacitance potentiometer, and this is obviously the only form of potentiometer that can be used without introducing too much distortion. Our experiments with capacitance potentiometers have shown that if they are calibrated with 50-cycle current they must have a very high insulation resistance, of the order of hundreds of megohms. I am interested in the authors' methods for taking photographs outside the oscillograph, thus avoiding the necessity of putting the plates inside the vacuum chamber.

**Mr. R. V. Whelpton:** During the discussion on a paper entitled "The Technique of the High-Speed Cathode-Ray Oscillograph,"* read by the late Mr. F. P. Burch and myself, Dr. Miller emphasized the difficulties of overcoming the various types of distortion likely to occur in a high-speed oscillogram. In the two papers now presented we have ample proof that he has overcome these troubles. I shall confine my remarks to the design of the cathode-ray oscillograph, dealt with in the second paper. The portability and compactness of the apparatus are to be commended. The actual oscillograph has been shortened by means of three expedients. First, the focusing coil has not been placed midway between anode and film surface, a condition demanded by the Busch formulæ for the sharpest focus when using a beam of large angular aperture. Such a beam is necessary for the very highest writing speed, and it is interesting that the authors can use a focusing coil very much nearer to the anode than to the film surface. They probably had in mind a reduction of the overall length of the oscillograph when they decided to use a single-stage beam trap. I have never made a thorough trial of such a beam trap, but have usually employed a 2-stage trap with deflection plates cross-connected, partly because of the completeness of trapping of stray electrons but mainly because a small residual voltage on the plates gives no resultant deflection and thus minimizes beam-trap oscillations, which are a very common complaint in high-speed oscillography. It would be interesting to see a high-speed oscillogram, with full-scale deflection of, say, $10^{-6}$ sec., taken by the authors and showing the start of the record immediately after the beam is de-trapped. The size of the insulators shown in Fig. 2 gives the impression that voltages of the order of several kilovolts are applied to the transient deflection-plates. This can effect a reduction in the length of the instrument at the expense of reduced sensitivity, but we have found it convenient to apply about 500 volts to the voltage plates, giving a 1-in. deflection on the film. This low voltage is very suitable for local experiments, such as those connected with travelling waves on cables. Should the authors apply asymmetrical voltages of several kilovolts to the transient plates, it might be better to put the transient plates above the time plates. The latter are symmetrically electrified during the time-sweep, and a large movement of the electron beam perpendicular to their lines of force will not cause the same distortion as will a large sweep parallel to the asymmetrically-electrified transient plates. The authors have taken considerable pains to develop a method of external photography and have photographed transients of fair writing speed, but I doubt whether the method of pressing a photographic plate against a thin transparent window coated with fluorescent material is the best. Dodds* has used a wide-aperture lens and a camera to photograph the fluorescent screen, and has taken excellent records of 1-metre oscillations, the writing speed being 42 000 km per sec. on the screen. He used a metal discharge tube and a cathode voltage of about 90 kV, passing several milliamps. I doubt whether the authors' method of external photography would give such good results, even at this power. Their device appears to require a skill in assembly which is not warranted by the results illustrated in Fig. 4. External photography of any type is perhaps an unnecessary refinement. The authors themselves use a large-bore tap between the pumping plant and the oscillograph, and high pumping speeds, so that to change the film inside the vacuum is only a few minutes' work. My colleagues and I have taken some 3 500 oscillograms, by the electron blackening method, without experiencing sufficient inconvenience to cause us to think seriously about external photography. The authors are to be commended on their researches on discharge tubes. The idea of giving the cathode a vertical adjustment is very useful. Has it been found necessary to alter the cathode-anode distance as the discharge current is varied? We have shared their experience that there is a certain minimum distance between cathode and anode necessary for a stable discharge, and that it is important to get the correct ratio of this distance to the discharge-tube diameter. Thermal instability is of small consequence in a metal discharge tube.

**Mr. J. O. Knowles:** I should like to emphasize the desirability of associating mathematical theory with engineering in continuous stages, namely from pure mathematics to applied mathematics, from engineering theory to formulæ, and from formulæ to commercial application. I hope that as a result of the reading and publication of the first paper the mathematical treatment of transient wave-forms as the difference between two or three exponential functions will add practical interest to the studies of those who have still the opportunity to learn their mathematics as the mathematics of electrical

* *Journal I.E.E.*, 1932, vol. 71, p. 380.

* J. M. DODDS: *Archiv für Elektrotechnik*, 1933, vol. 27, p. 531.

theory. I also hope that the paper will help others to investigate more successfully some of the perplexing problems associated with arc extinction in switchgear, where changes of potential occurring in a few microseconds are obviously to be studied much more closely in the future. I suggest that the order in which the paper is presented might be altered for the benefit of those who can rarely obtain more than an hour's uninterrupted concentration in the course of their working day. Would it not be desirable to segregate entirely the paragraphs dealing with circuits at the end of a line from those dealing with the junctions of two or more lines, and would it not be clearer to plot on one graph first the transmitted voltage-waves corresponding to an infinite rectangular wave for various combinations of capacitance, inductance, and resistance; then to compare the transmitted waves corresponding to initial waveform No. 2, and so on. The Table of Contents might indicate the salient types of alterations to initial waveforms which are caused in the circuits described. In conclusion, I should like to ask whether the author has studied the breakdown of insulation in microseconds by means of the cathode-ray oscillograph—a subject which might be productive of much useful information. If he knows of published research work already carried out on these lines, I should be glad of any information in regard to it.

**Mr. A. K. Nuttall:** With regard to the first paper, the most striking point is the very close agreement between the effects of the test wave on the various circuits, firstly as computed by operational analysis, and secondly as shown experimentally by means of the author's cathode-ray oscillograph. The power of the operational method has been established for some time, and this paper should do much towards reconciling many engineers, hitherto mistrustful of it, to the validity of results obtained by this means of attack. One point in connection with the oscillograms shown in the paper appears to require a little explanation. Figs. 32(*a*), 36(*a*), 37(*a*), and 37(*c*), show discontinuities of slope of the voltage wave in all cases at instants roughly corresponding to 1 and 2 microseconds respectively after the zero point of the oscillogram. The nature of these discontinuities suggests that they are due to reflections of some kind, and their persistence indicates that they all arise from some feature inherent in the oscillograph circuit. It would be interesting to know the cause of these discontinuities.

Turning to the second paper, it appears that the cathode shield is free to assume its own potential. It is possible that the potential thus assumed represents the optimum value for operation of the discharge tube. It would be interesting to know whether the authors have measured this potential, or have determined the effect of controlling it. I notice that the photographic system employs plates; has any use been made of roll film, and, if so, what are its disadvantages as compared with plates?

**Dr. J. C. Prescott:** The first paper emphasizes the simplification which can be effected in the treatment of transmission-line problems by the use of the operational calculus. This, by introducing limiting conditions at an early stage in the solution, makes it possible to solve comparatively briefly the problems which when treated by formal mathematics would lead to very cumbersome equations. I think, however, that the paper would have been clearer if the equations had been stated in the formal manner in the Appendix, and the operational equivalents had been derived from them. On page 479 it is stated that "$Z_2$ can represent the equivalent impedance of any number of outgoing lines, so long as there is no series impedance in them." Am I correct in my assumption that this "series impedance" implies concentrated impedance? With regard to the discussion of the voltage stress between turns (page 480, col. 2), it would seem that an infinite rectangular wave could set up this stress between two points of the same turn.

**Mr. F. W. Taylor:** The study of transient phenomena, whether from the purely mathematical standpoint or from the research or experimental side, is of extreme importance to engineers to-day, and particularly to the designers of heavy engineering equipment. At one time, as the authors mention, the cause of a large number of breakdowns on system apparatus was either unknown or put down frequently to faulty equipment. Although the study of transient phenomena has only claimed the attention of engineers for a short time, quite a large percentage of these breakdowns is definitely attributable to lightning and associated effects. Whilst a specialist branch of engineers is necessary to study the problem fully, the designer is fortunate in that another factor controlling the behaviour of insulation in service has been discovered and is being analysed. Dealing with the first paper, it is interesting to learn that more and more complicated circuits are being interpreted mathematically, enabling us to foretell the results of the imposition of transient phenomena on apparatus involving these circuits. Figs. 32 to 41, and the slides shown by the authors, speak well for their success in this direction. I am particularly struck by the oscillograms shown in these figures. The absence of fog and the clearness of the line throughout its length distinguish them from some of the oscillograms I have seen in various other engineering publications.

Turning to the second paper, the authors' oscillograph seems to be a fairly simple and straightforward piece of apparatus. Even if one is successful in constructing an instrument which will trace a line on a photographic plate, however, a considerable amount of research work and experience is necessary before engineers will have as much faith in the results which it gives as in the results given by the Duddell type of oscillograph. It is stated in the Introduction that the sealed-off type of tube is not suitable for the measurement of transient phenomena because the trace is insufficient. Considering the great advantage of portability possessed by this type of tube, I think that if this were the only difficulty it would not be long before an intensifying screen (such as is used in X-ray radiography) was developed to make possible a photographic record on a highly sensitive plate. It is also mentioned in the Introduction that to produce the necessary electrons a high voltage is required between the cathode and the anode. I gather that this means a d.c. voltage, presumably obtained by rectifying high-voltage alternating current. It is admittedly a very simple matter to smooth out the ripple when the load is only 1 milliamp., the figure given on page 514,

but it would be interesting to know just how important smoothing is, and what would be the effect of the lack of it on the record. On page 513 the authors mention the necessity of maintaining a higher vacuum in the main body than in the discharge tube: perhaps they could give the reason for this. In Fig. 6 it appears that the length of the air path outside the glass tube is short compared with that in the vacuum. I should like to know whether the authors have experienced any trouble, when trying out the various electrodes from the optimum writing-spot point of view, due to corona discharges outside the glass tube. Have they ever detected the production of X-rays, and, if so, what means have they used to protect the operators from harmful effects? On page 515 the authors discuss the deflecting plates and the considerations affecting their width. It is clear from the paper that the longer the plate the greater will be the sensitivity of the instrument, because the beam will have the deflecting force applied to it for a longer time. This element of time, which must be short compared with that of the transient, will no doubt provide one limit to the writing speed of the oscillograph.

My final point concerns the fact that the field at the edges of the plates is different from that in the middle. Whilst the plates are purposely made wide enough to embrace the beam throughout its deflection, what is the effect of the distorted field at the top and bottom edges?

[The authors' reply to this discussion will be found on page 528.]

## North-Eastern Centre, at Newcastle, 26th February, 1934.

**Prof. W. M. Thornton:** The first paper is the most complete that I know on the subject, having regard to the American and Continental literature. Figs. 32 to 41 are the finest results I have seen in connection with microtime oscillography. I cannot imagine better agreement between theory and practice than that shown by these curves. With regard to the Appendix, I take it that if one is willing to accept equation (1A) then all the rest follows. A student of mine, Mr. Crawford, has recently verified all the equations in the Appendix.

With regard to the second paper, I said many years ago that every works would one day have to have an oscillograph, and it seems now that all will have to have a cathode-ray oscillograph of the powerful type described by the authors. Nowadays engineers demand impulse tests, and there is no more certain way to gauge the efficiency of such tests than by an oscillograph. I think manufacturers will have to regard the rather high cost of such an instrument as a necessary outlay. We are installing a Finch cathode-ray oscillograph, of the same general type as that of the authors, and the information given in the paper will be most useful to us.

**Mr. J. A. Harle:** The external photography shown by the authors is somewhat on the lines of a scheme developed by Dr. Knoll in Germany, but in his case, while adopting a similar arrangement of metal supporting-bars, he used a cellon window over the grid. This will, I think, be easier to work with than the glass used by the authors. It is known, I believe, as a Lenard window, although the original Lenard window consisted of a thin sheet of foil. The scheme of coating the inside of the glass with a fluorescent material, while satisfactory for recording surges of the nature shown in the paper, may present difficulties if recurrent variable surges are to be recorded; I should appreciate information as to the time-lag before the fluorescence disappears. I should also like to know the nature of the cement joint in the discharge tube shown in Fig. 6. The use of the tap in the vacuum circuit appears to be a very sound scheme if the tap itself proves mechanically satisfactory in service. Has this method been free from trouble? The stabilizing resistance in the anode circuit is certainly much simpler than the saturated diode, but can the authors give some idea as to the relative smoothing obtained by the two schemes under the same conditions of stabilizing condenser? In view of the possibility of the combined characteristic of the saturated diode and the tube giving rise to instability, the resistance scheme would appear to possess advantages in the event of more than one oscillograph having to be operated from a common d.c. source. It is interesting to note that the authors tend to favour capacitance potential-dividers; but, when it can be used in conjunction with a length of cable and balancing resistances, etc., the resistance divider appears to be a useful device if it is desired to record the surge some distance away from the test point.

**Mr. P. J. Ryle:** At the top of page 475 of the first paper it is stated that a 132-kV line in a certain district might be expected to experience 3 surges per annum of the order of 1 000 kV. Too frequently, estimates of the reliability of transmission-line operation are given in which either the unit length of line or the unit of operation time is omitted, and this is a case where the length of line taken as the basis should be added; for instance, the statement should read "3 surges of the order of 1 000 kV per 100 miles per annum" (or whatever the mileage should be). At the top of page 481 the author refers to the series and earth capacitances of transformers. It would be of general interest if he could append a diagram illustrating qualitatively the physical nature of series capacitance in a transformer winding. Could he give approximate figures for the series and earth capacitances of, say, a 60 000-kVA 132-kV transformer? At the bottom of page 481 the author estimates that earth capacitances of the order of 3 000 micro-microfarads begin to be effective in flattening the surge wave. It may be pointed out that the capacitance to earth of 1 000 ft. of line conductor may be of this order, which tends to substantiate the accepted idea that surges originating, say, a mile out from the substation, will be very appreciably flattened before they reach the substation. It has been sometimes suggested that the addition of extra earth conductors on transmission lines near a substation would be of value. It is conceivable that the use of, say, three earth conductors instead of one for the first mile would quite appreciably increase the capacitance to earth of the line conductors and hence their surge-reduction efficiency. I should be glad of the author's views on this subject. I can find no reference in either the paper

or the Bibliography to Mr. L. C. Grant, the inventor of the surge absorber.

**Mr. W. D. Horsley:** The Introduction to the first paper states that there appears to be a great deal of conflicting evidence as to the relative magnitude and frequency of occurrence of direct and induced lightning strokes on high-tension overhead lines, but that engineers are practically agreed that the direct stroke is the more important. This is confirmed by a recent paper* describing an important investigation which was carried out in South Africa. The investigation was very complete and the conclusions were very definite. In the system on which the observations were made, lower-voltage lines (40 kV) run parallel with the 132-kV lines, and the difference in behaviour between these lines showed that induction is the cause of very few faults. This paper would be well worth adding to the Bibliography. Investigations both in this country and in America have shown that a surge applied to the primary winding of a transformer is transmitted to the secondary winding. It has been found in a number of experiments that the maximum voltage on the secondary winding is much greater than that calculated from the transformer ratio. A surge with a steep wave-front will probably be initially transmitted electrostatically from one winding to the other. If the primary and secondary windings are concentric, with the high-tension windings surrounding the low-tension secondary winding, then the value of the surge voltage on the latter will depend upon the relative capacitance between the primary and secondary windings, and between the secondary windings and the core. The capacitances in turn will depend upon the thickness of insulation between the primary and secondary windings, and between the secondary winding and the core. The thickness of insulation will naturally bear some relation to the voltage ratio of the transformer; it is probable, however, that in many designs the insulation between the secondary winding and the core will be thicker than is actually required to withstand the voltage. The ratio of the capacitances will thus be less than the voltage ratio, and the surge voltage on the secondary will be proportionately higher than that on the primary winding. In illustration of the paper the author showed an oscillograph record of the surge voltage on the secondary winding of a transformer when a surge of the standard wave-form shown in Fig. 32 is applied to the primary winding. The maximum voltage on the secondary reached approximately 6 kV, and it would be of interest to know the transformer ratio, seeing that the author later mentioned that this voltage was nearly 2½ times that which would be calculated from the transformer turns ratio. It seems clear from these results that apparatus on the low-tension side of transformers connected to high-voltage transmission lines is subjected to surges which are relatively greater than the surges on the lines. Breakdowns in rotating machines due to these causes are rare, and experience therefore shows that rotating machines are quite as resistant to surges as transformers, if not more so. This conclusion is not in agreement with the statement made by the author in the last paragraph of Section (5), and it would be of interest to have his views upon it.

* E. F. Rendell and H. D. Gaff: "An Analysis of the Lightning Faulting Characteristics of the 132-kV Lines of the Victoria Falls and Transvaal Power Co., Ltd.," *Transactions of the South African Institute of Electrical Engineers*, 1933, vol. 24, p. 258.

**Mr. G. D. Clothier:** With regard to the first paper, the author showed some extremely interesting slides illustrating the effect of series reactors and surge absorbers upon the wave-front of a surge generated under laboratory conditions. He pointed out how an absorber slopes off the wave-front, and so reduces the puncture stress upon the end turns of the transformer windings. As a result of, and perhaps as a part of, extensive investigation in other countries on actual working installations, the principle of discharging the surge energy to earth has become widely used at transforming stations to protect against surges set up by lightning. This type of protection obviously will not reduce the steepness of the initial part of the wave, but it is intended to cut off the wave and discharge it to earth before it can cause damage to other insulation nearby. The speed with which such a discharge can reduce the voltage of the surge was demonstrated by one of the author's slides showing the complete collapse, with subsequent smaller oscillations, of the surge when discharging to earth over an insulator. There will be a certain time-delay which will allow the surge voltage to exceed the 50-cycle flash-over value of the discharger by a fairly considerable degree, but is it not right to say that it will take considerably longer for the end turns of the windings, or the phase-to-earth insulation of the system, to fail? This can be expected, partly because the 50-cycle flash-over value of these is much greater than that of the discharger, and partly because the inherent time characteristics of the discharger are, by design, less than those of even an ordinary gap across an insulator, and considerably less than is required for puncture of solid insulation. For this reason, some information (and if possible an illustration) showing the effect of the discharger upon the form and magnitude of the surge would add much to the value of this part of the paper. In the discussion upon the recent paper by Mr. C. W. Marshall, Mr. R. W. L. Harris* referred to a device that is used in France which combines the function of a surge absorber with that of a discharger to earth. Perhaps some such combination will have the merits of both types.

**Mr. H. V. Field:** The author stated, in the course of his remarks relative to Fig. 1, that steep-fronted waves of types (i) and (ii) tend to become like type (iii) waves as they progress along the line, the steepness of the wave-front being reduced owing to the effect of corona on the high-voltage portions of the wave, which produces a reduced transmission speed as compared with the low-voltage portions. I presume that this effect is mainly due to the increase of capacitance caused by the corona effect, without any corresponding change in the inductance; otherwise there could be no change in velocity or wave-form. In connection with the focusing of the electron beam, is any appreciable assistance obtained from ionization along the beam path as in low-voltage tubes, or is focusing almost wholly dependent on the focusing-coil current?

[The authors' reply to this discussion will be found on page 528.]

* See page 137.

## Dundee Sub-Centre, at Dundee, 8th March, 1934.

**Mr. W. Woodiwiss:** The authors of the second paper are to be congratulated on their ingenuity in perfecting the cathode-ray oscillograph so as to give graphs of phenomena occurring in 0·000005 sec. The agreement between the actual records and the calculated graphs shows how very thorough they have been in their investigations. I should be interested to know whether the authors have used their oscillograph to check the performance of radio receivers. The reproduction of radio receivers has not yet reached perfection; there is something lacking in the quality. It is well known that a musical note is composed of a fundamental and harmonics, and that it is the harmonics which give a note its character. Thus the ear is able to discriminate between the middle C notes produced by the piano, violin, and organ, simply because, although for each instrument the fundamental is the same, the harmonics are different in degree and intensity. I should like to know whether the authors have used their oscillograph to take the wave-form of, say, the piano top C, directly from the piano, and the wave-form of this same note when reproduced through a radio receiver after being broadcast; if so, how did they arrange for the sound waves to operate the oscillograph so that the conversion apparatus did not suppress or introduce harmonics? Did they find a great difference between the "real" wave-form and the "reproduced" wave-form? The authors give some distribution engineers cause for anxiety by showing what enormous stresses may be imposed on the insulation of a transformer winding when the transformer is joined to a long overhead transmission line and a lightning storm prevails. I suppose that the authors have designed filters for reducing these stresses. Could they give some indication as to the specification that a distribution engineer should prepare prior to purchasing a filter for limiting the stress on his transformer to the safe limits? What installation test would the authors recommend for such a filter?

**Mr. D. H. Bishop:** It is extremely interesting, although rather disconcerting to a distribution engineer, to see oscillograms showing to what heights voltage surges on an ordinary system may rise and also how often they may occur, even though their presence may be entirely unsuspected owing to the absence of external indications of damage. It is rather much, however, to expect that they can be often repeated without eventually causing trouble. What length of underground cable is it necessary to interpose between an overhead line and a transformer in order to reduce voltage surges to a harmless amount? It would be interesting to know whether surges are transmitted from the high-tension winding of a transformer to the low-tension winding by electrostatic or electromagnetic means. In other words, does the turns ratio of the transformer have any influence? Is it practically correct to assume that all dangerous surges are due to lightning, and that no others need be feared on 6 600-volt underground systems?

**Mr. W. M. Mackay:** Apparatus for connection to transmission lines has been designed chiefly with reference to operation at low frequency. Its characteristics as regards high frequency were initially largely a matter of accident. As the existence of special risk due to oscillatory conditions and surges has become more fully recognized, modifications suggested by theory and supported by empirical results have been introduced with success. While measurement of the quantities concerned remained crude and uncertain, however, there was no finality about the results. The papers well mark the great advance which has been made possible in the understanding of transient conditions owing to the development of a recording device of the necessary range in voltage and speed. I should like to ask whether the oscillatory voltage produced in the transformer secondary shown on the lantern slide during the reading of the first paper is considered to have been transferred from primary to secondary by electrostatic or by electromagnetic means. Also, in view of the fact that increasing the end-turn insulation decreases the inter-turn capacitance of transformers and so aggravates the voltage risk under surge conditions, does the author regard the provisions of B.S.S. No. 422—1931 (Transformer Inter-Turn Insulation) as satisfactory from this point of view?

**Mr. G. F. Moore:** With regard to the time-sweeping circuit, is it necessary, in view of the small intervals of time which are being dealt with, to measure the capacitance of the condenser and its associated wiring *in situ*, in order to determine the intervals of time which form the abscissæ of the oscillograms? Further, how can these small intervals be checked?

## The Authors' Reply to the Discussions at London, Liverpool, Newcastle, and Dundee.

**Dr. J. L. Miller** and **Mr. J. E. L. Robinson** (*in reply*): We are very gratified with the reception accorded the two papers and we are particularly pleased to note how many and diverse are the fields of electrical science represented by those who have participated in the discussion. This evidence of widespread interest in a difficult but important subject is in itself sufficient justification for the production of the papers and we hope, as Mr. de Ferranti has stated, that by bringing about a greater degree of co-operation between academicians, manufacturers, and users, this general interest will expedite progress towards the achievement of a cheap and absolutely reliable electricity supply throughout the country.

Although the risk of lightning damage is discounted by many, and while we in this country have few dangerous storms compared with certain other countries, nevertheless even here lightning is, as Mr. Watson Watt has also pointed out, sufficiently potent a factor to deserve consideration in the design of transmission equipment, even if only in such instances as tower footing resistance and single-circuit versus double-circuit lines.

The intended purpose of the first paper, apart from demonstrating the formation of tractable equations,

was to make available—with a backing of experimental evidence sufficient to demonstrate their premises—some of our own experience and conclusions on high-voltage-transient phenomena, and generally to contrast and compare them with those of other workers. Opinions on many aspects of the problems discussed are diverse and, on occasion, we have found ourselves in disagreement with some of our contemporaries, but in general the discussion has not presented any serious challenge to our views. Two other interesting aspects of the discussion are the emphasis laid by many speakers, including Prof. Marchant, Mr. Taylor, Dr. Prescott, Mr. Nuttall, Prof. Thornton, and Mr. Thomson, on the agreement which can be obtained in work of this nature between observed and mathematically-formulated results, and the general appreciation of the simplification resulting from the employment of Heavisidian mathematics. Such agreement should, as one of the speakers suggested, go a long way towards reconciling those engineers who are a little mistrustful both of the high-speed oscillograph and of operational mathematics. It is this harmony between theory and practice which is the measure of the confidence that can be placed in prediction of performance, and, with Mr. Watson Watt, we realize that prediction is an essential weapon in the engineer's mathematical armoury.

Mr. Goodlet appears to have less use for mathematics, and therefore, so far as his opinion restricts its use to the more menial task of interpolation of knowledge, we cannot agree with his attitude. He comments on the orderly presentation and clarity of the first paper, but his subsequent remarks on the absence of new mathematical methods and the general trend of his criticism lead us to infer that he has overlooked the expressed purpose of this paper. He also makes reference to L. V. Bewley's book; while his further remarks, as we can only construe them, are dismissed by our comments near the beginning of this reply, we would also remind him that the book became available only after the submission of the paper. Actually, this book comprises the substance of Bewley's own admirable papers and those of his associates, and we have made full acknowledgment of their work. Mr. Goodlet will know, of course, that much of Bewley's work on transmission lines, as he himself implies in his introduction to the book, has also been covered by Continental workers. Bewley, however, by applying Heaviside's methods to single finite wave problems, has made both great advances in technique and contributions to mathematical discovery. Mr. Goodlet's statement that "the author has allowed his mathematics to run away with him" merely reflects his reactionary attitude towards the employment of mathematics, which has been commented on by another speaker and to which we have already referred. In any case, this remark might be regarded as a partial contradiction of his earlier statement regarding our lack of originality. In his reference to the second paper we feel that Mr. Goodlet has quite inadvertently created a wrong impression. It is desirable to point out that the construction of the oscillograph described in the paper was contemporary with, and not subsequent to, that described by the late Mr. F. P. Burch and Mr. R. V. Whelpton.*

* See Bibliography, (4) and (7), of the second paper.

On the question of "controlled versus uncontrolled" transients, also referred to by Prof. Howe, the object of treating our transients as "controlled" is, as is explained in Section 7(*a*) of the second paper, to avoid the greater loading of a cable delay. We find that the disadvantage cited by Mr. Goodlet does not arise in practice, and an examination of Fig. 5 will indicate the certainty of tripping.

We agree that there is a great deal to be said for resistance potentiometers, but like the capacitance potentiometers they need considerable space or careful screening if distortion is to be avoided. Actually, we have not used resistance potentiometers above 200 kV, but have employed capacitance potentiometers up to very high voltages both for straightforward impulse measurements and for experiments on energized transformers, where ordinary resistance potentiometers cannot be employed. We are, however, interested in Mr. Goodlet's remark that capacitance errors are not serious in the resistance potentiometer, and we are gratified by his congratulations on the agreement shown in Fig. 10. Two identical potentiometers with the usual damping resistances were used, the oscillograph plates being connected across the intermediate electrodes. The balancing has, of course, to be checked under a variety of conditions.

We agree with the remarks of Mr. Watson Watt and Prof. MacGregor-Morris as to the shortcomings of the sealed-off oscillograph. Regarding the former's remark on the distortion of Cartesian co-ordinates, in practice the amount of bending due to the focusing coil is slight.

In regard to the characteristic wave-shape of lightning, we agree with Mr. Watson Watt that the most damage is caused when a direct stroke occurs on transmission apparatus, and so far no one has been able to say what the voltage is or how rapid is its rise under these conditions; in fact, the whole phenomenon is still nebulous and probably the only reliable data available are the approximate current values. It was for this reason that it was only possible in the paper to consider disturbances which had actually entered the system, and while they most certainly cover a large number of important cases it is very patent that complete investigations of the conditions existing in the lightning stroke itself are necessary. There is no doubt that the failure of many discharge-type lightning arrestors is due to the incidence of a direct stroke.

We agree with Prof. MacGregor-Morris that oscillograms should be standardized as regards direction. The focusing coil was built up in sections for various reasons, and the size of the section now used is 2 in. long by 6 in. outside diameter. Fig. B shows an experimental relationship between cathode volts and ampere-turns in the coil to give correct focusing. While we have certain views on the effect of beam intensity on photographic action obtained from observations when experimenting on discharge tubes, we have never carried out a full investigation and therefore cannot give experimental verification of Dr. Wood's theories. Regarding vacuum measurements in the oscillograph, we find that the pressure in the discharge tube is of the order of $10^{-2}$ mm. We have, however, never made absolute measurements of pressure in the lower part of the

oscillograph. The pressure here must not exceed that at which the greatest deflection field produces ionization. There is no unique critical pressure at which this condition obtains; it is controlled by the disposition of the electrode system. On this score, therefore, it is only important that the pressure should be below this critical value. In our case it is much lower than this, and approaches the limiting pressure of the vacuum system. Prof. MacGregor Morris refers to Malsch's recent paper, which we had not seen until the second paper was in final form. In reply we would say that in calculating the figure of 260 microamperes per $mm^2$ Prof. MacGregor-Morris assumes that the beam is homogeneous over the whole cross-section of the anode aperture. This is not so, the current density being greater in the centre. Malsch himself has shown that there is considerable falling-off in electron current towards the edges, and we were aware of this fact. Further, Malsch employed pre-anode magnetic focusing to obtain his high current densities. It must also be remembered that he made his

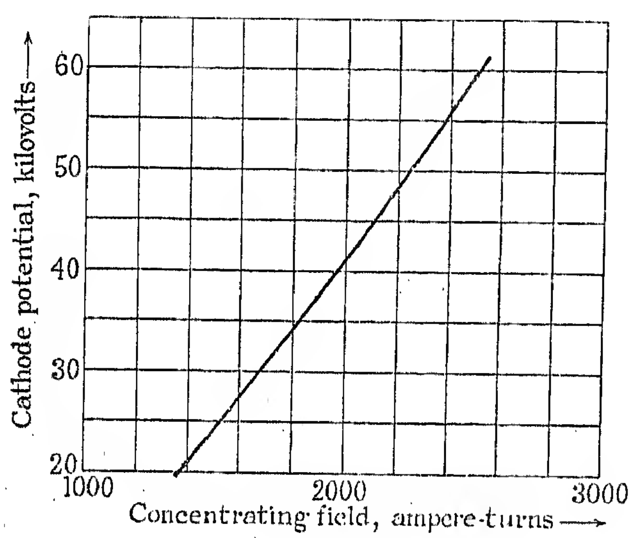



FIG. B.

measurements close to the anode of a skeleton oscillograph, and it does not follow that measurements at this point are any criterion of a satisfactory writing spot. In our case, with a smaller anode aperture than that stated on page 514, the post-anode current density is great enough to cause a bright red glow over a pin-point area on the metal cover of the plate holder, and of course such a beam immediately breaks the glass window used for external photography if the beam remains stationary on it for a moment. We would assure Prof. MacGregor-Morris, therefore, that the values of current and current density we employ do all that is required, and normally for oscillography a few microamperes only in the writing beam is necessary. If high efficiency is employed (in the past we have employed and discarded efficiencies of the order of 35 per cent) it necessarily requires the discharge tube to deliver a few microamperes rather than 0·1 to 0·5 mA, and on occasion this may present difficulties. If he has the saving of power in mind, this is negligible when all the auxiliaries are taken into account. If he has heating difficulties in mind, we would say that our discharge tube will run continuously at 0·5 mA without water cooling of the anode.

With reference to discharge tubes, perhaps we have been rather cursory in dismissing our theories and experimental results on the dependence of spot size on cathode assembly, but the subject is too extensive to be discussed in a brief reply. In connection with the inquiry of Prof. MacGregor-Morris about the positions of the lines of electrostatic force and of the equipotential lines in the discharge tube, while these can be drawn out readily enough under static conditions, the system is completely different when current flows. We have never found it necessary to go into this aspect of the question deeply enough to obtain information which would be of value to him. His assumption that the potential of the shield varies is not correct, however; it is determined by the internal discharge, and for a given cathode potential and discharge current it assumes a value which is of the order of 20 per cent less than the cathode voltage. We have used tapped d.c. supplies to control the shield potential, and this had remarkable effects on the focused spot. After due consideration and experiment, however, we found it best to allow the shield to float. (This also answers Mr. Nuttall's question on the same point.)

In reply to Prof. Howe, we agree that the article by Rogowski might well have been added to the Bibliography.

We think that Col. Lee is a little pessimistic in regard to the interpretation of Mr. Watson Watt's remarks, and we do not agree that the practical applications of the travelling-wave phenomena discussed are as limited as he states. Mr. Watson Watt implied that direct stroke conditions are still more onerous than the travelling-wave conditions dealt with in the paper, and must be worked out in the future. As Col. Lee points out, the equations given in the paper are not related to the lightning stroke itself, but since they are based on wave-shapes that have been shown by means of the high-speed oscillograph to exist immediately after the lightning stroke, they therefore cover a very large percentage of over-voltage conditions. The lightning-protective properties of high metal structures referred to by Col. Lee have been noticed by various observers. We agree with the need for low values of the tower footing resistances, and efforts are usually made by supply engineers to maintain these at a small value.

Dr. Rayner raises several interesting points in connection with the oscillograph. As he states, phenomena written at a speed of 200 km per sec. and greater can be seen at the high accelerating potentials used. It is our experience, however, that where there is considerable visual contrast in writing speeds in a particular trace, such as results from the superposition of high-frequency oscillations on a slower transient, the eye tends to ignore the faster regions of the trace, although the maximum writing speed here may be considerably less than the figure mentioned by him. In connection with visual inspection before recording, we find that the operation of the oscillograph is so certain that it is often unnecessary to examine the trace before photographing; this is of importance in measurements on solid insulation, where it is often not permissible to impress more than one surge at any particular voltage owing to the definite damage that would result. As Dr. Rayner states, the pumping time to a usable state of vacuum is of the order of a few minutes when using plates. Ordinate bias control enables more than one picture to be made on a

plate, and as we use holders containing several plates—transferable from outside the vacuum—a considerable number of records can be taken before reloading. In some work it is often desirable to develop a single plate before deciding on the next measurement, and this is a decided point in favour of the use of plates rather than roll films. The other advantage of plates is that the pumping time to a satisfactory degree of vacuum is less than with film, even when the covering paper is removed. (This also answers Mr. Nuttall's query.) We agree with Dr. Rayner that fairly slow plates for inside photography are quite satisfactory. Lantern plates and bromide paper are, in general, too slow, although we have satisfactorily recorded slower transients on these. We are of the opinion, on the other hand, that the use of fast plates is unnecessary. The system of exhausting the air leak through the main chamber has no disadvantage in practice. Regarding the allowable ripple in the d.c. cathode supply, while this voltage can be considered constant during the recording of one impulse, it is necessary to employ as smooth a voltage as possible since any ripple causes corresponding change in deflection sensitivity from record to record. (This also answers Mr. Taylor's query on this point.) Dr. Rayner refers to the position of the focusing coil; as Mr. Whelpton points out, the Busch formulæ theoretically demand that it be placed midway between the anode and the screen, but we have noted experimentally that within certain limits it is quite safe to depart from this requirement. Examination of Fig. 2 (Fig. 1 is not to scale) shows that the lower part of the coil (it is in three sections, and the lower one is normally used) is just a little above the midway position, since the screen is about 3 in. above the bottom cover-plate. In building a small and transportable oscillograph there are, of course, other factors to be taken into account in positioning the coil. (This also answers Mr. Whelpton's comments on this point.) Regarding Dr. Rayner's remark on symmetrical time-sweep motions, we find that for large deflections the use of a symmetrical field allows a more compact deflection-chamber design. We have not found it necessary so far to employ other than spark-gap tripping circuits. We decidedly agree with Dr. Rayner on the need for careful screening of many parts of the circuits, and on occasion of the oscillograph itself when operating at high voltages. Different troubles and different laboratory lay-outs require different treatments, and therefore it is difficult in general to make constructive suggestions. We would say, however, that expanded metal of fine mesh can be satisfactory, and non-magnetic shields are suitable for most cases. Great care is required in laying out magnetic shielding.

Mr. Norris and Prof. Marchant bring out an important fact in regard to the first paper. In very many cases the values of the constants of the circuits under transient conditions are different from those under a.c. or d.c. conditions, and before calculations can be made it is frequently necessary to explore these variations with the oscillograph. We appreciate with Mr. Norris that, in general, no mathematics is yet available for the theoretical elucidation of these variations.

We are entirely at variance with Mr. Stritzl's views concerning surge protection. A decision on the relative hazards of transformer breakdown to earth or between turns does not rest with technical demonstration, but rather is determined by operating experience and by the results of laboratory investigations. While Dr. Willheim may have shown to Mr. Stritzl's satisfaction that the peak value of the surge is the true factor endangering transformer windings, our experience is quite to the contrary. In the laboratory, during the course of our investigations we have surged transformers (the surge being synchronized with the crest of the 50-cycle wave) with long-backed waves of 100 microseconds and more, and breakdown has only resulted between turns. It is thus the steepness of the wave-front which is the danger, and on this point there are many who will agree with us. Turning to Mr. Stritzl's summary of the advantages of the discharge-type arrestor, while the modern forms of such apparatus may be satisfactory and give good protection when handling the currents associated with travelling-wave over-voltages, we do not think that anyone would claim that they can stand up to local direct-stroke conditions where the current is now known to be of the order of hundreds of kiloamperes. As is stressed also by Mr. Watson Watt and Col. Lee, it is just these conditions, where as yet the rate of rise of voltage is unknown, which are the most dangerous to equipment. Complete protection can thus only be assured by apparatus which, while withstanding these conditions, will sufficiently reduce inter-turn stresses in windings by minimizing the rate of voltage-rise transmitted to them. This is the function of the apparatus referred to by Mr. Stritzl, and which he erroneously calls a "choke coil." Other possible troubles due to the time-lag of dischargers are mentioned in our reply to Mr. Clothier.

As Mr. Thomson points out, it is more convenient to solve operational equations having more than three degrees of freedom individually by means of the Expansion Theorem. While general solutions for operational equations of higher orders could be written down in terms of the roots of $Z(p) = 0$, the resulting equations would be too cumbersome. Cardan's method for the solution of algebraic equations of the third degree is relatively speedy, but the approximate methods for higher orders are laborious and, except in rare cases, we endeavour to simplify the circuit to one of three degrees of freedom. We thank Mr. Thomson for drawing attention to the fact that repeated reflections on short lines can be readily taken into account. The only difficulty is that of keeping account of the reflections, and Bewley overcomes this very neatly by plotting the various waves in the form of a lattice. If empirical data on attenuation are available, very accurate results can be obtained.

We are very glad Prof. Marchant has drawn attention to the fact that the first paper and also most of the papers published on the same subject in the United States are fundamentally based on the pioneer work of the Englishman, Oliver Heaviside. As Prof. Marchant, Mr. Thomson, and Dr. Prescott point out, Heaviside's methods lead to great simplification. Prof. Marchant mentions that several small transformers may give more flattening than an equivalent large one. While this may

often be true, the amount of flattening in the former case is certainly not sufficiently great to warrant the use of smaller transformers. Regarding Prof. Marchant's query as to the limiting values of inductance to prevent oscillation with transformers, so much depends on the variation of capacitance of the latter with different designs, sizes, and voltages, that it is difficult to give figures to meet general cases. Further, even if the voltage is oscillatory, it does not always follow that high potentials will be attained. If $C_2$ is of the order of $1\,000\,\mu\mu$F, however, values of $L$ greater than $100\,\mu$H will in general give rise to some oscillation. Prof. Marchant also refers to a remark regarding neutral voltages, which was made in the course of the reading of the first paper. With an open-neutral transformer it is possible for the neutral voltage to earth to rise to twice the incident-wave voltage, and when the neutral is earthed through reactors this voltage may attain greater values. Under certain circumstances excessive inter-turn voltages may also be found at this point, with both earthed and unearthed neutrals. We are ever, that our external window is very little more difficult to make than other ground surfaces, and that once made it is extremely simple to use. In criticizing the examples shown in the paper (Fig. 4, page 513) it must be remembered that values of the beam current of the same order as those used in internal photography were employed, and these records were included to show what could be done under these adverse conditions. Normally we would employ greater values of current, and when a large number of records are required in a short interval of time for routine experiments the method has its advantages of simplicity. We thank Mr. Whelpton for commending our work on discharge tubes; in practice we do not alter the cathode-anode distance as the beam current is varied.

In reply to Mr. Knowles we would say that we have used the oscillograph for the microtime examination of insulation breakdown. Very little information is available on solid insulation, but Peek and Torok have published various papers in the *Transactions* of the American Institute of Electrical Engineers that might be

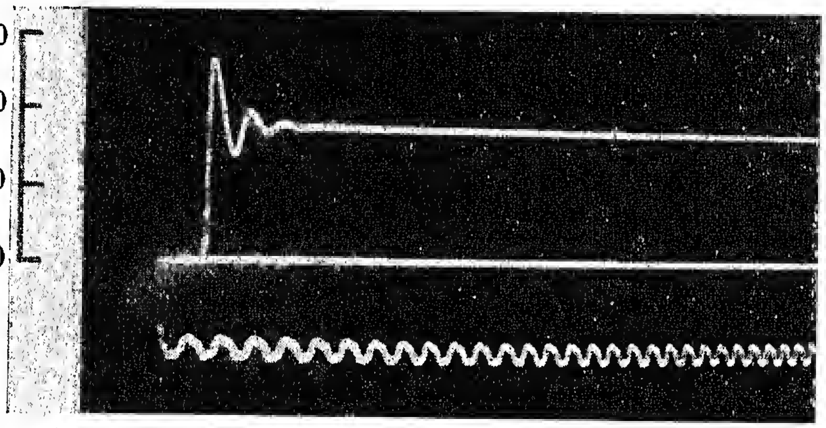



FIG. C.—Oscillogram illustrating negligible oscillation at de-trapping. Time scale given by 500-kilocycle oscillation.

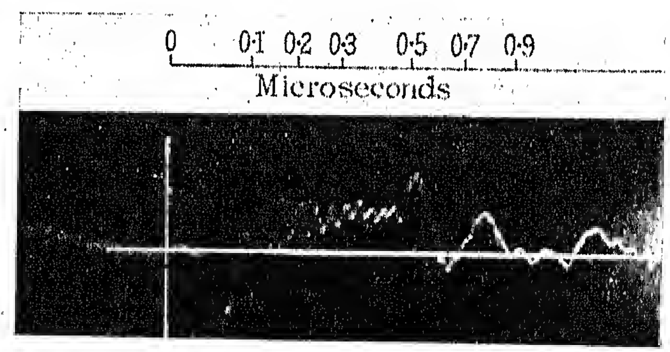



FIG. D.—Oscillogram illustrating negligible oscillation at de-trapping. Sweeping time of the order of 1 microsecond.

interested in Prof. Marchant's comments on his experience with condenser potentiometers, with which we agree.

Mr. Whelpton's remarks are very welcome. The compactness of the oscillograph described in the second paper, on which he comments favourably, minimizes transport difficulties. As he points out, we have been able to shorten the oscillograph by means of three expedients. The first, concerned with the position of the focusing coil, has already been dealt with in our reply to Dr. Rayner. The second is the use of a single-stage beam trap. Although, as stated in the paper, the double-stage trap has many advantages, we find our single-stage trap quite satisfactory in practice. In reply to Mr. Whelpton's request to see a high-speed oscillogram in order to enable him to examine the effect of beam-trap oscillations, we would submit Figs. C and D. In the first case the voltage is high and rises rapidly, and in the second case the duration of time-sweeping is short, yet no appreciable oscillation results at de-trapping. The third expedient is the relatively short distance between deflection plates and the screen; we have, however, never felt the want of greater deflection sensitivity. His remarks on outside photography with a lens and camera are interesting; we have never tried this type of photography. We would assure him, how- of assistance to him. We appreciate that the high-speed oscillograph will be of considerable use in problems associated with arc extinction in switchgear.

In connection with Mr. Nuttall's query concerning the small discontinuities of slope in certain of the oscillograms, we would state that these are caused by reflection at the turn-back of the overhead line used in those particular experiments. The variation in the position of the discontinuity is due to the fact that at different times different lengths of line have been employed.

We agree with Dr. Prescott that the inclusion of some of the formal equations before the derivation of the operational ones would have imparted further clarity to the paper; some formal equations were in fact included in the first presentation of the paper, but they were deleted later. Since we have dealt with concentrated circuits almost entirely, his assumption that series impedance implies concentrated impedance is quite correct. We agree that a truly infinite rectangular wave could set up maximum stress between two points on the same turn, but naturally no insulation breakdown would ensue.

We are glad that Mr. Taylor has stressed the importance to all heavy-engineering designers of both experimental and mathematical transient research, and

we thank him for his favourable remarks on the appearance of the oscillograms. He is correct in stating that it is one thing to produce an oscillograph capable of drawing a line and quite another thing to obtain distortionless results; the technique is certainly not simple, particularly at the higher voltages. We agree that fluorescent materials are being improved very rapidly, but it must be remembered that at the higher writing speeds considerable difficulties would arise with the small sealed-off oscillographs, even if the trace could be photographed. This has been referred to by Prof. MacGregor-Morris and Mr. Watson Watt. The purpose of having a higher degree of vacuum in the lower part of the oscillograph is principally to maintain the insulation of the deflection plates and to prevent beam scattering. The discharge tube shown in Fig. 6 is free from corona on the outside up to 70 kV; corona outside the tube above this voltage does not appear to affect the stability of the beam, although, of course, it is only satisfactory to design for no corona. The concave construction of the anode prevents the operator receiving the harmful effects of X-rays. The limit of writing speed due to the finite size of the deflection plates suggested by Mr. Taylor is correct. Mr. Taylor's final question is very pertinent; to reduce the effect to a minimum, the plates are made long.

It is very gratifying to have Prof. Thornton's appreciation of the oscillogram illustrations of the first paper. As he points out in reference to the Appendix, when equation (1A) has been formed the rest follows naturally once the reader has become familiar with the Expansion Theorem and the Shifting Theorem. Other equations for the total and reflected voltages similar to equation (1A) can be derived, and the later equations also apply to these. The author's thanks are due to Mr. Crawford for examining the equations in the Appendix. We agree with Prof. Thornton's remarks on the need for high-speed oscillographs in works manufacturing many types of electrical equipment. As he points out, impulse tests on energized transformers, for instance, may eventually become part of acceptance-test routine in this country. We have done a great deal of work on these lines. It is necessary suitably to synchronize the impulses with the 50-cycle wave so that power-frequency follow-up current will indicate and mark any breakdown, and an oscillograph is the only means of monitoring and measuring the applied impulses and the subsequent behaviour. We congratulate Prof. Thornton on his foresight in being the first to purchase a continuously evacuated oscillograph for a university laboratory in this country.

In reply to Mr. Harle's remark regarding external photography we mention that the system we use depends on the translation of electron energy to light energy, whereas, in general, cellon windows are employed for the transmission of electrons outside the vacuum. Thus our glass windows can be thicker than those used for outside electron-blackening photography, with certain mechanical advantages. The earlier Lenard windows comprised thin aluminium foil, but we believe that minute pin-holes gave rise to some inconvenience from the vacuum point of view. With the normal current density in the writing beam appropriate to the writing speed required, no trouble whatever is experienced from after-glow. When a writing-beam current vastly in excess of this value has been employed we have noticed on occasion the luminous trace still persisting slightly on the screen some minutes later in the dark-room. With the very generous writing-beam currents of the order of 500 microamperes used in our early experiments, the surges actually became permanently engraved on the fluorescent coating of the viewing screen. The cement joints used in the construction of the oscillograph are made with picein wax; we have also employed glycolphthalic anhydride resin. The vacuum tap operates perfectly satisfactorily. We have no information available on the relative smoothing effected by resistances and by saturated diodes, but of course either method may be designed to give any desired degree of smoothing. We would refer Mr. Harle to Dr. Rayner's contribution to the discussion on this point.

In reply to Mr. Ryle, we agree that the unit of operating experience should always be stipulated. This was inferred, though not explicitly stated, in the example he quotes, since it is mentioned that 73 surges, over 8 times normal, were recorded on 5 systems over a period of 5 years, thus giving approximately 3 surges per system per annum. What is, however, just as important as the time and length units is the specification of the type of locality to which the experience pertains, and as this is not expressible in a simple index, a description of the soil conditions, country traversed, and frequency of thunderstorms, should, strictly speaking, be included. With regard to Mr. Ryle's remark that a surge originating some distance from a terminus becomes appreciably flattened as it progresses, he is quite correct in respect of high over-voltages, but it must be remembered that the capacitance of the line cannot really be segregated and is the measure of one component only of its surge impedance and propagation velocity. The flattening is therefore not directly due, as he surmises, to the capacitance of the line, but primarily results from corona and other line losses; or, more generally, is occasioned by any line-distorting characteristics. Corona, of course, increases the line capacitance, but this has the effect of slowing-up the high-voltage portions of the wave in the manner mentioned by Mr. Field. In view of these remarks Mr. Ryle will appreciate that extra earth wires cannot be of much value in the manner he suggests. Earth wires have their well-known applications, but the advantage of wires additional to the first is only slight, except for the purpose of reducing the hazards of direct hit on to conductors—as they do if they are high and if the earth resistances are low. We cannot in a short reply deal fully with Mr. Ryle's question as to the physical nature of the series and earth capacitances of a transformer winding, but we would say that the latter refers to the total capacitance of the winding to earth-potential conductors assuming zero series inductance, while the former is that between ends of the coil —regarding its inductance as infinite. For a transformer of the size he mentions, these capacitances might respectively be of the order of 5 000 and 10 $\mu\mu$F.

We are indebted to Mr. Horsley for drawing attention to the paper by Rendell and Gaff, which adds confirmation to the statement (in the Introduction of the first

paper) that induced lightning strokes are of no importance on lines operating above, say, 11 kV. Mr. Horsley refers to the propagation of surges from the h.t. to the l.t. windings of transformers. Such propagation is due not only to electrostatic induction, however, but also in a greater measure to electromagnetic induction, and the relative magnitudes of these depend on the factors enumerated by Mr. Horsley, on the mutual inductance between windings, and on the secondary terminal connections; and the more so is the latter true as regards the voltage developed at the secondary terminals, to which Mr. Horsley particularly points his argument. In the slide shown during the reading of the paper, the turns ratio of the single-phase transformer used in making the oscillogram was 22 : 1, and measurements on other transformers have shown us that the later electromagnetic portion of the secondary transient is sometimes twice as great as that given by the turn-to-turn ratio, which is in accordance with theory. (This also answers Mr. Bishop's query on this point in part, and Mr. Mackay's query completely.) Mr. Horsley also refers to the breakdown of rotating machines. Here there are two sets of circumstances to be considered, those where the machines are connected directly to lines or long cables and those where they are connected directly to the l.t. windings of transformers. It is the former which are considered in the paper, and the latter which are dealt with by Mr. Horsley. Our reply to his query is that, in the event of a steep-fronted long-backed wave striking the h.t. winding of an unprotected transformer, breakdown may occur between turns or coils but not to earth. The surge transmitted to the l.t. winding may have a relatively high amplitude, but certainly will have only a relatively slow rate of voltage-rise. Thus there will probably be no danger of the inter-turn type of breakdown in the machines, and there is only left the possibility of breakdown to core. Machines under such circumstances may only rarely break down, but transformer l.t. windings never do. Therefore, it cannot be argued that a machine is stronger than a transformer, particularly as the latter has borne the brunt of the impact; in other words, in this case the machine is never called upon to withstand a steep wave-front, and, as we implied in the paper, the stresses occasioned by the fronts of waves are more onerous than those set up by their amplitudes.

Mr. Clothier's remarks are very interesting and we would first refer him to part of our reply to Mr. Stritzl. As Mr. Clothier states, even when a lightning arrestor is installed the transformer will be subjected to some transient voltage owing to the time-lag of the protective device, but his own argument shows that under such conditions the inter-turn insulation is excessively overstrained and breakdown is only obviated by reason of the briefness of the overstress. It is now well known that insulation exhibits electrical fatigue and that overstress therefore permanently damages the insulation, which, though it may not fail under the first shock, is progressively weakened by successive over-voltages, until breakdown results.

Mr. Field's assumption that the flattening of wave-fronts is due to a reduced propagation velocity caused by corona on the high-voltage portions of the wave, is essentially correct. As regards his other point, focusing action due to ionization is not obtained, the focusing coil being relied on entirely.

We thank Mr. Woodiwiss for his kind comments, and appreciate the greater impression he conveys by writing out the small time-interval as a decimal, but as a matter of interest we would point out that some of our previously-published oscillograms justify the addition of at least two ciphers to the long row. The application of the oscillograph to radio measurements, which Mr. Woodiwiss mentions, is normally the sphere of the smaller glass sealed-off type. Such measurements usually consist in comparing the output and input waveforms, and while theoretically nearly faithful reproduction can be obtained, it is unfortunate that considerations of cost prohibit full advantage being taken of this in commercially-available apparatus. Comparisons between sound input and output are very difficult and are rarely carried out. Filter apparatus such as he mentions is commercially available in the form of the surge absorber for effecting reduction in wave-front steepness. We cannot here give a full reply to his request for the specification which such apparatus should meet. We can only say that the transformer and surge absorber should withstand an impulse test with the steepest possible voltage wave of magnitude depending on the transmission system insulation. The possibility of such acceptance tests being eventually standardized, and the form they should assume, have been referred to in our reply to Prof. Thornton.

With regard to Mr. Bishop's fears of overstrain resulting from over-voltages, he is quite correct in surmising that trouble may only be postponed. So far as we can reply generally to Mr. Bishop's second point, lengths of at least several hundred yards of cable are necessary to give the protection he indicates. The initial wave entering the cable suffers considerable reduction in amplitude, but successive to-and-fro reflections allow subsequent build-up almost to the peak voltage. There is thus no reduction in voltage amplitude unless the wave is short or the cable long, but the front of the wave is effectively flattened by an amount depending on the length of, and the propagation velocity through, the cable, and the values of the various surge impedances.

It is not possible to reply in general to Mr. Bishop's last point without knowing all the circumstances of the case. If the system is connected through transformers to an overhead system which is subjected to lightning, there is certainly the possibility of surges being transmitted into the cable system. Owing to the fact, however, that the ratio of primary line surge-impedance to secondary cable surge-impedance may be of the order of 8 : 1, the amplitudes of the secondary surges will be small. In addition, as we have pointed out in our reply to Mr. Horsley, the fronts of these surges are not steep. These facts probably account for the small amount of trouble experienced from these causes on the 6·6-kV cable systems Mr. Bishop has in mind.

Mr. Mackay's interesting opening remarks call for no comment. Our reply to Mr. Horsley covers his query regarding voltages in the secondaries of transformers. Mr. Mackay also mentions the fact that increase in inter-

turn insulation at the line ends of transformer windings does not give a proportionate increase in surge strength. This is quite true but is not always appreciated, and we are glad it has been mentioned. We cannot here reply to his final question.

Normally the capacitance of the condenser in the time-sweeping circuit referred to by Mr. Moore is very much greater than the stray capacitances, and it is therefore unnecessary to measure this capacitance *in situ*. There are available three simple methods for calibrating time-abscissæ motions. In the first, as a rough guide the product of capacitance and resistance and plotting of the appropriate exponential is used. The other methods consist of superimposing on the uncalibrated oscillogram an oscillation produced by a standard-frequency oscillator or the to-and-fro reflections in a short transmission line whose length is known and whose propagation velocity is that of light.

# DISCUSSION ON "THE APPLICATION OF AUTOMATIC VOLTAGE AND SWITCH CONTROL TO ELECTRICAL DISTRIBUTION SYSTEMS."*

## West Wales (Swansea) Sub-Centre, at Swansea, 25th January, 1934.

**Mr. C. G. Richards:** The subject dealt with by the authors is of particular interest just now, since this month we are advised of the Electricity Commissioners' new regulations for electricity supply by authorized undertakings. While these new regulations increase the permissible limits of voltage variation, they contain clauses which will make it more difficult to maintain the voltage within even the wider limits of variation which are now permitted. It does not necessarily follow that the limits of voltage variation laid down by the new regulations will be those to which supply authorities will desire to adhere, because consumers are prone to complain of variation even within the limits of $\pm$ 4 per cent permitted by the old regulations. The increasing use of electric motors in business premises, and the heavier loadings of distributors arising from heating and cooking, will make it necessary to give very much more attention to the voltage regulation of l.t. distribution systems. The paper refers to the system of voltage control which has been adopted by a very large undertaking, but, in view of the increasing use of electricity for domestic purposes, special apparatus for the automatic control of voltage will in the near future be necessary in towns whose populations are very small compared with that of Manchester. In the case of l.t. distribution networks without automatic voltage-regulating apparatus, the maximum possible loading is determined by the permissible voltage drop and not by the thermal limits of the cables. It would be interesting to know whether the application of automatic voltage control has enabled the authors to increase the loading of their distributors so as to bring it nearer to the thermal limits, and, if so, to what extent. It would appear to be reasonable to expect that the capital charges and maintenance costs of automatic voltage-control equipment could be readily saved on heavily loaded networks by the increase in the loading of the distributors, apart from the improvement in the service. It has been my practice to provide for manually-operated on-load tap-changing equipment on all distribution transformers. This enables one to provide the voltage regulation necessary at the various seasons of the year. Turning to the question of supervisory control, the present sphere of application of this appears to be limited to thickly populated areas, but 10 years from now supervisory-control systems will probably have to be considered by many undertakings in order to ensure continuity of supply and avoidance of delay when failures occur. I should be glad to know whether the authors have adopted separate cables for their supervisory-control system, or whether Post Office telephone lines have been used. If the latter policy has been adopted it would be interesting to know the results obtained, and also the particulars of the costs.

[The authors' reply to this discussion will be found on page 541.]

## Scottish Centre, at Edinburgh, 13th February, 1934.

**Mr. P. Butler:** Referring to Fig. 11, it is doubtful what exactly the authors wish to stress; it would appear that the voltage should be the same in both cases, for a fair basis of comparison. Does not Fig. 11 tend to make the excitation loss seem as high as possible? The low-tension voltage is taken much higher in Fig. 12, and I should like to know the reason for this. It is evident from the increase of excitation loss that the authors have taken the case of very badly-designed transformers. On the other hand, it may be due to the old trouble that users of transformers ask for tappings for high-tension variation, but use them for a different purpose, namely to vary the low-tension voltage. Such a misuse of the transformer results in variation of the induction in the iron, with a consequent variation in excitation loss and kVA. With regard to tap-changers, it is not quite true that with the resistor type the circuit broken is non-inductive. I should like some further information as to the speed of breaking the circuit since, according to the paper, this operation takes place in the first half-cycle. On some of the types of tap-changers illustrated in the

* Paper by Messrs. W. Kidd and J. L Carr (see page 285).

paper such a speed will certainly not be attained. It is to be hoped that the introduction of all these complications into the structure of the transformer will not affect the main design of what is really a very reliable piece of apparatus.

**Mr. J. Eccles:** The authors have a very favourable system for illustrating the advantages of automatic voltage control. It comprises a 33-kV network feeding a 6·6-kV network, which in turn feeds the usual distribution system. Some undertakings dispense with the 33-kV stage and start with 11 kV or 6·6 kV; in such cases every substation is a main substation in the sense that the authors use that term, and close voltage regulation becomes much more expensive. Since the paper was written, the permissible voltage variation for new supplies has been altered to $\pm$ 6 per cent. This wider range might obviate the necessity for installing voltage regulators in certain cases, but I am of opinion that we should aim at giving the highest possible service and therefore should not necessarily avail ourselves of the extra allowance. Touching on the point about transformers raised by Mr. Butler, there is no doubt that the kVA curves shown in Fig. 11 must have been obtained on a transformer that was reasonably saturated initially. It has been the practice of the Edinburgh undertaking, for some time past, to specify that at normal voltage the flux density in any part of the iron circuit shall not exceed 11 000 lines per cm$^2$. It was recently advocated* that this value of limiting density should be adopted by all distribution authorities; there is something to be said for this recommendation, not only from the point of view of voltage control but also from that of losses and noise. I should like to know what the authors' experience has been with regard to voltage surges during tap-changing, and particularly as to the relative merits of the choke and resistance buffers used in the changing of taps. Perhaps they will say which they prefer, and why.

**Mr. J. Bentley:** It is not clear from the paper how the voltage is regulated when governing with the object of keeping a constant voltage; some variation is necessary to obtain power to operate the tap-changing devices described by the authors. Having been concerned with the design side of direct-current gear, I realize how difficult it is to do this with electromagnetic devices.

**Mr. F. C. W. Clark:** On page 295 the authors describe an automatic voltage-control equipment which operates tap-changing gear. On this type of equipment the actual operation of the tapping switch is carried out by the energy stored in the flywheel of the motor, after this motor has been disconnected from the line. It appears to me that this method has an inherent weakness in that the amount of travel of the switch blades over, or through, the contacts, depends upon the amount of friction between the switch blade and its contacts. Should the contacts be over-lubricated there may be a danger of the blades passing right through them; on the other hand if the contacts are very dry the blades may only just enter them, thereby causing local heating. It would be interesting to know whether the operating circuits of this apparatus are supplied from a battery or from the mains through suitable transformers. In Table 1 the authors quote a resistance voltage-drop of 3·95 per cent and a reactance voltage-drop of 1·55 per cent (total 4·24 per cent) from the distributor to the constant-voltage point. This corresponds to 17 volts (approximately) on a 400-volt 3-phase supply, and apparently represents the drop on both l.t. feeders and l.t. distributors. At a loading of 1 000 amps. per sq. in., corresponding to peak-load conditions, this would seem to limit the total length of l.t. cable to 600 yards. I suggest that this is rather an uneconomical arrangement and that a more suitable position for the regulators would be at the 6 600/400-volt substations.

**Prof. F. G. Baily:** With regard to the point raised by Mr. Eccles, namely that it would be expensive to adopt automatic voltage control on a system which only had distribution transformers, I think that in the future all systems will have both main substations and distribution substations. Load densities are at present very low compared with what they will eventually be, and higher load densities mean shorter distances between distribution-transformer substations. Thus although the ideas contained in the paper may at present apply only to a limited number of distribution systems, they will very soon apply to all towns. I do not know that I quite agree with the authors' list of apparatus which requires regulation. I notice that cookers are put down as very sensitive pieces of apparatus; but one can cook a dinner on a coal fire, which has no thermostatic control, or in a gas oven subject to fluctuating gas pressure. I think the electric oven, even with the present crude method of regulation, compares favourably with its competitors. Lamps are less sensitive, as the metal-filament lamp more or less regulates itself.

[The authors' reply to this discussion will be found on page 541.]

## North Midland Centre, at Leeds, 20th February, 1934.

**Mr. S. R. Siviour:** Whilst the raising of the permissible voltage variation to $\pm$ 6 per cent will assist undertakers operating in rural areas, and in other difficult cases, my view is that it would have been better if this variation had been limited to an interim period of development. We shall only obtain and retain the cooking and heating load in the domestic realm by providing a supply within closer limits than those actually permissible, and I advocate designing the network on the basis of a regulation not exceeding $\pm$ 4 per cent, leaving the remaining 2 per cent to take care of contingencies. With regard to 75 kW being the limiting load for supply at low voltage, whilst this may be a fair figure to take for any large city or town, I think the figure is much lower in rural and some urban areas. The cities have much more copper in the mains and can probably take up to 75 kW, but in rural work with smaller average copper and scattered premises the limit for l.t. supply is generally of the order of 40 kW. I find little to disagree with in the paper, even when comparing the Manchester problem with that of supply over a large area. The authority with which I am

* E. Seddon and J. Eccles: "The Design, Equipment, and Operation of Static Substations," *Proceedings of the Incorporated Municipal Electrical Association*, 1933, p. 26.

associated has exactly the same conditions to meet and is dealing with them in the same way, i.e. by providing regulation at the main distribution substations where we step down from 66 kV or 33 kV to the secondary distribution at 11 kV. By arranging the secondary distribution and networks with suitable constants, reasonable limits of regulation can be attained. I should like to ask the authors whether they prefer the resistor to the reactor type of tap-changing equipment, and whether they can give the results of any extended experience with these two types.

**Mr. R. M. Longman:** The methods of voltage regulation which have been employed in the past fall into four stages: (1) Regulation at the power station by increasing the busbar voltage as the load increases. (2) Similar regulation at substations, chiefly where transformation to direct current occurs. (3) A combination of (1) and (2). (4) Regulation at the power station and also at static substations by automatic or remote-controlled apparatus with a compounding effect (the scheme adopted by the authors). On page 285 it is suggested that concentrated loads exceeding 75 kW should be supplied direct from the high-voltage feeders. This seems to be rather a low figure, but of course the actual value is largely dependent upon the loading in the neighbourhood and the size of the l.t. mains already laid, or the ease with which such could be provided. With reference to Fig. 1, the distances between the new substations 1–2, 2–3, 3–4, and 4–1, are approximately 1 500, 900, 700, and 1 600 yards respectively, by direct measurement. It is perhaps a pity that the authors do not show the area adjacent to the square indicated in the figure, as presumably the substations which are located on the boundaries also feed areas all round them. It is not to be expected that the regulation provided by the grid will meet the distribution requirements of the supply authorities. The figures given in Table 1 are of particular interest, and in the paragraph below I note the remark that the basic voltage on distribution transformers is adjusted by selection of the most suitable tappings. I should like to know what range of tappings the authors normally recommend. In my experience tappings of $\pm 2\frac{1}{2}$ per cent and $\pm 5$ per cent have generally proved sufficient, and in many cases the $-5$ per cent tapping could be omitted, the tendency being to keep the main transmission voltage as high as possible. On page 295 it is suggested that the operation of boosters during periods of short-circuit would tend to cause abnormal stresses in the equipment; I am very doubtful whether the special relays provided to meet such conditions will prove effective, as short-circuits occur without warning and the rapidity of action is probably greater than that of the protective arrangements. A few years ago I had experience of two such instances, as the result of which some auxiliary items of the regulating apparatus were damaged. I found that if real protection was to be obtained all tapping and auxiliary apparatus of a reactive nature inserted during tap-changing had to be designed to withstand any of the stresses to which it might be subjected. Under short-circuit conditions a choke coil or an auxiliary transformer may quite possibly be subject to 10 times its normal voltage. It is of interest that on the grid tap-changing equipments no case of failure or trouble has arisen owing to a fault occurring whilst tap-changing has been in progress.

**Mr. W. Dundas:** The difficulties of maintaining a 4 per cent voltage variation over a large network appear to have been responsible for the increased tolerance now allowable under the revised regulations. I am inclined to think that this is a step in the wrong direction, which may retard development in the use of electricity as a heating medium, especially in the field of domestic service. Regulation to closer limits than $\pm 6$ per cent should be effected as far as possible. The somewhat poor inherent regulation on many networks is due to the existence of a number of small feeder cables laid in the past; more copper has had to be provided on that account and in order to allow for future development. Again, a standard voltage of 6·6 kV for primary distribution is no longer sufficient on large systems, and it will become necessary in many cases to consider superimposing a higher transmission voltage and subdividing the areas as indicated in the paper. I do not agree with the authors that 75 kW is the lower limit of load for direct high-tension supply. In densely loaded areas, adequate space on consumers' premises is usually difficult to obtain, and the provision of high rupturing-capacity switchgear and of the necessary precautions and safeguards makes the cost prohibitive. In large stores and buildings, duplicate transformers would be required to guarantee supply, thereby further increasing the cost and space involved. The problem is admittedly difficult and is closely associated with the average length of the l.t. distributors, which is given in the paper as 600 yards. With shorter distances, say 400 yards, it would appear to be a more economical proposition to take supplies at low tension from a nearby transformer chamber.

**Mr. H. J. H. Nethersole:** I should like to emphasize that some undertakers consider that the C.E.B. voltage-control apparatus can be used to regulate the voltage of their own supply networks. This gear is designed to keep the voltage on the undertaker's busbars constant under all conditions up to full load of the C.E.B. feeders, and the undertakers should therefore make their own provision for voltage regulation. With regard to the authors' remarks on the control of the automatic regulators, have they ever attempted to control the automatic apparatus from a relay at the remote end of a distributor, e.g. at the works of a large consumer? I am rather surprised at their statement that remote control for voltage regulation is uneconomical, in view of their wholesale adoption of it for supervisory control gear. I should have thought that controlling the regulating gear from the main supervisory point would have been cheaper, and probably even more reliable, than automatic regulation. Turning to the subject of supervisory gear, I do not think the authors' method for tripping a circuit breaker is ideal. The operator should be able to trip any switch in one operation, instead of having to wait for the check signals to come back to him before he can finally open the switch. Consider an ordinary remote-control board where the operator has to push only one button in order to trip a switch; mistakes very seldom occur, and I therefore cannot see

why the supervisory board, once the operator has become used to the gear, should not be just as simple to handle as any other type. There should be no difficulty in choosing the right button to push. I presume the idea at the back of the author's mind in having more than one stage per operation is to guard against mistakes in selection, but I feel that it will only be a matter of time before supervisory control gear becomes as common as ordinary remote-control gear, and unnecessary stages in operation will then be eliminated.

**Mr. P. M. Maxwell:** I consider that the regulation of the Electricity Commissioners which increases the permissible voltage variation from $\pm 4$ to $\pm 6$ per cent represents a retrograde step. Although the former variation of $\pm 4$ per cent was often exceeded, to the detriment of the supply industry, the increased figures introduce the danger of going from bad to worse. In regard to the voltage regulation, the size of the system (i.e. the lengths of feeders) determines to a great extent the position where regulation should be effected, but I am inclined to agree with the authors' opinion that generally the best place is the substation. In a comparatively small supply area of from 6 to 8 square miles like the one with which I am connected, the question of regulation is not serious if the provision of copper has been on generous lines.

**Mr. I. H. Hedley:** I am in agreement with the authors that it is desirable to make on-load tap-changing gear for transformers entirely automatic and also to use it on large units only. There seems to be a school of thought which proposes on-load tap-changing gear for all distribution transformers, but as the average size of such transformers appears to be about 150 kVA I do not think such a development will be satisfactory, apart from its prohibitive cost. The necessity for voltage regulation in order to give the maximum voltage at the point of maximum load is clearly shown by reference to Fig. 11, from which it will be seen that, in order to get slightly more than normal voltage at periods of peak load, it is necessary to have 8 per cent over-voltage at periods of light load. This over-voltage on light loads gives rise to a very large increase in the magnetizing current, the value given by the curves being 40 per cent, and this occurs when it is least desirable, that is, when the load current is of approximately the same value as the magnetizing current. This brings me to the importance of transformers having a low flux density; in my opinion no distribution transformer should have a higher flux density than 12 500 lines per cm$^2$ on normal voltage and tapping. I shall be glad to know whether the authors can confirm, from experience, the statement at the bottom of page 293, that with the resistor method of on-load tap-changing gear there is less wear than with reactive methods.

[The authors' reply to this discussion will be found on page 541.]

## SOUTH MIDLAND CENTRE, AT BIRMINGHAM, 26TH FEBRUARY, 1934.

**Mr. J. Goodman:** In Table 1, figures are given for the resistance and reactance; I should like to know the size in kVA to which these figures refer. In this connection is there any tendency in rural districts where the usual size of transformer is, say, 15 to 20 kVA, to revert to impedances of the order which held sway some 15 to 20 years ago, namely 2 per cent? Such a change would tend to ease the regulation question on the lower power factors. Turning to Fig. 11, the authors state that it is essential that the distribution transformers should be designed with reasonably low flux densities. I should be interested to know whether they have any figure in mind for the maximum induction in the core, as obviously the higher this is the greater the possibility of trouble due to high harmonics. I gather that the authors have met with trouble due to harmonics; I should be glad to know with which harmonics they have had trouble, and what form the trouble took. With regard to tap-changing on load, this is fairly common practice and transformer manufacturers can meet any usual demands. The authors consider that the tap-change equipment should be designed so that no harm arises in the event of an operation not being completed. I prefer that it should be a feature of the design that once the tap-change movement has commenced it cannot fail to be completed. Stability of the equipment is also very necessary. With regard to Fig. 16, showing the boosting arrangement, I should be glad if the authors would confirm that, with the arrangement shown, phase shift will be introduced. With regard to the utility of on-load tap-changing, the authors consider that this should be applied to the main transformers only, the distribution transformers being adjusted on a particular tapping prior to installation. Would this apply if there were long feeders in the district? As an analogy, one might consider the installation of static condensers for the improvement of power factor. If one large unit alone is installed at one position the correction obtained is not so good as when several smaller condensers are placed at selected points. Does not a similar idea apply to some extent to voltage regulators? On-load tap-changing gear is possibly not uneconomical in the neighbourhood of 500 kVA, but with smaller sizes of transformers the cost of such gear is very much out of proportion to the price of the transformer. I should be glad if the authors would enlarge upon their remark about the noise caused by the operation of a tap-changing movement. If the transformers fitted with on-load tap-change equipment are installed in the main substation the noise which is usually experienced is not serious.

**Mr. F. Barlow:** There is just one point I should like to make in connection with automatic voltage regulation on medium-voltage feeders. About four years ago we had a very cold snap at this time of the year, and in a town very near Manchester where residential consumers were supplied through the medium of 50-kVA transformers it was found that owing to the increase in the installation of radiators the local transformers became very much overloaded, and in consequence the voltage regulation was very bad. The result was that the size of the transformers had immediately to be doubled, and a

system of automatic voltage regulation is now being installed at the suburban substations. Perhaps the authors could tell me whether they met with a similar occurrence in Manchester, and, if so, whether it was found necessary to deal specially with the situation.

**Mr. J. A. Sumner**: I agree with the authors that the wireless and lighting load is a class of demand in which the voltage regulation is very critical, and I suggest that the introduction of "all-mains" wireless sets, and of television sets, is causing a very serious problem. I should like to mention that in rural areas the difficulties of voltage control are much greater than in the towns. Where the mains are of comparatively small section—the copper in the mains is more or less proportional to the density of the area they have to cover—a 3-kW radiator or an 8-kW cooker is a very serious proposition. In view of the smallness of the revenue obtained in rural areas, one generally has to wait until receiving complaints about the voltage variation before installing voltage-control equipment. One of the difficulties is to know when steps should be taken to introduce such equipment. Should one wait until the voltage has fallen by 5 per cent, say, 3 times in a year? I do not agree with the statement (page 293) that steady voltages increase consumption and revenue. Suppose a rather mean consumer has a 25- or 40-watt lamp in a 12 ft. × 12 ft. room, and is just able to read when the voltage is normal. When the voltage drops he generally puts in a 60-watt lamp; low voltage does not decrease revenue in this case, but it may increase it. Referring to Figs. 4, 5, and 6, I should like to ask the authors whether these are the actual recorded curves. As I see it, the effect of regulating was to bring the voltage down to a more steady, but lower, level.

**Mr. J. D. Parker**: I am particularly interested in the supervisory gear, and I should like to ask the authors whether they have tried the carrier-current system. This would overcome the difficulty of carrying on a telephone conversation and at the same time effecting supervisory operation. I understand that the Post Office employ a type of gear called a "routine tester" which tests the supervisory apparatus at a high speed and with such a degree of accuracy that faults are detected before they affect the apparatus; have the authors considered the installation of such apparatus? The Post Office have had trouble with the relays on their selector mechanism, which is of a similar type to that used by the authors, due to oil accumulating on the relay contacts. I have been informed that this caused a great deal of incorrect operation in Post Office selector circuits until the fault was finally traced and corrected.

**Mr. W. Wilson**: The authors deal with their subject largely from the supply point of view, and set out very completely the requirements to be met and the results which have been obtained. It will be of interest to refer briefly to the design of the apparatus whereby the operations specified by them have been carried out. One of the principal conditions that they lay down is that a tap-change must be infallibly carried out when the starting impulse has been received. The reason for this is, first, that several transformers are almost invariably connected in parallel, and that if one is working on a different tapping from the others heavy circulating currents will be produced which will quickly cause serious overheating; and secondly, during the actual tap-change, a number of components having short ratings are brought into circuit, which would be speedily burnt out if the operation were halted mid-way. Since failure of the supply voltage is a possible cause of such a halt, it is necessary that the gear should store sufficient energy to complete the whole operation with the supply cut off. Such storage is best effected mechanically, and there are three available methods, namely those employing a flywheel, a falling weight, and a spring. Each of these devices has its proper field, the flywheel being appropriate for the largest pieces of equipment, the falling weight for the intermediate sizes, and the spring-controlled gear for the smallest regular substations. The production of the apparatus has required considerable ingenuity, and it is gratifying to have the authors' statement as to the satisfactory working of the designs that have been got out. As a typical cycle of operations, that appropriate for the largest class of apparatus will be described. In the first place, a squirrel-cage motor of about 3 h.p. accelerates a flywheel to nearly synchronous speed (i.e. about 1 400 r.p.m.) in the course of from 20 to 25 revolutions, and the motor is then cut off from the line. Secondly, the speed is tested in the course of the next 3 to 5 revolutions by means of a fly-ball relay. Thirdly, the speed being in order, a mechanical clutch is operated and the tap-changing gear completes its operation, the flywheel giving up not more than 200 r.p.m. in supplying the energy. Fourthly, a mechanical brake is applied to bring the flywheel to rest within a few revolutions; and fifthly, the motor is brought back to its exact starting position ready for the next tap-change, by means of automatic control gear resembling that employed for "decking" automatic lifts. Regulation of rural distribution lines possesses a special significance, in that first the expense of tap-changing gear as described above would be quite out of the question, and secondly the cost of a high-tension transmission line is frequently not justified. Regulation is required in the case of many communities taking a much smaller load than the 15 kVA which has been mentioned by a previous speaker, and I consider 6 kVA to be a very useful size for such purposes. When small villages of about this size are first connected it is a great advantage for them to be supplied from low-voltage lines, which will involve a voltage variation of anything up to ± 10 or ± 15 per cent. This can be corrected by means of a cheap and simple automatic regulator, which should not cost more than about £25, and which can be removed for use in another locality when the load has grown sufficiently to justify the installation of high-voltage transmission. The problem of producing such regulators is a special one, but it can be solved by designing them in the form of "overgrown" relays in which the one device not only measures the correction required but also furnishes sufficient energy to carry out the operation.

**Mr. R. H. Rawll**: Supply engineers have been very interested in the recent amended regulations of the Electricity Commissioners, wherein supply authorities are now given a latitude of 6 per cent, instead of 4 per cent, in their variation of voltage. I should think that this must be considered by most supply engineers to be

a retrograde step, as is borne out by the specific statement in the early part of the paper that, with the ever-increasing development of special applications as regards electrical apparatus, it is necessary that very close regulation of voltage should be maintained on supply networks. Several speakers have pointed out that the necessity for regulating the voltage on a large e.h.t. system usually involves regulation at key points, and I notice that the paper refers to automatic regulation. It is the usual practice of supply authorities who are responsible for large networks to install voltage-regulation apparatus at key points, but it is not always automatic. I can therefore quite appreciate the point of view of the authors that if the full benefits of automatic regulation are to be realized all regulation must be by automatic means, instead of the equipment at some sub-stations being arranged for automatic control while others are manually operated. The Birmingham undertaking have a number of regulating points in connection with the main 33-kV trunk cables, and the opportunity of the general upheaval caused by the frequency-change is being taken to introduce additional regulation at certain other centres. I should like to ask the authors whether they have on the Manchester system any form of regulation on the trunks at the input end? So far as I can gather from the paper, all the regulation is done at the 33/6·6-kV transformer where it is stepping down. Consider the case of a 33-kV trunk from a generating station or a grid substation which is looped in to feed a 33-kV substation where step-down transformers are installed, and then goes on to feed another similar 33-kV substation. It is quite possible in such circumstances that two regulating points could not be justified, and that it would be necessary to install one regulating point only at the input end. I should like to ask the authors whether such a case arises in the Manchester area. They give some interesting figures regarding the cost per kVA of installing automatic regulation apparatus, and refer to the pilot wires. I presume that these were already installed; the cost of installing pilot wires, especially in an urban district, is rather heavy. It would be interesting to know how long it took for the complete automatic system to be installed and put into commission. With regard to the 6·6-kV distribution, what is the avereage number of substations, and what is the loading and length per ring main? In Birmingham, where we have a very dense industrial area towards the centre of the city, we have some very heavily loaded e.h.t. ring mains with a considerable number of consumers, and, like the e.h.t. networks of other large cities, these were originally laid down in the days when the generating stations were comparatively near the loads which they fed. As the load increased, additional generating stations were erected on the outskirts of the city, and, by means of 33-kV trunks, main substations have had to be introduced to feed into that network. It will be appreciated that the voltage regulation of such a heavily loaded e.h.t. network becomes quite a problem when not only are long ring-mains feeding numerous industrial consumers concerned, but, in addition, the main substations to which these ring mains are connected also have rotary-convertor plant installed for supplying d.c. low-tension networks. It would be interesting to learn from the authors whether a similar condition of affairs prevails in any area in Manchester, and whether automatic voltage control on the lines suggested by them is successful in such circumstances.

**Mr. F. S. Naylor:** I intend to confine my remarks to the question of loss of revenue due to poor regulation. I take it that the authors are advocating automatic voltage regulation, firstly at consumers' premises, and, if this is not possible, then at distribution substations. Having described the field of these two possibilities, they go on to say that it is better to install it, for reasons of economy, in the main substations. Their grounds for advocating automatic voltage regulation are better service, less excitation loss, and increase in revenue. The authors have not produced a very strong argument for installing complicated apparatus of this nature, particularly in rural areas. It should be borne in mind that the type of argument that would appeal to a supply engineer is a statement of the economies that would be effected by installing such gear. I think the argument lies in the question of loss of revenue owing to poor regulation. The energy consumed is undoubtedly increased by keeping up the voltage of the supply to consumers, but incidentally the maximum demand on the system is increased. In the case of energy consumed it can be shown that at 100 per cent load factor—which, of course, is never obtained—the increase in consumption effected by maintaining the voltage variation between the limits of + 4 per cent and declared voltage, as against ± 4 per cent of the declared voltage, is 8·51 per cent. There is also an increase in the maximum demand on the system by the same amount, i.e. 8·51 per cent. Will the revenue derived from the increased consumption be outweighed by the increased cost of the bulk supply on maximum-demand charges? I should like to ask the authors whether they have considered this point, which has struck me as being of sufficient importance to warrant a close investigation of a particular case, namely that of a 50-kVA transformer in the Shropshire distribution area. This transformer has a load factor of 16·2 per cent, and the revenue from it amounts to £943 per annum. The cost of the bulk supply to provide the energy consumed from this transformer is £162 10s. (fixed charge), plus £55 per annum (running charge), based on £3 5s. per kVA plus 0·185d. per unit. On running charges only the difference between the revenue that was obtained and the cost of the units is £943 — £55, i.e. £888 per annum. By taking the annual load curve of the transformer and replotting it on the assumption that the voltage varied between + 4 per cent and the declared voltage, instead of ± 4 per cent, it was found that the amount of energy supplied would have been increased by 4·8 per cent. Taking into account all the factors that I have mentioned, it was found that the increase in revenue outweighed the increase in bulk-supply costs, and that there was a net increase in profit of £28 per annum. Automatic voltage-regulator equipment would cost about £100–£150 for a 50-kVA transformer, and the capital charges on such equipment might be approximately £13 (based on 8·5 per cent per annum for interest and sinking-fund charges). It will be seen, therefore, that in order to obtain an

increase in revenue of £28 one could afford to spend £13 in providing voltage-regulator equipment. This case illustrates the type of economic argument which appeals to the supply engineer.

**Mr. W. Burton:** It would be true to say that as regards voltage regulation what is correct practice for an urban district is in many instances correct practice for an area comprising urban and rural districts. Whereas in Manchester or Birmingham one 500-kVA transformer is employed per substation, in rural areas transformers of 15 to 25 kVA are largely used, and as the authors state that it would not be a practical proposition to provide regulating equipment for each small substation in urban areas, it obviously would not be economical to provide it in a rural district. I would like to support the authors' contention that regulation should be provided at certain strategic points. Some years ago, when areas were small, the only equipment installed for purposes of regulation—whether a.c. or d.c.—was at the power station, but this would not be sufficient to-day. Regulating equipment for each substation would theoretically be very satisfactory, but it would be costly, and I do not think it would necessarily be the best system to adopt. In the first place, we get the advantages of diversity by regulating at key points: the high power-factor load helps the low power-factor load. Moreover, a 500-kVA regulator will deal with 1 000 or 1 500 kVA on various feeders, owing to the effect of the diversity factor. A point which arises more particularly on rural supplies is the sudden voltage-drop which occurs when a piece of apparatus is switched on. For example, on a 15-kVA supply to one, two, or three private houses, a considerable drop will occur when a big cooker, radiator, or geyser, is switched on, whereas with a 500-kVA unit this would not matter so much. Assuming that voltage regulation is carried out at the key points, the effect of this sudden drop is small. I think that in rural areas it would be easier to maintain a constant voltage if the makers would supply transformers with low regulation. The objection will be raised to this that such transformers are not so suitable for dealing with short-circuits; but rural transformers are generally installed so far away from the power stations that I think we could do something along these lines. It is unwise to reduce the copper cross-section too much, as there frequently arises an emergency when the demand is temporarily increased, e.g. a very cold snap will increase the supply taken for heating. In such an emergency the chief consideration will be the current-carrying capacity of the mains.

## The Authors' Reply to the Discussions at Swansea, Edinburgh, Leeds, and Birmingham.

**Messrs. W. Kidd and J. L. Carr** (*in reply*): Many of the comments made in these discussions are covered by our earlier reply.* We propose to deal with the remainder according to their subject-matter.

*Working Limits of Voltage.*

We agree with Mr. Siviour that it would have been preferable to apply the increased limit of permissible voltage variation to special cases only.

In reply to Mr. Richards we would observe that, since the adoption of automatic voltage control, complaints regarding voltage conditions have been reduced by approximately 50 per cent. Of these, only a small percentage were found to be justified, and a number of these cases were satisfactorily met by overhauling relay settings. Expense in distribution alterations has therefore been reduced very considerably.

*Selection of Voltage-Regulating Position.*

With reference to a point raised by Mr. Rawll, on the system described in the paper no voltage adjustment is made at the input to the main transmission system. Where this is possible, the variation required at other points is naturally reduced. The whole of the automatic equipment was installed within a year.

The case cited by Mr. Maxwell probably lends itself admirably to voltage adjustment at main busbars, provided that the amount of distribution copper is adequate.

We are pleased to note that Mr. Hedley supports our argument regarding the location of tap-changers.

*Urban Distribution Methods.*

Replying to Messrs. Clark, Wilson, Rawll, and Longman; as is described in the paper, distribution networks (i.e. without medium-voltage feeders) are installed. The maximum permissible voltage-drop in distributors is the same under all conditions, and is not governed by the location of regulators at different points.

Main transformers are not operated in parallel on the system referred to, but may be paralleled during system changes for short periods. At such times, however, the taps are adjusted manually.

As stated earlier, the average number of consumers' and distribution substations connected to a 6 600-volt feeder is 4. The loading varies considerably according to the location and state of development. Converting substations are also connected, and usually constitute important switching centres. Automatic voltage control has been successfully applied to the whole system. The substations shown in Fig. 1 supply neighbouring districts also.

*Load Densities and Development.*

In response to the remarks of Mr. Eccles we would observe that the system described has been designed to meet future requirements: the necessity for this is not always appreciated.

We are in complete agreement with Prof. Baily's forecast of future development.

It is impossible to give a general limit of load for high-voltage supply. Much depends on local conditions; and the methods suggested by Mr. Dundas appear quite satisfactory.

*Effect of Seasonal Load.*

We would inform Mr. Barlow that during the severe weather referred to, certain substation plant was heavily taxed; no drastic overhaul of distribution plant was, however, necessary.

* See page 317.

*Voltage Charts.*

We would inform Mr. Sumner that the charts referred to were plotted from actual voltage records to show total variations. The result is to reduce the voltage at light-load and increase it at heavy-load periods.

*Increase in Consumption due to Better Voltage Conditions.*

If the lighting load is increased on account of the voltage being low, the drop is accentuated. We would refer Mr. Sumner to remarks made by other speakers on this point in earlier discussions.

The particulars given by Mr. Naylor for Shropshire are interesting and valuable: increase in revenue does not, however, constitute the main argument for automatic voltage control. This point is dealt with more fully in our earlier reply.

*Transformer Losses.*

Dealing with the remarks of Messrs. Butler and Eccles, we would observe that, in a system without voltage control, the tappings of distribution transformers are arranged to give approximately the declared voltage at times of heavy load. At other times the distribution voltage increases because of reduction in voltage drop in the component parts of the system. On a controlled system the voltage is adjusted so as to give the declared value on distributors at light load, and to increase by the amount required to compensate for distributor and other drops at the maximum load: the average voltage is therefore lower. Figs. 11 and 12 were prepared from actual voltage measurements, and the remaining particulars computed from reliable transformer data.

*Distribution Transformers.*

The following remarks cover points raised by Messrs. Butler, Eccles, Goodman, Hedley, and Longman.

The transformers are designed with a flux density of 12 500 lines per $cm^2$ at rated voltage, and are the products of reputable makers. The limitation of flux density at rated voltage is highly desirable, although the value mentioned by Mr. Eccles is, if anything, rather lower than is essential on a controlled system. The importance of this point is also stressed by Mr. Hedley. The average size of distribution transformer employed is between 250 and 500 kVA. The adoption of very low-reactance transformers might necessitate the provision of additional reactors at other points, because of short-circuit considerations. Distribution-transformer tappings are now specified to be $\pm 2\frac{1}{2}$ per cent and also $+ 5$ per cent in addition to normal.

*Compensators.*

The difficulties attending remote control by pilot wires from the distributor, as suggested by Mr. Nethersole, are considerable. With an isolated distributor and a carefully adjusted compensator the effect would, of course, be the same. The regulator, however, caters for average conditions over a wider area; and, provided that due care has been paid to the lay-out of the local system, control by means of a compensator is satisfactory.

*Voltage Variation to Initiate a Tap-Change.*

Replying to Mr. Bentley, although the system is intended to maintain approximately constant voltage at consumers' premises, minor alterations must obviously occur before the apparatus will operate.

*Effect of Short-Circuit.*

Dealing with a point raised by Mr. Eccles, we would remark that no trouble has yet been experienced due to tap-changing during system faults. Faults on the 6 600-volt system are comparatively rare, and the probability that these may synchronize with tap-changes is remote.

*Harmonics.*

We should like to correct Mr. Goodman's impression regarding harmonics. No trouble from this cause has been experienced on the supply system described; but difficulties are not unknown in other places. In our view, the maximum flux density should not exceed 13 000 lines per $cm^2$.

*Time of Operation for Tap-Change.*

In reply to Mr. Butler, the time of 1 half-cycle for interruption of the current refers to the actual contactor and not to the complete tap-change.

*Tapping Switch.*

A point raised by Mr. Clark is explained by the fact that the tap-changers are provided with definite stops, for the purpose of securing accurate registration of contacts. Operation is by means of mains supply.

*Supervisory Gear.*

We would inform Mr. Parker that since metallic circuits are available, no necessity has arisen for the use of the carrier system. The installation of a routine tester for supervisory equipment has not yet been found necessary; wear and tear is naturally much less than in a busy automatic telephone exchange.

We would inform Mr. Nethersole that the object of the double operation before switching is to provide a delay so as to reduce the possibility of incorrect selection. In the early stages, at least, this has been found to be a wise precaution.

In reply to Mr. Richards's inquiry regarding the supervisory system, lines have not been rented from the Post Office.

*Maintenance and Reliability.*

Dealing with the remark made by Mr. Butler, we would observe that although tap-changers remove the completely static character of the transformer, experience does not indicate any considerable impairment of reliability.

The information given by Mr. Longman is very encouraging.

# AN INVESTIGATION INTO THE FACTORS CONTROLLING THE ECONOMIC DESIGN OF BEAM ARRAYS.

By T. WALMSLEY, Ph.D.

(*Paper first received* 10*th August, and in final form* 25*th October*, 1933; *read before the* WIRELESS SECTION 3*rd January*, 1934.)


SUMMARY.

The engineer, faced with the problem of designing beam arrays for transmission and reception purposes, should satisfy himself that his proposals are technically and economically sound. He thus requires to know the anticipated performance and cost of each section of complete array systems, to enable him to arrange an equable balance between gain in distant field strength and capital outlay needed to obtain this gain. The factors concerned in the solution of the problem are:—

(1) the angle of elevation of the axis of the main lobe of radiation of the beam to give the highest gain in field;

(2) the economical design, fabrication, and erection, of structures to support the array;

(3) the most economical type of feeders and transmission lines to use, having regard to the energy losses and pick-up of the lines.

These three factors are investigated in the paper, and, from the results of the investigation, deductions are drawn regarding the gain of a horizontal type of array composed of numbers of half-wave radiators grouped in various ways. It is shown that the cost of array systems increases rapidly not only with wavelength but also with the angle to the vertical at which the radio energy is required to be projected or received.


TABLE OF CONTENTS.



LIST OF SYMBOLS.

$a$ = outer radius of inner conductor of tube transmission line, in cm.
$b$ = inner radius of outer conductor of tube transmission line, in cm.
$b_p$ = breadth of section of post.
$b_s$ = width of section of bracing.
$c$ = velocity of propagation of electromagnetic waves in space.
$D$ = separation of wires of transmission line.
$d_1$ and $d_2$ = constants depending on $\sigma$, $f$, $\kappa$, and $\theta$.
$f$ = frequency, in cycles per sec.
$f_c$ = critical frequency.
$h$ = height above the earth.
$I$ = current.
$j = \sqrt{(-1)}$.
$K$ and $K_1$ = constants depending on $d_1$, $d_2$, and $\theta$.
$k_1$, $k_2$ = constants depending on wind load.
$l$ = length.

LIST OF SYMBOLS—*continued*.

$M$ = number of rows of radiators in array.
$N$ = number of radiators per row.
$n$ = number of wavelengths.
$P$ = top horizontal pull on tower, in tons.
$P_1$ = power radiated by standard aerial.
$P_N$ = power radiated by compared aerial.
$R$ = equivalent resistance of tube transmission line per cm, in ohms.
$R_1$ = resistance of single standard half-wave radiator.
$R_N$ = resistance of individual half-wave elements of an array consisting of $N$ radiators.
$r$, $r_0$ = great distances from radiators.
$r_g$ = minimum radius of gyration of steel section of strut.
$t$ = time, in seconds.
$W$ = mass of tower, in tons.
$x$ = (height of apex of ray above earth)/(radius of earth).
$\beta$ = angle between the projection of the ray on the $zx$ plane and the $z$ (or vertical) axis.
$\epsilon$, $\epsilon_1$, $\epsilon_2$ = amplitudes of electric fields at receiving measuring set.
$\epsilon_{1\beta}$, $\epsilon_{N\beta}$ = electric fields due to standard and compared aerials respectively.
$\theta$ = angle between ray and vertical axis.
$\theta_1$ = angle between direction of radiator and direction of ray.
$\theta_c$ = critical angle.
$\kappa$ = dielectric constant.
$\lambda$ = wavelength.
$\mu$ = magnetic permeability.
$\rho$ = resistivity of transmission-line wire or tube.
$\sigma$ = conductivity of soil, in electrostatic units.
$\omega = 2\pi f$.

## Part 1. MEASUREMENT OF THE BEST ANGLE OF PROJECTION OF BEAMS OF RADIATION.

### INTRODUCTION.

Although in the design and lay-out of beam aerials great care is taken to ensure that the axis of the beam lies in the great-circle path joining the transmitting and receiving stations, little attention has been given to the correct angle of elevation of the axis of the beam.

The opinion in general is that in the case of long-distance propagation a low angle of elevation to the earth is productive of the best results. This conclusion was reached both by Meissner and by Eckersley. The former conducted tests between Berlin and Buenos Aires by rotating a parabolic reflector about an energized

horizontal antenna placed along the focal axis of the reflector curtain. By rotating the parabola the beam was projected at different angles of elevation, and Meissner* concluded that low angles gave the highest field at the distant point. Since the width of the beam in the vertical plane was considerable, and since, owing to reflection from the earth, the direction of the maximum field made a large angle with the horizontal, Meissner's tests yielded no definite conclusion.

T. L. Eckersley† conducted tests by suspending three short-wave vertical aerials at different heights above the earth. Since stronger signals were received when the higher aerials were used, he concluded that radiation making a low angle with the horizontal was the most effective. His results, like those of Meissner, lacked definiteness.

A few years ago the author, on behalf of the Radio Section of the Post Office Engineering Department, made tests between Rugby and New York and between Rugby and Australia with the idea of ascertaining what improvement could be expected by projecting a beam with low-angle radiation. Three aerials were built for the purpose. The first consisted of 19 tiers of half-wave horizontal radiators spaced half a wavelength apart between adjacent tiers, the top radiators being 10 wavelengths above the earth; the second consisted of 64 horizontal half-wave radiators, arranged in 8 tiers one above the other spaced half a wavelength apart, the top tier being 4 wavelengths high; the third comprised 112 half-wavelength vertical radiators arranged in vertical lines of 7 radiators, the top of the uppermost radiators being 4 wavelengths above the earth. The first aerial was designed for a wavelength of 16·11 m and was directed on New York; the second and third were designed for 28·6 m and were directed on Sydney, Australia. Tests made on all three aerials gave no outstanding results. They served to demonstrate, however, that during the few days on which the tests were made there was no advantage in the particular low angle at which the maximum energy was emitted.

In the tests outlined, inability to vary the angle of maximum radiation was a marked disadvantage, and to remove this defect arrangements were made to build aerials that could be raised and lowered. This provided a ready means of varying the angles of radiation in the vertical plane. The purpose of Part 1 of this paper is to give an account of the experiments made by the author to ascertain the optimum angle of projection of a "beam" of radiation.

## Outline of Method.

The general method of procedure was to raise step by step small arrays consisting of groups of half-wave radiators, and alternately to energize a fixed standard aerial and the raised aerial and measure the field strength at the distant receiving station. A large number of readings were taken of the resulting field strength for each position of the raised aerial. By this means it was possible to ascertain the ratio of the field strengths due to the fixed aerial and to the movable aerial for each height of the latter. By plotting curves (in a manner to be explained in detail later in the paper) of the calculated ratio of the radiated fields emitted in directions making various angles with the vertical, and comparing with the ratio of the received fields, it was possible to form an opinion upon the best angle of propagation. To supplement the tests made by raising aerials, tests were also carried out by comparing the received field strengths due to a number of horizontal aerials fixed at different heights above ground. The relative values of the received field strengths were then used to ascertain the angle of projection. The theoretical and practical aspects of the problems associated with both methods of test are dealt with in detail later in the paper. The majority of tests were conducted over a period of 1 year between the transmitting station situated at Rugby and receiving stations at Netcong and Holmdel, U.S.A. Engineers of the American Telegraph and Telephone Co. and the Bell Telephone Laboratories made the receiving measurements. Tests were also made between Rugby and Teneriffe, and Rugby and Berlin, with the assistance of the Campania Telefonica National de Espana, and the Reichspostzentralamt. In most cases the tests were made during the period when daylight prevailed throughout the route, since during this period—as far as radio telephone services are concerned—the greatest volume of traffic is passed. This period is therefore of the most importance on economic grounds. Throughout the tests horizontal and vertical aerials were employed at the receiving stations. Horizontal aerials have the great advantage over vertical aerials that their radiation properties are less dependent upon the earth constants. In all calculations of radiation characteristics, however, the measured values of the earth constants were used.

* See Bibliography, (1).

† *Ibid.*, (2).

## Calculation of Radiation-Characteristic Diagrams.

The field strength due to a doublet $I\delta l$ is the resultant of the direct field and the field reflected from the earth. The reflected field can be regarded as arising from an image of the doublet of strength $(K - jK_1)I\delta l$, where $K$ and $K_1$ are constants depending on the dielectric constant and conductivity of the earth and the angle of incidence of the reflected ray. Thus in Fig. 1(*a*) the field due to the horizontal half-wave radiator and its image at a point P, whose distance $r$ from the origin O is very great compared with the distance ($2h$) between object and image, is proportional to

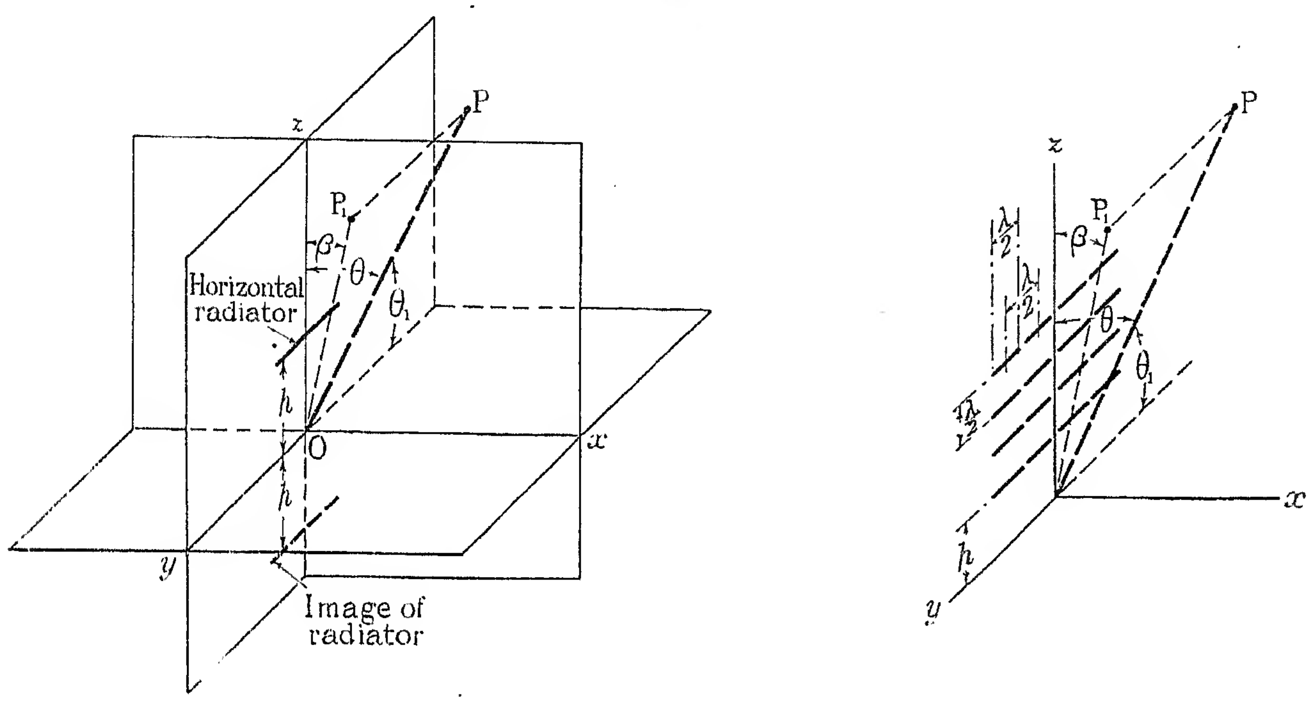
$$\frac{I}{r}\frac{\cos(\frac{1}{2}\pi\cos\theta_1)}{\sin\theta_1}\left[\sin\left(\omega t + h\frac{2\pi}{\lambda}\sin\theta_1\cos\beta\right) + \sqrt{(K^2+K_1^2)}\sin\left(\omega t - h\frac{2\pi}{\lambda}\sin\theta_1\cos\beta - \arctan\frac{K_1}{K}\right)\right]$$

where $\theta_1$ is the angle between the direction of the radiation (i.e. the $y$ axis) and the direction of the ray OP; $\beta$ is the angle between the projection of OP on the $zx$ plane and the $z$ axis; $\lambda$ is the wavelength; $\cos(\frac{1}{2}\pi\cos\theta_1)/\sin\theta_1$ is a factor arising from the assumed sinusoidal current distribution in the radiator; and $\omega t = 2\pi ft$ has the usual reference to frequency and time. If, instead of being a single horizontal half-wave radiator, the array consists of $N$ similar radiators in a row at equal distance above the earth and $M$ such rows

separated by ½ wavelength and placed in a vertical plane (Fig. 1*b*), the value of the resultant field is that due to a single radiator multiplied by the array factor:—

$$\frac{\sin\left(\frac{1}{2}\pi N \cos\theta_1\right)}{\sin\left(\frac{1}{2}\pi\cos\theta_1\right)}\cdot\frac{\sin\left(\frac{1}{2}\pi M\sin\theta_1\cos\beta\right)}{\sin\left(\frac{1}{2}\pi\sin\theta_1\cos\beta\right)}$$

Thus, since the height $h$ of the elementary system of Fig. 1(*a*) must now be replaced by the height




(*a*) (*b*)

Fig. 1.

$h + \frac{1}{2}(M - 1)\lambda/2$ of the array system, the complete expression for the amplitude of the field at a distant point becomes proportional to:—

$$\frac{I}{r}\sqrt{\left\{1 + K^2 + K_1^2 + 2(K^2 + K_1^2)^{\frac{1}{2}}\cos\left[2\pi\left(\frac{2h}{\lambda} + \frac{M-1}{2}\right)\sin\theta_1\cos\beta + \arc\tan\frac{K_1}{K}\right]\right\}}$$
$$\times\frac{\cos\left(\frac{1}{2}\pi\cos\theta_1\right)}{\sin\theta_1}\cdot\frac{\sin\left(\frac{1}{2}\pi N\cos\theta_1\right)}{\sin\left(\frac{1}{2}\pi\cos\theta_1\right)}\cdot\frac{\sin\left(\frac{1}{2}\pi M\sin\theta_1\cos\beta\right)}{\sin\left(\frac{1}{2}\pi\sin\theta_1\cos\beta\right)} \quad . \quad (1)$$

The values of $K$ and $K_1$ may be obtained from the expressions

$$K = -\frac{d_1^2 + d_2^2 - \cos^2\theta}{d_1^2 + d_2^2 + \cos^2\theta + 2d_1\cos\theta};$$

$$K_1 = \frac{2d_2\cos\theta}{d_1^2 + d_2^2 + \cos^2\theta + 2d_1\cos\theta};$$

$$d_1^2 = \tfrac{1}{2}\left\{(\kappa - \sin^2\theta) + \sqrt{\left[(\kappa - \sin^2\theta)^2 + 4(\sigma^2/f^2)\right]}\right\};$$

and

$$d_2^2 = \tfrac{1}{2}\left\{-(\kappa - \sin^2\theta) + \sqrt{\left[(\kappa - \sin^2\theta)^2 + 4(\sigma^2/f^2)\right]}\right\};$$

the negative root being taken for the value of $d_2$.*

Also $\sigma$ = conductivity, in electrostatic units; $f$ = frequency, in cycles per sec.; $\kappa$ = dielectric constant; and $\theta$ = angle between the vertical axis and the direction OP.

Regarding the value of $\kappa$, Strutt† gives 10 for fairly dry soil and 15 for soil after rain, at a frequency of $2 \times 10^4$ kilocycles per sec. Ratcliffe and White‡ give $48 \cdot 5$ to $5 \cdot 3$ at a frequency of $2 \times 10^4$ kilocycles per sec., according to whether the soil is wet or dry.

More recently, tests were made at the National Physical Laboratory on samples of soil taken from Rugby radio station. The value of $\kappa$ was found to vary considerably, one sample of loam and clay with a moisture content of 33 per cent having a value of 43 and another sample with a moisture content of 13 per cent having a value of 21. The frequency used for these tests was $10^4$ kilocycles per sec. Fortunately, considerable variation in the dielectric constant is possible without there being any appreciable effect on the value of the field strength, and since the majority of the propagation tests to be described later were made under dry conditions the value $\kappa = 10$ has been assumed in all calculations. Like the dielectric constant, the conductivity varies considerably with moisture content. For example, at a frequency of $10^4$ kilocycles per sec. the average value of the conductivity of the surface soil at Rugby to a depth of a little over 1 ft. varied from $7 \cdot 4 \times 10^8$ to $1 \cdot 25 \times 10^8$ electrostatic units as the average moisture content changed from 46 per cent to $17 \cdot 5$ per cent. The mean of these two values agrees well with the figure given by Barfield, who calculated from his tests by the tilt method that the Rugby soil had a conductivity of $4 \cdot 4 \times 10^8$ electrostatic units at a frequency of 8 450 kilocycles per sec. and $4 \cdot 1 \times 10^8$ electrostatic units at 12 700 kilocycles per sec. The moisture content of the earth was not measured during Barfield's tests, but it is known that the soil was rather damper than when the majority of the propagation tests described later in this paper were made. Having regard to the weather con-

* See Bibliography, (13). † *Ibid.*, (3). ‡ *Ibid.*, (4).

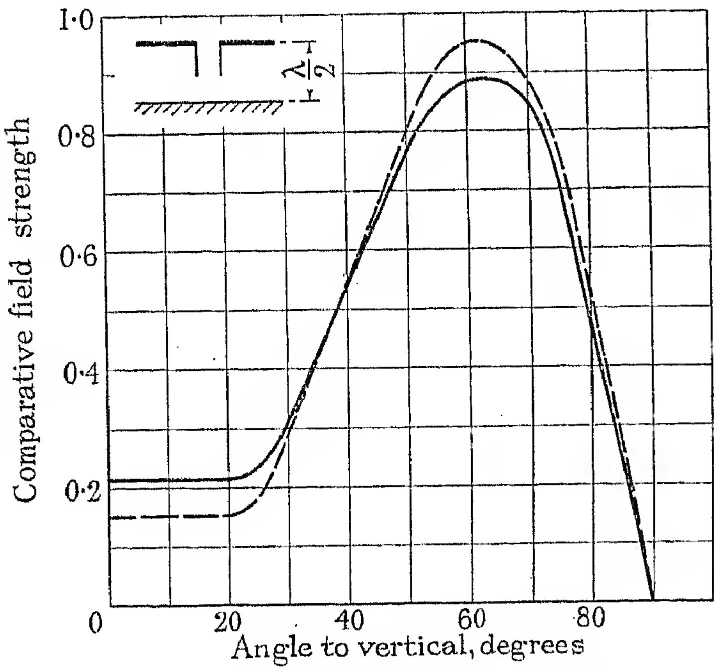



FIG. 2.—Comparative field strength due to single horizontal radiator ½ wavelength above earth in vertical plane normal to radiator.

- - - - $\kappa = 10, \sigma/f = 20$.
——— $\kappa = 10, \sigma/f = 5$.

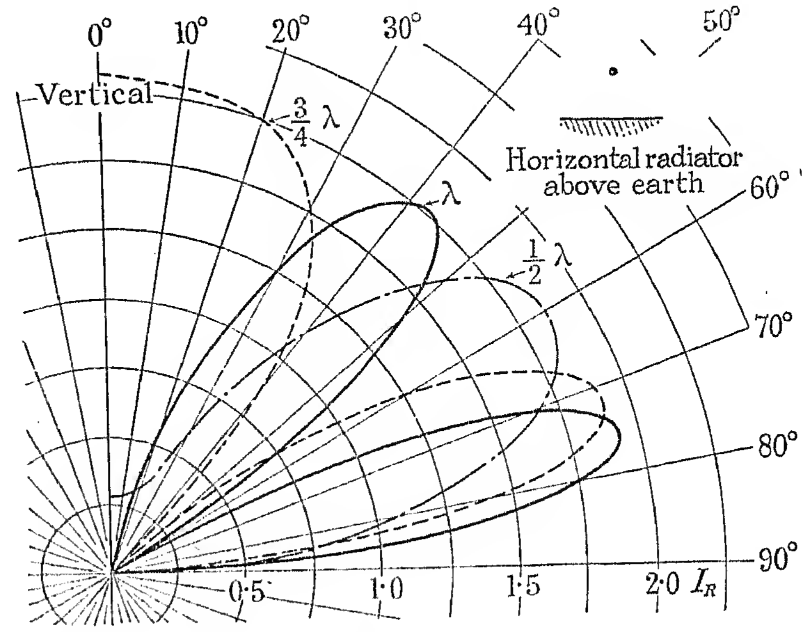



FIG. 3.—Vertical polar diagram of horizontal 1-tier aerial in normal vertical plane.

——— Radiator 1 wavelength above earth.
– – – Radiator ¾ wavelength above earth.
—·— Radiator ½ wavelength above earth.

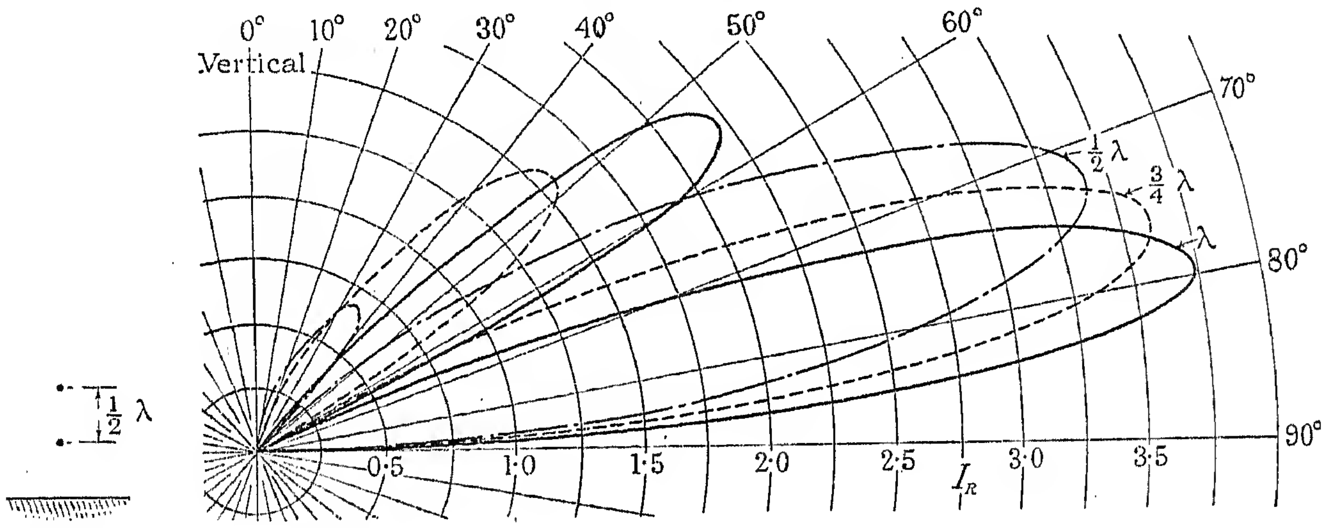



FIG. 4.—Vertical polar diagram of horizontal 2-tier aerial in normal vertical plane. (The sketch on left represents horizontal radiators above earth.)

——— Lower radiator 1 wavelength above earth.
– – – Lower radiator ¾ wavelength above earth.
—·— Lower radiator ½ wavelength above earth.

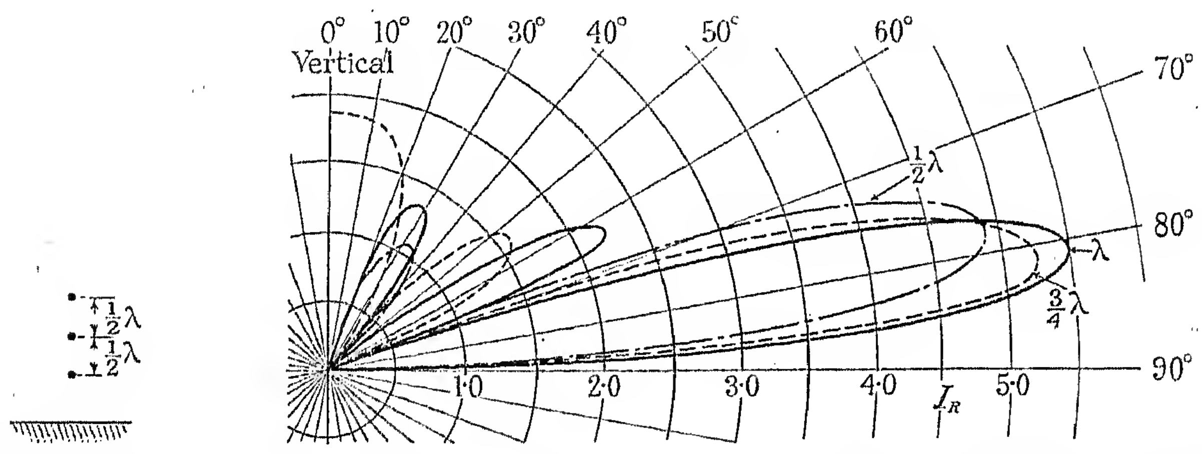



FIG. 5.—Vertical polar diagram of horizontal 3-tier aerial in normal vertical plane. (The sketch on left represents horizontal radiators above earth.)

——— Lowest radiator 1 wavelength above earth
– – – Lowest radiator ¾ wavelength above earth
—·— Lowest radiator ½ wavelength above earth
$\sigma/f = 20, \kappa = 10$.

ditions prevailing during these tests, the value $\sigma/f = 20$ has been assumed in all calculations. Since the frequencies used in the tests to be described varied between 15 000 and 10 000 kilocycles per sec., the assumed value

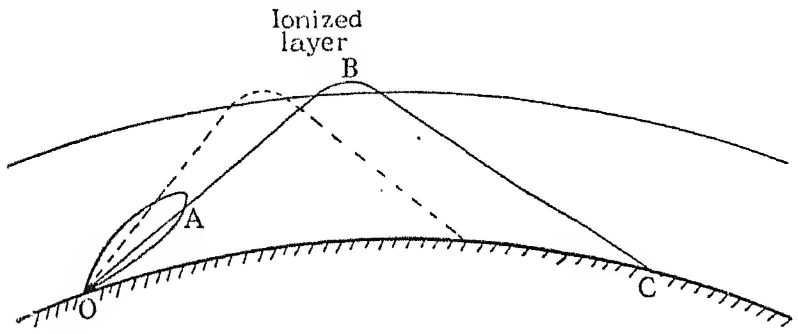



Fig. 6.

of $\sigma$ varied between $3 \times 10^8$ and $2 \times 10^8$ electrostatic units. The extent to which the conductivity affects radiation is illustrated in Fig. 2, which gives the comparative field in the vertical plane normal to the hori-

### Method of Computing Angle of Projection of Beam.

Assuming that the distant received signal is the result of energy propagated along one path only, e.g. the path OABC of Fig. 6, then, as the test aerial is raised, the received field in the ideal case should vary in intensity proportionally to the amplitude of the field emitted along the favoured direction. Suppose, for instance, that the received ray leaves the transmitting end in a direction making an angle of 80° with the vertical. If now a curve is plotted giving the comparative calculated amplitude of the field along this direction as the aerial is raised, the received field strengths should also vary in the manner indicated by the curve. Curves of this type, calculated from equation (1), have been plotted in Figs. 7 and 8 for different angles to the vertical. Fig. 7 refers to an aerial comprising two tiers of horizontal radiators arranged one above the other, the current in the radiators being equal, whilst Fig. 8 refers to a similar type of aerial having three tiers instead of two, the

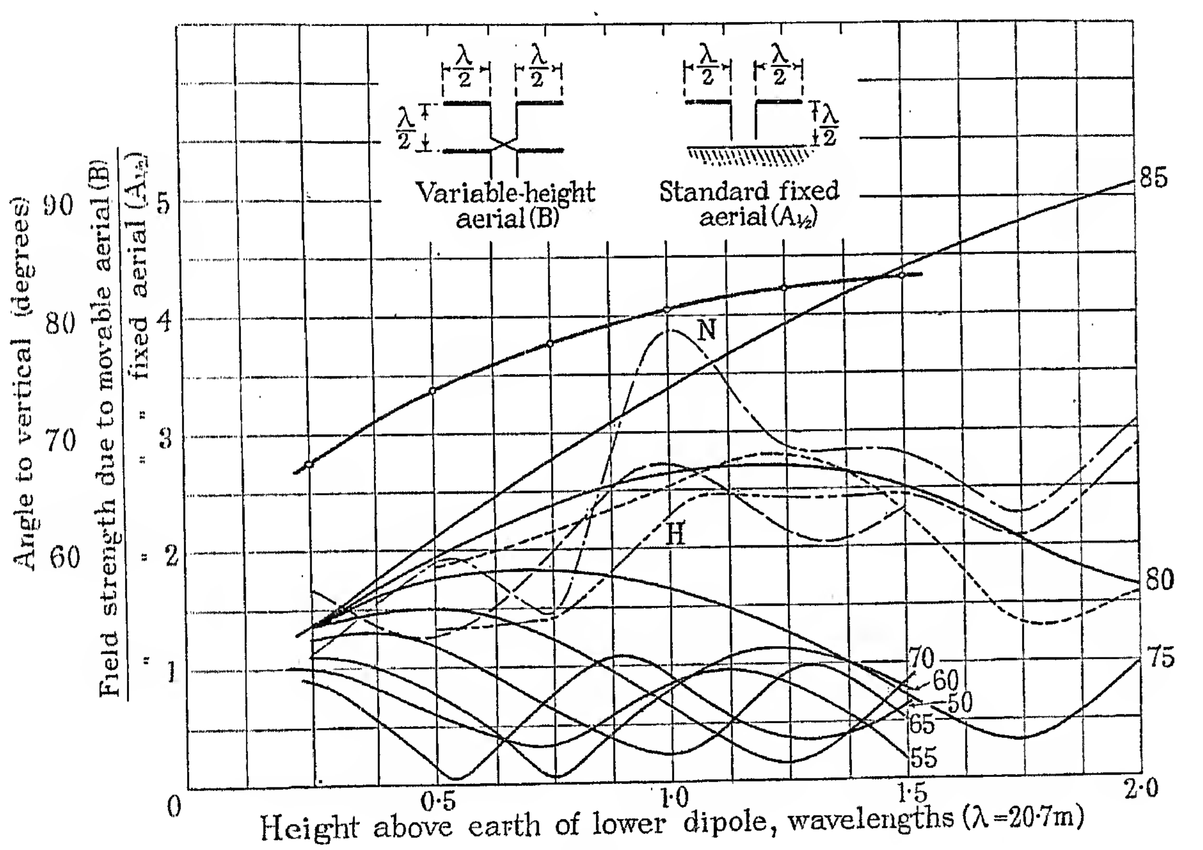



Fig. 7.

Calculated strength of fields due to aerial (B), at various angles to vertical, assuming $\kappa = 10$, $\sigma/f = 20$.
Strength of field measured at Netcong, U.S.A., between 25th and 27th January, 1932, 1300–1600 G.M.T.
Strength of field measured at Netcong, U.S.A., between 7th, 8th, 10th, and 11th June, 1932, 1400–1900 G.M.T.
Strength of field measured at Netcong, U.S.A., between 21st and 22nd May, 1932, 1425–2010 G.M.T.
Strength of field measured at Holmdel, U.S.A., between 21st and 22nd May, 1932, 1300–1600 G.M.T.
Angle to vertical of maximum radiation of primary loop.
All values are relative to those due to the fixed standard aerial and are based on equal power input.

zontal radiator situated ½ wavelength above earth. It will be seen that, particularly at high angles of incidence (with which this investigation is chiefly concerned), changing $\sigma/f$ from 20 to 5 does not greatly affect the value of the field strength.

From equation (1), the comparative values of field strength at points equidistant from the aerials in the principal vertical plane have been calculated. The resultant polar diagrams are typified in Figs. 3, 4, and 5, which show the vertical radiation characteristics of 1-, 2-, and 3-tier aerials having the same span raised ½, ¾, or 1 wavelength above earth.

separation between tiers being ½ wavelength in both cases. The vertical ordinates in all cases are given in terms of the field due to a single-tier aerial, fixed half a wavelength above the earth, and having the same power input as the 2- and 3-tier aerials. Comparison at short intervals with the field from some form of standard aerial is usually essential in long-distance field measurements, owing to the great variability of short-wave field strengths. In the ideal case, the curve of the received field-strength values should coincide with one of the family of curves of which examples are shown in Figs. 7 and 8. In the second method of test, usually three

aerials of the 2-tier horizontal-radiator type were employed. These were fixed so that their lower members were respectively $\frac{1}{2}$, $\frac{3}{4}$, and $1\frac{1}{4}$ wavelengths above the earth. By means of a rotary change-over switch, change-overs from one aerial to the other were made every few minutes over periods of several hours. Using the $\frac{1}{2}$-wavelength high aerial as a standard, by comparing the ratios of the received signals with the calculated ratios for various angles of propagation (plotted in Fig. 9) it should be possible to deduce the actual angle of propagation.

In both methods of test, however, the following two factors may operate to produce disagreement between the actual and the ideal. (1) The calculated vertical polar diagram and the actual polar diagram may not be similar. (2) The received field may be the resultant of energy projected at two or more angles of elevation and propagated along two or more paths.

With regard to (1), field-strength tests from aeroplanes flying above aerials have been made in America, England, and Germany. Probably the most thorough and convincing of these tests are those conducted in Germany by Baumler* and others. The investigators showed that there was a close agreement between the shapes of the calculated and of the measured vertical polar diagrams.

Tests were also devised by the author to provide a check upon the accuracy of the calculations of the vertical characteristics of the aerials, using horizontal-type radiators.

* See Bibliography, (5).

A 2-tier type of aerial was raised in $\frac{1}{4}$-wavelength steps from the height of $\frac{1}{4}$ wavelength to that of 2 wavelengths above earth. A single dipole with vertical twin open-wire feeders was fixed 1 300 ft. away at a height of 230 ft. Thus the angle of incidence of the received wave was approximately 80°. A field-measuring set of a type described later in this paper was fixed a few feet above the earth and connected to the twin feeder. By this means the relative values of the field were obtained as the transmitting aerial was raised.

The values of the field strength to be expected at the receiving dipole were calculated, due allowance being made for the difference in distance of the two transmitting-aerial radiators, and their images from the receiving aerial. These are plotted in Fig. 10, in which the measured values have also been inserted.

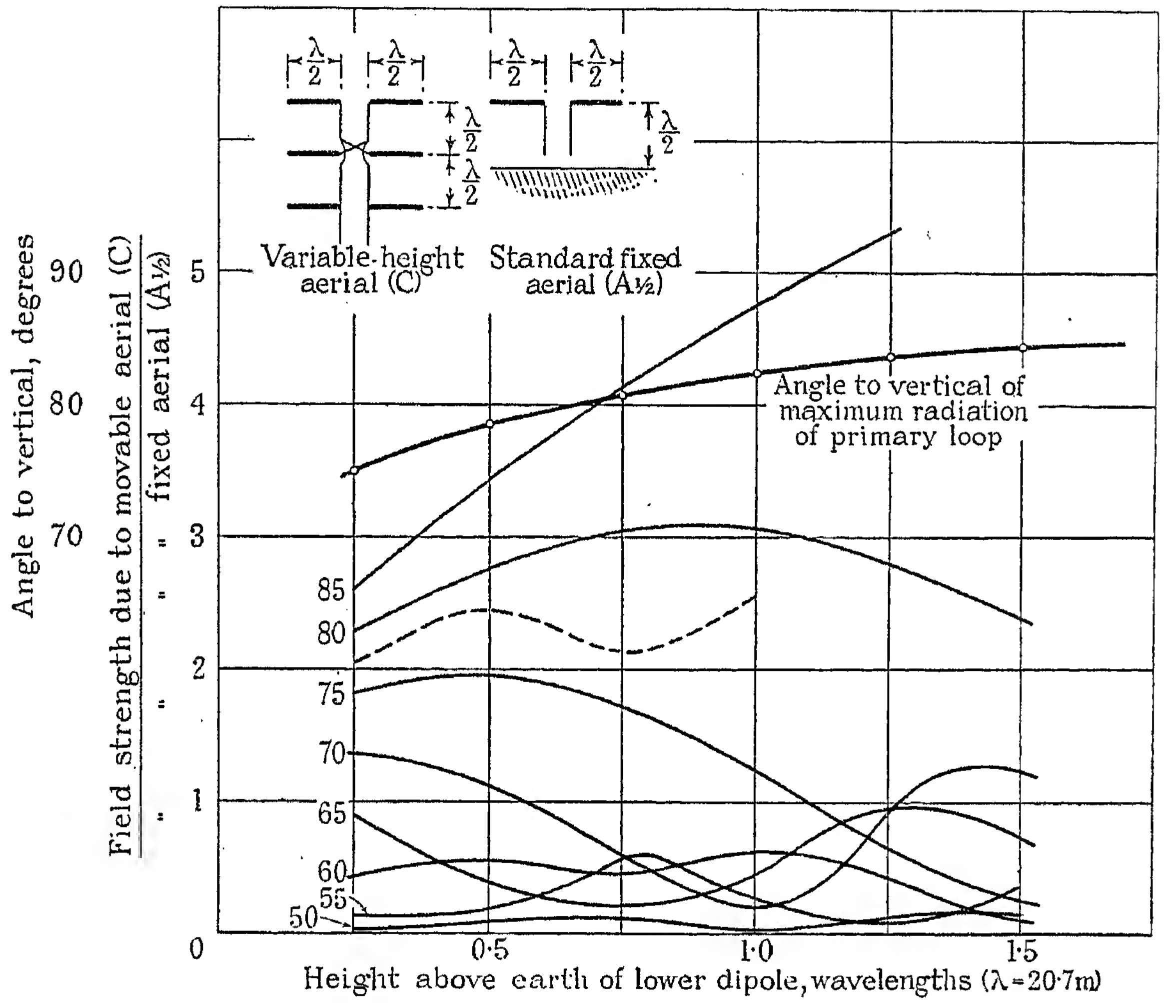



Fig. 8.

——— Calculated strength of field due to array (C), at various angles to vertical.
— — — Strength of field measured at Netcong, U.S.A., between 21st and 22nd January, 1932, 1300–1600 G.M.T.
All values are relative to those due to the fixed standard aerial and are based on equal power input.

The first fact of note is that the measured values lie very close to a smooth curve, which would certainly not be the case if appreciable reflections from masts, stays, and other objects, were obtained. The second fact is that whilst the aerial was being raised from 0·125 to 1·25 wavelengths high, the measured and calculated results were in almost entire agreement. The agreement was not so good for the greater heights of aerial. This discrepancy, however, is not such as to affect appreciably the deductions made in this paper.

A second check upon the accuracy of the calculated and measured vertical polar diagrams was made by comparing the fields due to a 2-tier aerial placed $\frac{1}{2}$ wavelength above earth and a 1-tier aerial fixed at the same

height. Reception was made in a manner similar to that already described for the previous test. The results of several observations showed the ratio of the two fields to be very constant and the agreement between the calculated and the measured ratios to be remarkably close. When the measured values had been corrected for the same antinodal current in all radiators, the measured ratio of the field strengths due to the two aerials was 1·8 : 1, whilst the calculated value was 1·81 : 1.

All the evidence points, therefore, to reasonable agreement between the calculated and measured values of the vertical polar characteristic of horizontal radiators.

With regard to the second possible source of error, namely that due to the propagation of the energy by more than one path, it is necessary to remember that even though there may be more than one path the relative phasing of the received fields will be constantly changing. Since only a slight variation in the length of any one path is needed for a complete reversal of phase, an average of a considerable number of measurements could be expected to reduce the error to reasonable dimensions. Consider, for example, two fields having amplitudes $\epsilon$ and $\epsilon_1$ at the input of the receiving measuring set. Assume that, owing to a progressive change in length of the path, there is a variable phase difference $\omega_1 t$ between the fields. The average amplitude of the resulting field is

$$\frac{1}{\pi}\int_{\omega_1 t=0}^{\omega_1 t=\pi} \sqrt{(\epsilon^2 + \epsilon_1^2 + 2\epsilon\epsilon_1 \cos \omega_1 t)}\,dt$$

By expanding the cosine terms, the value of this expression is obtained in the following form:—

$$\sqrt{(\epsilon^2+\epsilon_1^2)}\left(1 - \frac{1}{2^4}\cdot\frac{\epsilon^2\epsilon_1^2}{\epsilon^2+\epsilon_1^2} - \frac{3\cdot 5}{2^{10}}\cdot\frac{\epsilon^4\epsilon_1^4}{(\epsilon^2+\epsilon_1^2)^4}\cdots\right)$$
$$\doteqdot \sqrt{(\epsilon^2+\epsilon_1^2)}$$

Thus when the relative values of $\epsilon$ and $\epsilon_1$ are 2 : 1, i.e. when the maximum to minimum ratio is given by

$$\frac{\epsilon+\epsilon_1}{\epsilon-\epsilon_1} = \frac{3}{1}$$

(corresponding to 9·5 decibels), the measured strength of the combined fields will be 11·8 per cent (1·5 decibels) more than that of the greater single field. In the worst case, i.e. when the two fields have equal amplitude, the strength of the combined fields is 41 per cent (3 decibels) more than that of the greater single field. This condition assumes a ratio of maximum to minimum signals equal to infinity, a condition not experienced during the tests.

In the foregoing discussion the value of the frequency $f_1$ in the term $\omega_1 t = 2\pi f_1 t$ has been assumed to be constant. Even with haphazard fading, however, there is a strong probability that the received field strength, averaged over a considerable period so as to include a large number of fades, gives a reasonably accurate measure of the greatest of the two or more components. To test this matter, arrangements were made with the engineers of the Bell Telephone Laboratories to measure

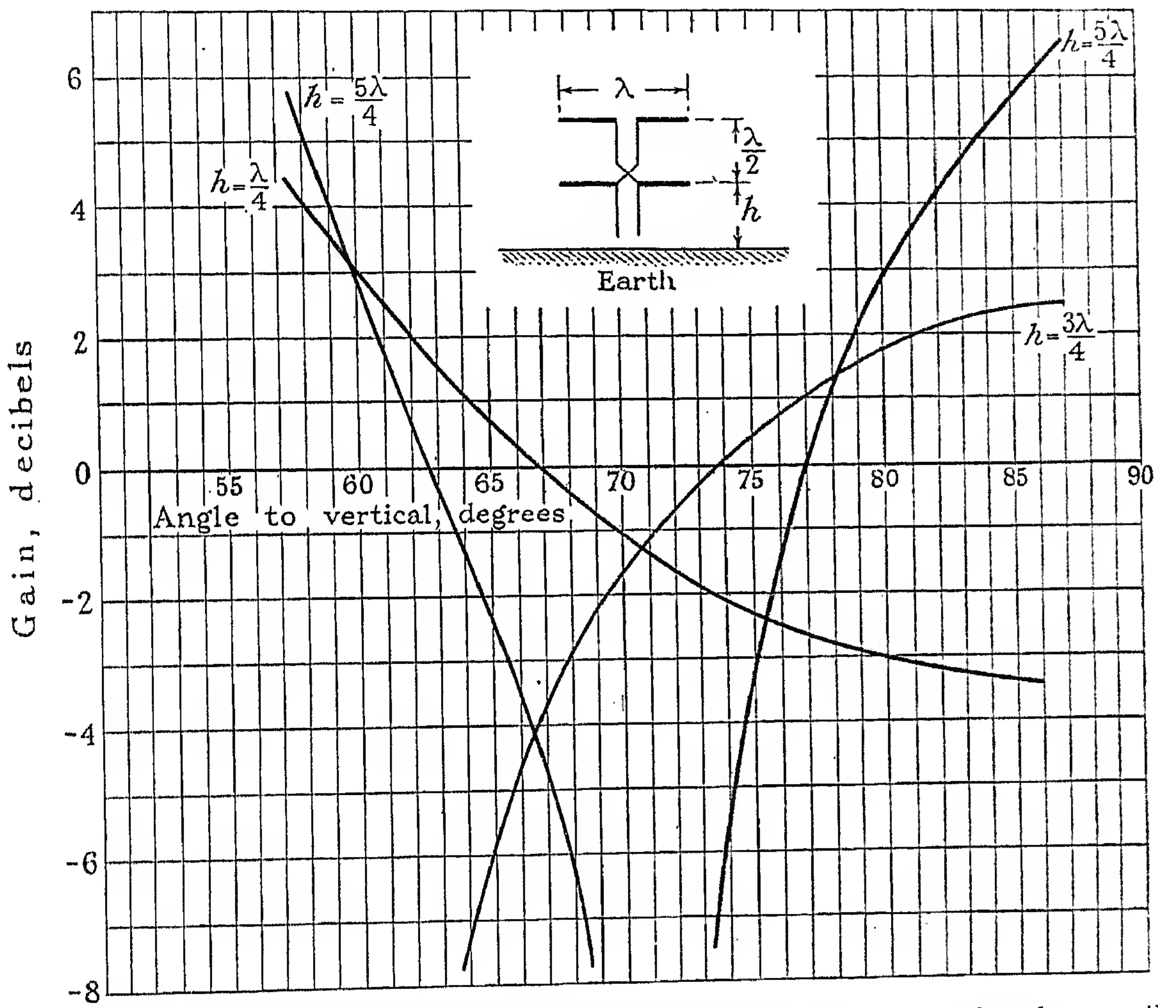



FIG. 9.—Gain, in decibels, of 2-tier system of horizontal radiators at various heights above earth, over the same system, having lower radiator ½ wavelength above earth. ($\kappa$ = 10, $\sigma/f$ = 20.)

the downcoming angles at the same time as the field-strength measurements of the Rugby emissions were made.

The tests were carried out by measuring the field strength simultaneously on two receivers, one being connected to a horizontal dipole $\frac{1}{4}$ wavelength above the earth and the other to a similar dipole 1 wavelength high. From the ratio of the readings on the two receivers, the downcoming angle could be deduced by making use of the vertical radiation characteristics of the aerials. The average downcoming angle measured over a period of a few minutes remained reasonably constant. Separate tests were also made by sending out from Rugby pulses of energy of 1/10 000 sec. duration 50 times per sec. and observing the result at Holmdel on a cathode-ray oscillograph.

Although the oscillograph record showed that several rays were arriving with different time-lags and therefore by different paths, the signal received by one path was frequently appreciably stronger than the signals received by other paths.

Furthermore, a large number of tests made between Rugby and Holmdel using transmitting aerials giving rise to beams of varying concentration having their maximum radiation at various angles of elevation, showed that the ratios of the received fields corresponded to that portion of the energy projected at a definite angle of elevation. The evidence that the measured field strength averaged over a period of several minutes gives a reasonably accurate value of the predominant component therefore seems to be satisfactory.

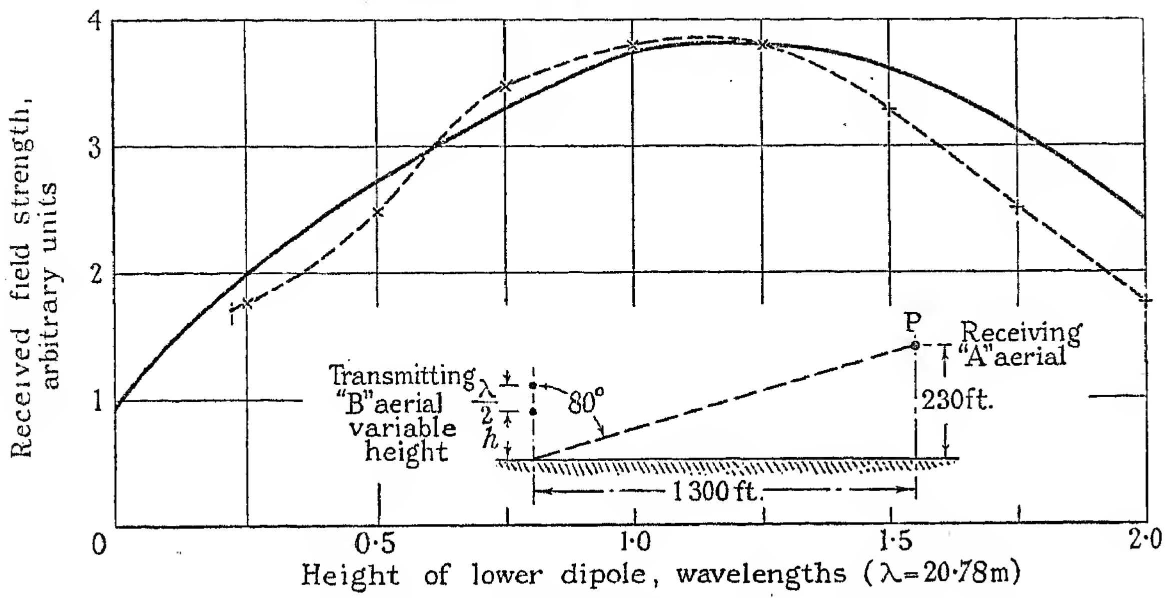



FIG. 10.—Received field strength at 80° to vertical from "B" aerial at various heights above earth.
———— Calculated value.
– – – – Measured value.

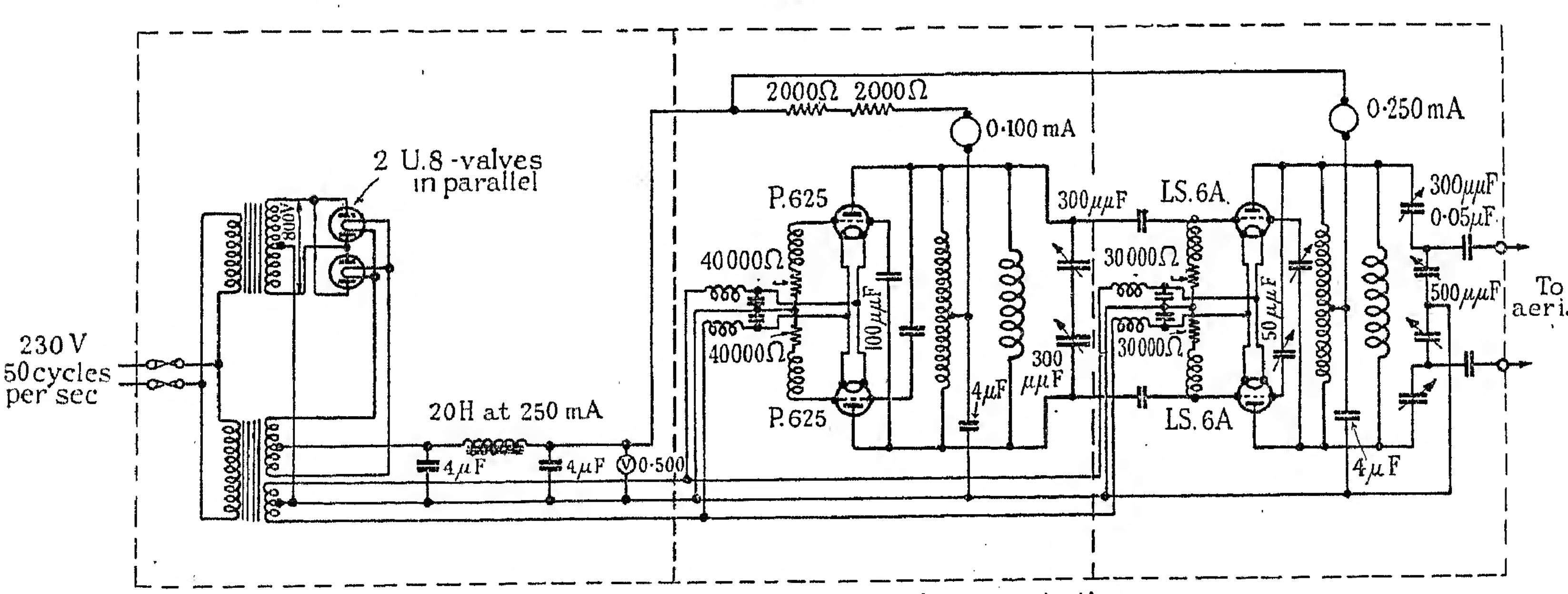



FIG. 11.—Short-wave transmitter for array testing.

## APPARATUS.

*Transmitters.*

(*a*) *Fixed.*—Throughout the tests the transmitters used for the overseas telephone service were employed. Frequency control was by means of a quartz crystal fixed in a temperature-controlled oven. The frequency constancy was remarkably good, the variation being less than ± 50 cycles per sec. in a million. The final output circuit comprised four water-cooled valves which delivered about 12–15 kW into the transmission lines, of which approximately 75 per cent was fed into the aerials.

(*b*) *Portable*.—For the purpose of carrying out certain tests on transmission lines which are mentioned later in the paper, and in order to line up the test aerials correctly (i.e. to match the array correctly to the impedance of the

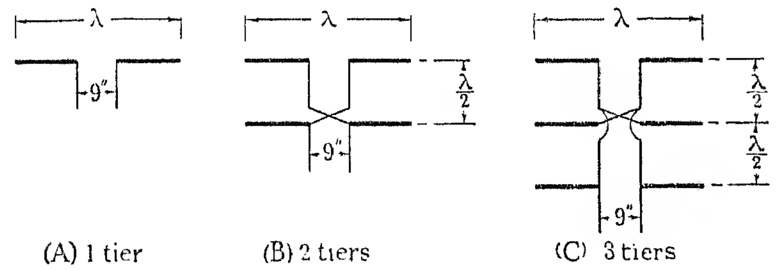



FIG. 12.

feeding lines), a small portable transmitter capable of giving an output of about 25 watts and a range of wavelengths from 10 to 60 m was built.

The set comprised two full-wave rectifying valves in parallel giving a d.c. output of 0·24 amp. at 400 volts, two oscillator valves in push-pull, and two amplifying valves in push-pull. The whole transmitter was built into a 3-compartment aluminium box 4 ft. × 1 ft. 4 in. × 1 ft. 4 in., and could be carried by two men. A diagram of connections is given in Fig. 11.

## AERIALS.

The types of aerials used during the tests are shown in Fig. 12. To avoid unnecessary repetition, the aerials will be referred to by a capital letter to denote the number of horizontal tiers and a number to denote the vertical height of the lower member above the ground. For example, A½ and B1 refer respectively to aerials consisting of one horizontal tier, located ½ wavelength above earth; and two horizontal tiers, the lower tier being 1 wavelength above earth. Similarly H½ refers to an aerial having 8 tiers—H being the eighth letter of the alphabet—the lowest tier being ½ wavelength above earth.

In the first tests the movable aerials were suspended between two lattice-steel towers 180 ft. high, spaced about 500 ft. apart. The method of suspension is shown in Fig. 13. Later, when greater heights were required, cables were attached to steel masts 820 ft. high and the aerials were raised by means of hemp ropes passing over pulleys attached to these cables. Care was taken to break up the cables by compression insulators, so that no section of any cable was a multiple or sub-multiple of a wavelength. All feeders to the aerials consisted of a pair of copper wires, 400 lb. to the mile, run on porcelain insulators fixed on wooden poles about 18 ft. above earth. The separation between wires was 9 inches, thus giving a surge impedance of 550 Ω.

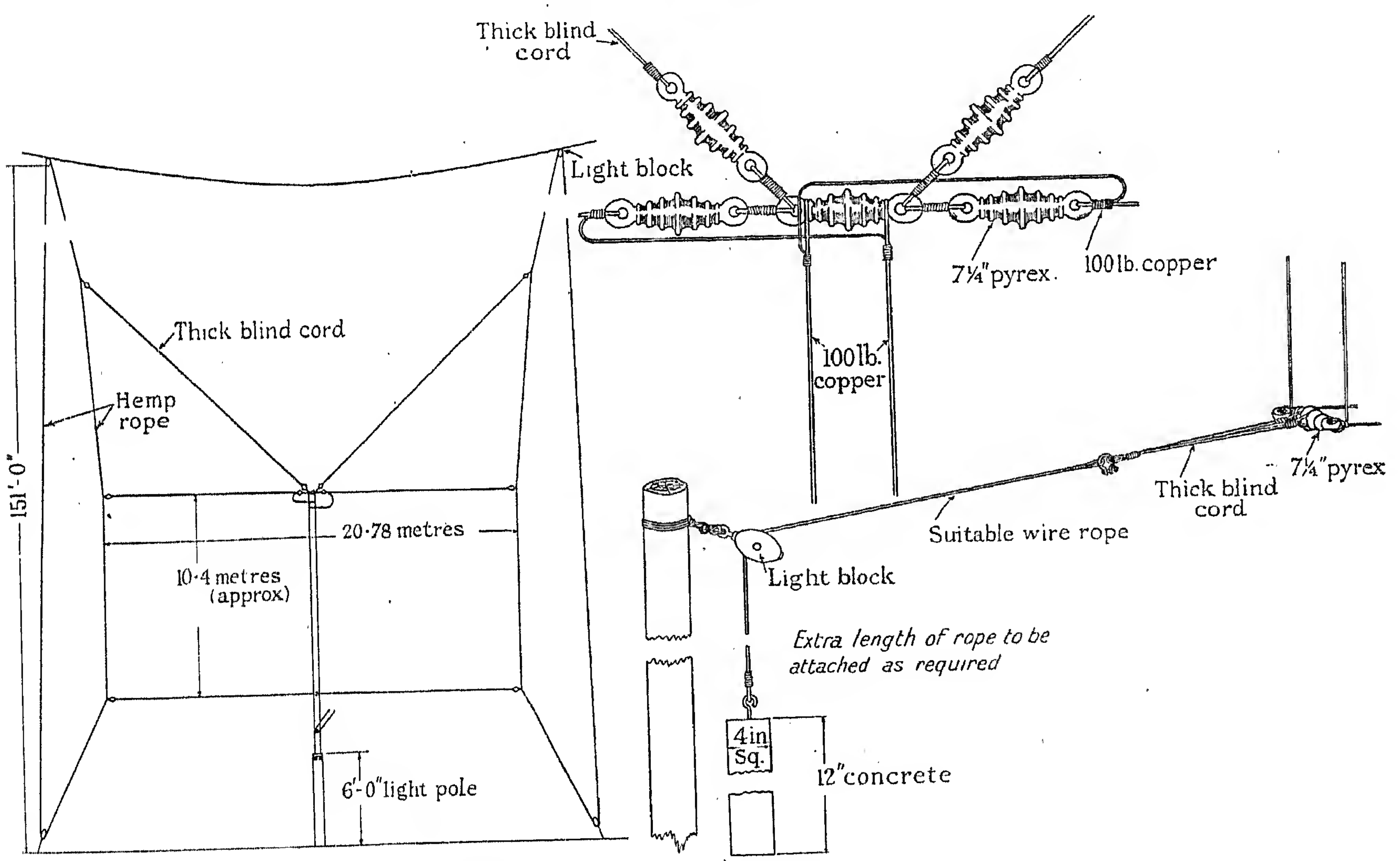



FIG. 13.—Experimental half-wave aerial.

## AMMETERS.

High-frequency ammeters of the thermo-couple type were used to measure current in radiators and feeders. These were clamped on the wire, thus making a shunt circuit. All ammeters were calibrated by placing a similar high-frequency ammeter, used as a standard, in series

with the line wire. A drawing of the ammeter and clamp is given in Fig. 14.

FIELD-STRENGTH MEASURING SETS AND AERIALS.

The measurements of the received fields were made by the expert staffs of the American, German, and Spanish authorities, and the methods used were of course quite outside the control of the author. Only a brief reference to the type of set and the method of measurement employed will therefore be made.

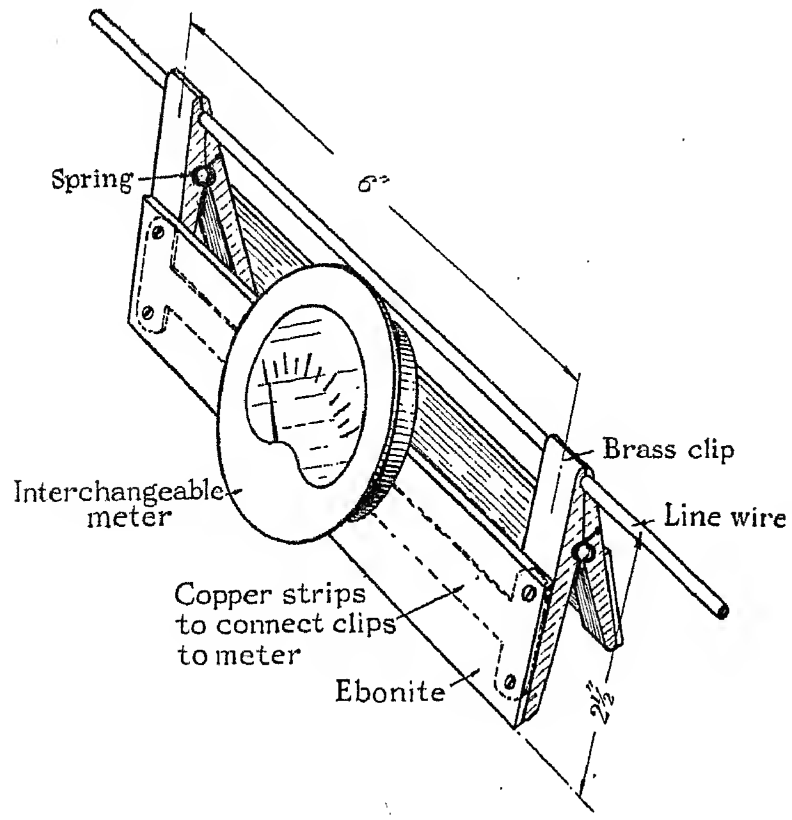



FIG. 14.—Thermo-ammeter and clip.

The American observers utilized the type of measuring set already described by Friis and Bruce.* The apparatus employs the principle of double detection and has a calibrated intermediate-frequency attenuator and a local-signal comparison oscillator. The comparison voltage is measured by a valve voltmeter. For a few of the measurements the readings were taken by observers, but in the majority of cases an automatic recorder of a modified Leeds and Northrup* type was employed. The recorder gives a record of the voltage transferred from the receiving aerial and integrates this voltage over a period of 10 sec. The receiving aerial used for a few of the earlier measurements was a single half-wave vertical dipole, but in later measurements two horizontal dipoles located 1 wavelength and ¼ wavelength above the earth respectively were used. Each aerial was connected to a separate set to enable the downcoming angle to be ascertained at the same time as the angle of projection at Rugby was observed.

The German measurements of field strength were made by comparing the rectified output of the receiver when connected to a vertical aerial 4 m long, with the output due to a local oscillator. In effect, the set was a valve voltmeter which gave a continuous record on an automatic recorder. The set was calibrated in terms of the current in the local oscillator circuit.

At Teneriffe the receiver was a Standard Telephones and Cables superheterodyne type, similar to that used for the American measurements, attenuation being introduced in one of the stages of the intermediate-frequency amplifier. The aerial consisted of a horizontal dipole fixed ½ wavelength above the earth.

A portable field-strength measuring set was used at Rugby for local measurements on field intensity due to transmitting aerials. The circuit diagram is given in Fig. 15, from which it will be seen that two valves in push-pull are fed from a tuned circuit. The grid bias is such that an increase in the signal voltage on the grids of the two valves causes the anode current to increase, giving rise to a larger voltage-drop across the 100 000-ohm resistance in the anode battery circuit. The anode voltage thus decreases, and hence the negative bias on the grid of the second-stage valve increases, causing its anode current to diminish. This decrease is registered on the milliammeter. Increased sensitivity is obtained by "backing off" the milliammeter by means of the

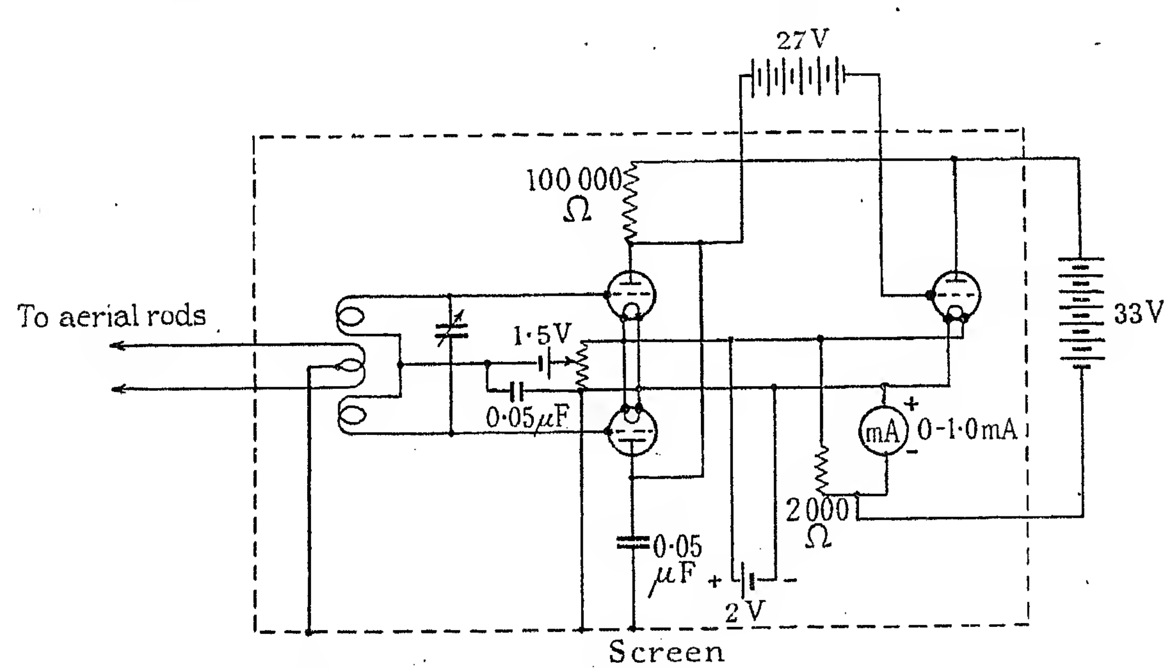



FIG. 15.—Sensitive field-strength measuring set. Range 0–0·5 volt.

* See Bibliography, (6).

* See Bibliography, (7).

filament battery. The whole set is contained in a screened box 11 in. × 11 in. × 9 in. approximately, and leads are taken through a short metal tube to a dipole which can be rotated in a vertical plane. The calibration curve of the set is shown in Fig. 16.

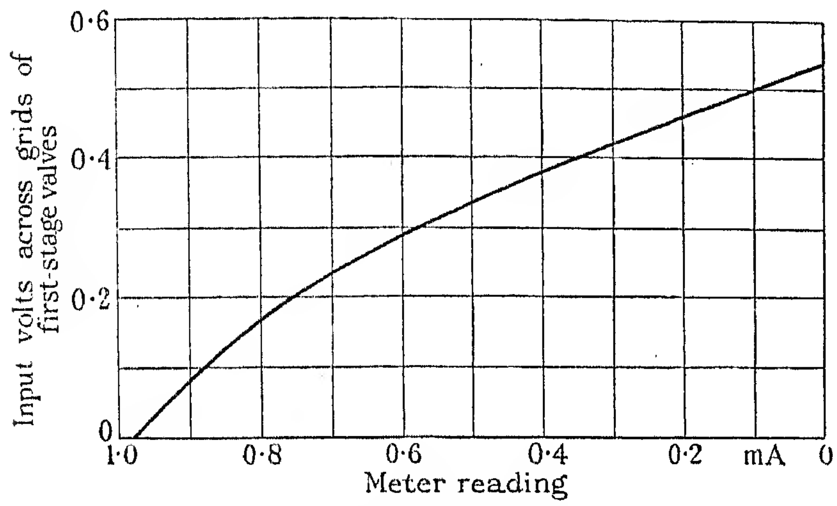



FIG. 16.—Calibration curve of sensitive 2-stage field-strength measuring set.

## TESTS.

For convenience, the tests are divided into two groups. In the first the field due to a fixed standard aerial was compared with the field due to an aerial raised to various heights, whilst in the second the fields due to several fixed aerials arranged at different heights were compared. The advantage of the movable-aerial method of test lies in the fact that it permits a continuous curve of received field strength against transmitting-aerial height to be plotted, and this enables a comparison to be made with the theoretical plotted curves as regards both shape and amplitude. Other points of interest are dwelt upon in the discussion on the results of the tests.

TABLE 1.

*Schedule of Tests for 21st and 22nd May, 1932.*

Call sign GBW; wavelength 20·78 m; tests extended from 1400 to 1858 G.M.T. on Saturday, 21.5.32, and continued throughout the corresponding period on Sunday, 22.5.32.

| Time (G.M.T.) | Remarks |
|---|---|
| *Saturday.* | |
| 1400–1410 | Tone and call sign on aerial A½ |
| 1415–1423 | Carrier on aerial A½ |
| 1425–1433 | Carrier on aerial B¼ |
| 1435–1443 | Carrier on aerial A½ |
| 1445–1453 | Carrier on aerial B¼ |
| 1455–1503 | Carrier on aerial A½ |
| 1505–1513 | Carrier on aerial B¼ |
| 1530–1628 | Tests on A½ and B½, as above |
| 1645–1743 | Tests on A½ and B¾, as above |
| 1800–1858 | Tests on A½ and B1, as above |
| *Sunday.* | Tests as on Saturday, aerials A½, B1¼, B1½, B1¾, B2 |

In the first series of tests the fixed standard aerial was a horizontal dipole consisting of one tier of two half-wave radiators, ½ wavelength above the earth (A½). The movable aerial was either a similar dipole or a combination of two or three similar tiers with ½-wavelength separation between adjacent tiers (B and C). Each test was begun by signalling the call sign on tone and then alternately transmitting on carrier for a few minutes at a time on the fixed and the movable aerials. Change-overs from one aerial to the other were made during a period of 1 hour, after which the movable aerial was raised to the next position and the procedure repeated. A typical schedule of tests is given in Table 1.

TABLE 2.

*Schedule of Tests for 27th September, 1932.*

Call sign GBW; wavelength 20·78 m.

| Time (G.M.T.) | Remarks |
|---|---|
| 1355–1400 | Tone and call sign on aerial B½ |
| 1400–1404 | Carrier on B½ |
| 1405–1409 | Carrier on B¾ |
| 1410–1414 | Carrier on B¼ |
| 1415–1419 | Carrier on B½ |
| 1420–1424 | Carrier on B¾ |
| 1425–1429 | Carrier on B1¼ |
| Tests continued as above until 1559 G.M.T. | |

In the second group of tests three similar 2-tier aerials were fixed at different heights above the earth and change-overs from one aerial to another were made at short intervals. A typical schedule of this type of test is given in Table 2.

During the course of the second group of tests a few observations were also made by alternately energizing a 2-tier (B½) and an 8-tier (H½) aerial and comparing the field strengths at Holmdel.

TABLE 3.

*Schedule of Test for 12th October, 1932.*

Call sign GBW; wavelength 20·78 m.

| Time (G.M.T.) | Remarks |
|---|---|
| 1355–1400 | Tone and call sign on B1 |
| 1400–1404 | Carrier on B1 |
| 1405–1409 | Pulses on B1 |
| 1410–1414 | Pulses on H½ |
| 1415–1419 | Carrier on H½ |
| 1420–1424 | Carrier on B1 |
| And so on until 1700 G.M.T. | |

During some of the tests in both groups the carrier was emitted in pulses of about 1/10 000 sec. duration, 50 times per sec., and the resulting pattern was examined at Holmdel on a cathode-ray tube, the object being to

ascertain to what extent the pattern of the received field was modified by aerials having different vertical characteristics. The procedure was to interpose plain carrier field-strength tests with pulse transmission in the manner shown in the following typical schedule (Table 3).

During the test periods the input to the aerials was maintained as steady as possible, and normally the variation was less than ± 10 per cent from the average. At intervals of a few minutes, readings of the current in the transmission lines close to the aerial were taken.

The ammeters were located at points of maximum and minimum current value separated by $\frac{1}{4}$ wavelength. It has already been shown by the author* that the power input to an aerial can be obtained by multiplying these two values of current by the line surge-impedance.

Before the propagation tests were undertaken, simultaneous measurements were also made of the current at the middle of each half-wave radiator and of the input from the feeder lines. By this means it was possible to make corrections for the aerial-current variation from observations of the feeder line current.

TABLE 4.

*Current Values in Radiators of* $H\frac{1}{2}$ *array.*

| No. of tier | Current value at middle of:— Left radiator | Current value at middle of:— Right radiator | Average current in both radiators |
|---|---|---|---|
| | amps. | amps. | amps. |
| 1 (top) | 3·6 | 3·6 | 3·60 |
| 2 | 3·9 | 3·3 | 3·60 |
| 3 | 3·6 | 3·6 | 3·60 |
| 4 | 3·6 | 3·1 | 3·35 |
| 5 | 3·8 | 3·6 | 3·70 |
| 6 | 3·7 | 3·1 | 3·40 |
| 7 | 3·6 | 3·6 | 3·60 |
| 8 | 3·9 | 3·4 | 3·65 |

The calculation of the vertical characteristics of each test aerial was made on the assumption of equal currents in its radiators. By careful construction and by exact adjustment of the length of the radiators, a reasonably even distribution of currents was obtained. The extent to which equality of currents was possible is shown in Table 4, which indicates that the variation from the average value was not more than ± 6 per cent. Tests on the B type of aerial, placed at varying heights above the earth, showed variations of the same order of magnitude.

## RESULTS OF AMERICAN TESTS.

The results of these tests are given in the curves of Figs. 7 and 8. In Fig. 7 the actual fields measured in Netcong and Holmdel as the test aerial (B) was raised are given as a ratio of the field due to the standard fixed aerial ($A\frac{1}{2}$). The full-line numbered curves give the calculated ratio of the fields due to rays leaving the aerials at various angles to the vertical. A curve has also been plotted giving the calculated angle of maximum radiation from the primary loop due to the (B) aerial, as the aerial is raised.

Examination of the curves shows that, for heights of aerial (B) greater than about $\frac{3}{4}$ wavelength, the measured curves follow the 80° calculated curve fairly closely. One marked divergence occurred on the 21st May, when aerial (B) was 1 wavelength high. At this point, a large increase was registered during a period when fades were rapid, rendering exact computation of signal strength rather difficult. It is significant, however, that the rise occurred when the calculated angle of maximum radiation from the aerial was 80° to the vertical. In Fig. 8 curves have been plotted for the case of the 3-tier (C) aerial. Here again the measured values are close to the 80° curve.

TABLE 5.

*Tests between Rugby and Holmdel, U.S.A., 13th and 14th August, 1932.*

Call sign GBW; wavelength 20·78 m.

| Series No. of tests | Period of tests (G.M.T.) | Transmitting aerial | Gain over $B\frac{1}{4}$ (averages during 8-min. periods) | Angle to vertical, deduced from Fig. 9 | Average angle to vertical |
|---|---|---|---|---|---|
| | *13th Aug.*, 1932 | | decibels | degrees | degrees |
| 1 | 1204–1240 | $B\frac{1}{4}$ | −1·5 | 71·5 | 74·5 |
| | | $B\frac{3}{4}$ | −0·4 | 72·5 | |
| | | $B1\frac{1}{4}$ | +2·5 | 79·5 | |
| 2 | 1244–1320 | $B\frac{1}{4}$ | −2·3 | 75·5 | 77·2 |
| | | $B\frac{3}{4}$ | −2·3 | 77·0 | |
| | | $B1\frac{1}{4}$ | +2·34 | 79·0 | |
| 3 | 1324–1400 | $B\frac{1}{4}$ | −2·3 | 75·5 | 78·8 |
| | | $B\frac{3}{4}$ | +2·0 | 81·0 | |
| | | $B1\frac{1}{4}$ | +4·8 | 83·0 | |
| 4 | 1404–1440 | $B\frac{1}{4}$ | −2·0 | 73·5 | 75·0 |
| | | $B\frac{3}{4}$ | +0·5 | 75·0 | |
| | | $B1\frac{1}{4}$ | −0·75 | 76·5 | |
| | | | | Average of all angles | =76·5 |
| | *14th Aug.*, 1932 | | | | |
| 5 | 1204–1240 | $B\frac{1}{4}$ | −0·7 | 69·0 | 72·2 |
| | | $B\frac{3}{4}$ | −3·6 | 67·0 | |
| | | $B1\frac{1}{4}$ | +2·72 | 80·5 | |
| 6 | 1244–1320 | $B\frac{1}{4}$ | −3·1 | 81·0 | 80·2 |
| | | $B\frac{3}{4}$ | −1·1 | 77·0 | |
| | | $B1\frac{1}{4}$ | +4·4 | 82·5 | |
| 7 | 1324–1400 | $B\frac{1}{4}$ | 0·0 | 67·0 | 75·3 |
| | | $B\frac{3}{4}$ | +1·7 | 79·5 | |
| | | $B1\frac{1}{4}$ | +2·6 | 79·5 | |
| 8 | 1404–1440 | $B\frac{1}{4}$ | −3·2 | 83·5 | 80·0 |
| | | $B\frac{3}{4}$ | +0·7 | 76·0 | |
| | | $B1\frac{1}{4}$ | +3·3 | 80·5 | |
| | | | | Average of all angles | =76·9 |

* See Bibliography, (8).

In the second series of tests 2-tier aerials (B) were erected, having their lower radiators $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, or $1\frac{1}{4}$ wave-

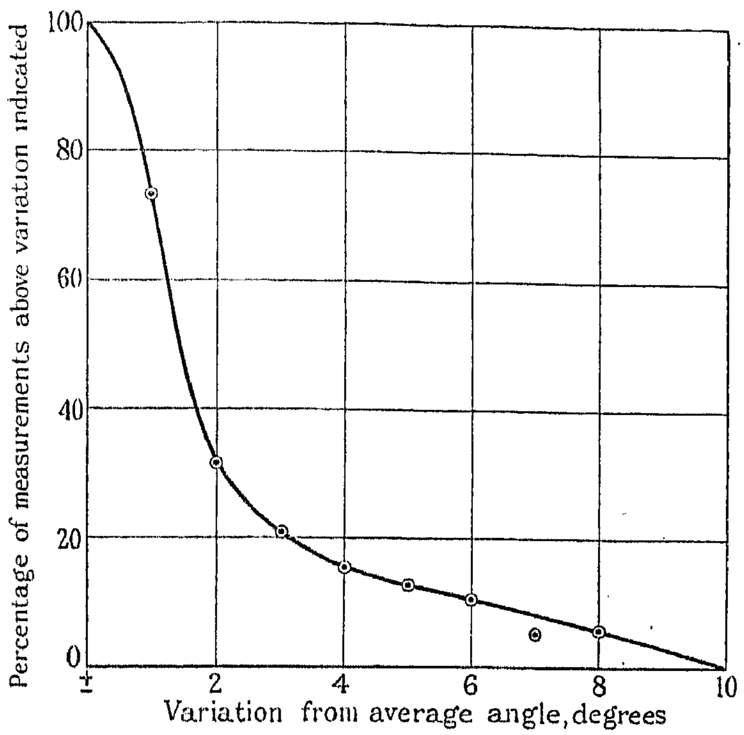



FIG. 17.—20·7-metre tests, Rugby–Netcong. Variation from average of individual angle measurements from December, 1931, to December, 1932. Average angle = 78·9°.

lengths above the earth. Curves were drawn (Fig. 9) giving the gain due to $B\frac{1}{4}$, $B\frac{3}{4}$, and $B1\frac{1}{4}$, over $B\frac{1}{2}$. Thus, by associating the measured values of field strength with the curves, it was possible to deduce the angle of projection. The method is explained in Table 5.

The tests made on the 13th and 14th August were divided into 8 series, each requiring 36 minutes to compare $B\frac{1}{4}$, $B\frac{3}{4}$, and $B1\frac{1}{4}$, with $B\frac{1}{2}$. In the fourth column of Table 5 the gain in received field strength due to the various aerials in comparison with $B\frac{1}{2}$ is given. The fifth column shows the angle of emission deduced from the curves of Fig. 9. The average angle deduced from the various aerials varies slightly, being 75·4° for $B\frac{1}{4}$, 75·6° for $B\frac{3}{4}$, and 80·1° for $B1\frac{1}{4}$.

The agreement between the average readings is considered to be good. The tests were repeated on several occasions throughout the year, but the aerial $B\frac{1}{4}$ was not used. The results of the tests on the angle of projection made through the period December, 1931, to December, 1932, are given in Fig. 17. The curve is based upon many hundreds of measurements made on 10 days spaced throughout the year. They show that not more than 32 per cent of the measurements of the angle of projection differed by more than ±2° from the average value (78·9°).

The evidence provided by the tests on both the movable and the fixed aerials seemed to indicate clearly that an angle of projection equal to about 79° was the best for propagation purposes. It was therefore decided to construct a 2-tier aerial with its lower member fixed 1 wavelength above the earth and to build an exactly similar aerial capable of being raised several wavelengths. The fixed aerial has its direction of maximum radiation at about 80° to the vertical, and it was therefore antici-

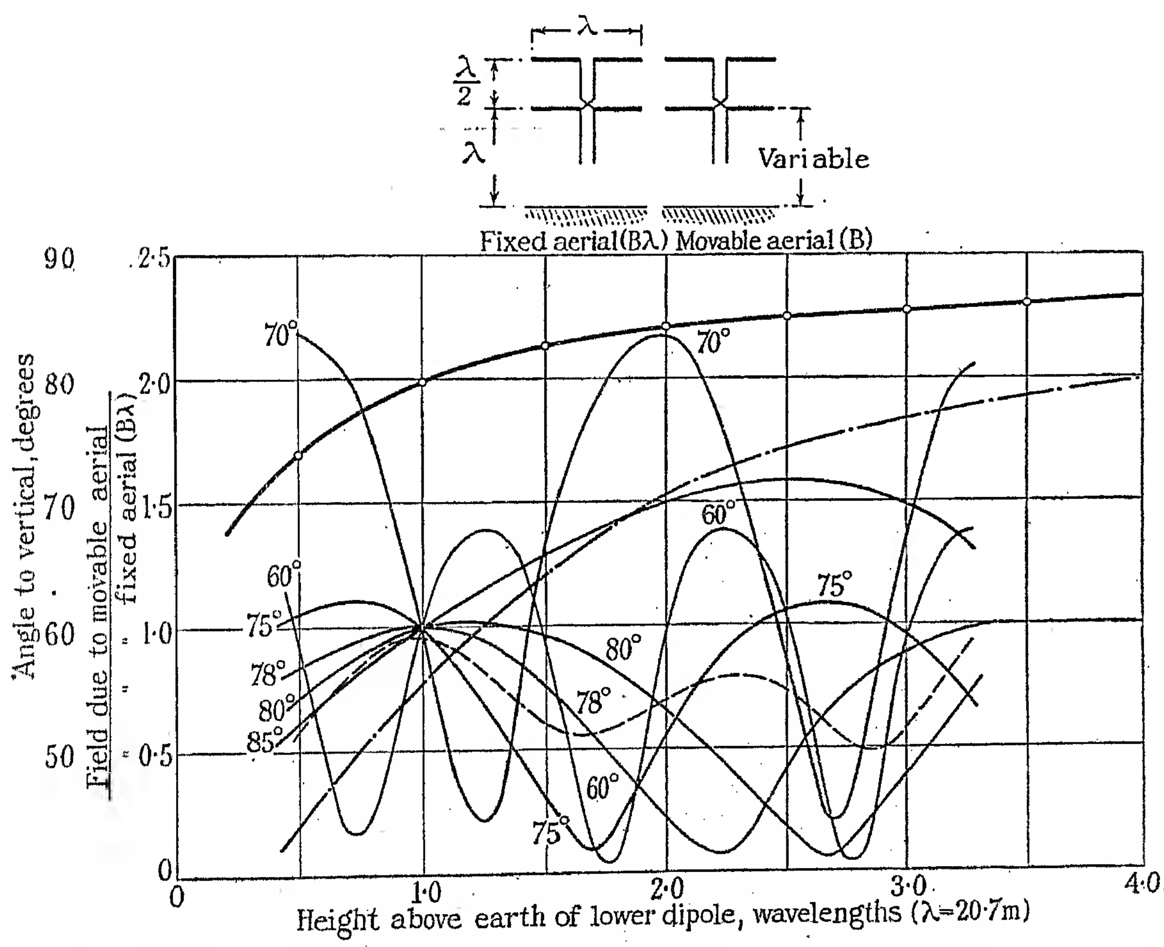



FIG. 18.—Radiation field due to two lines of horizontal radiators at various distances above earth, compared with field due to two similar lines of horizontal radiators having the lower radiator 1 wavelength above earth.

——— Calculated strength of field due to aerial (B), at various angles to vertical (for same power in aerial).
– – – – Strength of field measured at Netcong, U.S.A., 2nd and 3rd July, 1932
——o—— Angle of direction of maximum radiation in primary loop.
—·—·— Angle of direction of maximum radiation in secondary loop

pated from the evidence of the previous tests that the similar movable aerial would not give rise to equal field strengths except when its direction of maximum radiation coincided with that due to the fixed aerial.

The results of the tests of these aerials, given in Fig. 18, show that the anticipations were realized to a considerable extent. After the necessary corrections had been made, however, it was found that although the measured field due to the movable aerial never exceeded that due to the fixed aerial, the curve of received field strength did not coincide with the calculated curve for any particular angle at all stages in the hoisting. Several interesting deductions are possible from an examination of the received and calculated curves. The first point is that when the movable and fixed aerials were of the same height, namely 1 wavelength, the measured fields differed very slightly, thus forming a good check on the accuracy of both the transmitting and the receiving measurements. The calculated curves for angles of 80° to 85° when the aerial positions are low, are very close together. Thus the fact that the measured curve appears to follow closely the 85° curve when the aerial is between $\frac{1}{2}$ and 1 wavelength high cannot be regarded as evidence that this value represents the angle of projection of the received ray. Moreover, if the slight discrepancy between the measured fields when both the aerials were 1 wavelength high had been remedied, the measured wave would then have almost coincided with the 80° curve as the aerial was raised from $\frac{1}{2}$ to 1 wavelength. Between the heights of 1 wavelength and $1\frac{1}{2}$ wavelengths the corrected measured curve would almost coincide with the calculated 78° curve. Again, when the aerial was raised from $2\frac{3}{4}$ to $3\frac{1}{4}$ wavelengths the measured curve lay between the 78° and 80° calculated curves. Furthermore, maximum and equal values of field were recorded when the aerials were 1 wavelength and $3\frac{1}{4}$ wavelengths high. The curves in Fig. 18 giving the angles of maximum radiation of the primary and secondary lobes as the aerials are raised show that, at these heights, maxima occurred at 80° and 78° respectively. So far the argument in favour of angles of projection of 78° to 80° seems strong.

Whilst the aerial is being raised between $1\frac{3}{4}$ and $2\frac{3}{4}$ wavelengths, however, the divergence of the measured curve from the 78° or 80° curves appears at first sight to be very marked. Further consideration shows that only a slight change in angle during this stage was needed to produce a large increase in comparative field strength. The angles deduced from the curves whilst the aerials were raised from $1\frac{3}{4}$ to $2\frac{3}{4}$ wavelengths in $\frac{1}{4}$-wavelength steps were 80°, 75·5°, 77°, and 79° respectively, giving an average of 78°. The outstanding fact is that when both the primary and the secondary lobes of radiation had their directions of maximum value pointing at 78° to 80°, the resulting fields were a maximum.

In order to prove beyond doubt that energy radiated at very high angles to the vertical did not result in a large increase in gain at the receiving station, an 8-tier

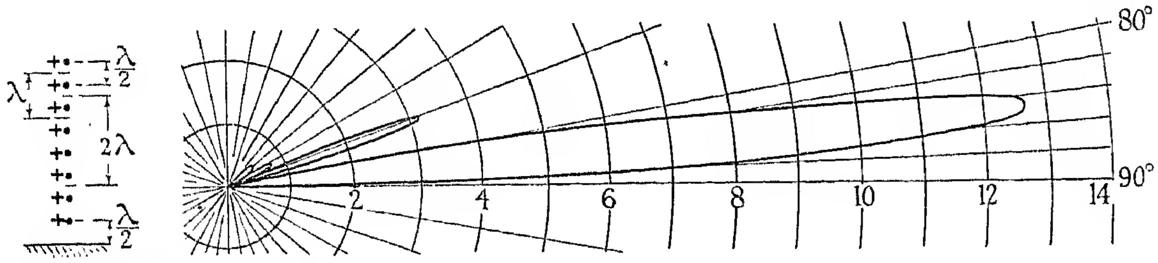



FIG. 19.—Vertical polar diagram of field strength for the 8-tier aerial shown on left (the dots represent horizontal radiators above earth).

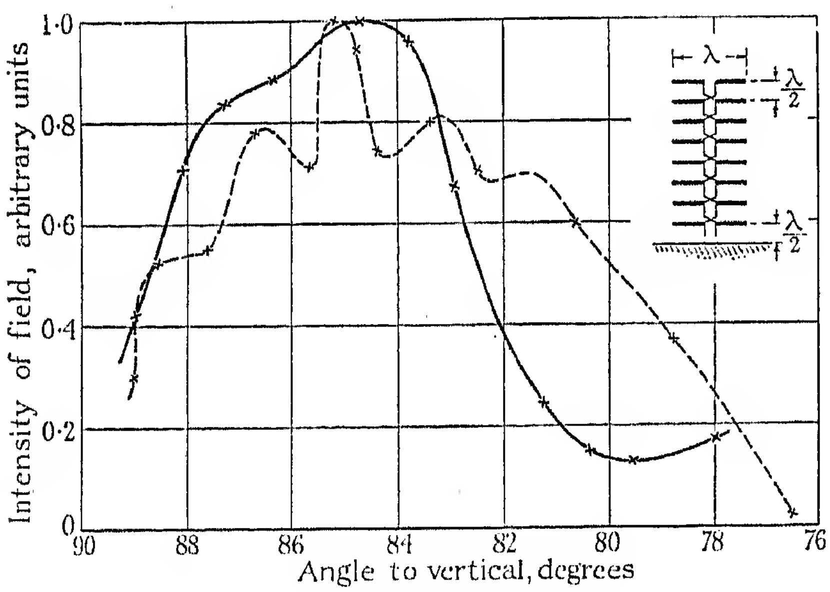



FIG. 20.—Intensity of field in vertical plane due to aerial of type shown ($H\frac{1}{2}$).

— — — Measured 10th October, 1932, at various heights up a vertical steel mast, separated from the aerial a horizontal distance of 2 200 ft.

——— Calculated on assumption of perfect earth.

TABLE 6.

*Tests between Rugby and Holmdel, U.S.A., 12th October,* **1932.**

Call sign GBW; wavelength 20·78 m.

| Period of tests | Gain due to $H\frac{1}{2}$ over B1 aerial, corrected for equal power input | Average angle of projection to vertical, deduced from Fig. 21 |
|---|---|---|
| G.M.T. | decibels | degrees |
| 1420–1438 | + 1·4 | 82·0 |
| 1440–1458 | − 0·2 | 81·5 |
| 1500–1518 | + 4·9 | 83·5 |
| 1520–1538 | − 1·3 | 81·1 |
| 1540–1558 | − 1·0 | 81·2 |
| 1600–1618 | + 2·0 | 82·3 |
| 1620–1638 | + 2·4 | 82·5 |
| 1640–1658 | + 1·5 | 82·0 |
| | Mean + 1·2 | Mean 82·0 |

aerial ($H\frac{1}{2}$) was erected having its lowest member $\frac{1}{2}$ wavelength above earth. The calculated polar diagram in a vertical plane is shown in Fig. 19. In order to obtain some idea of the actual polar diagram, a field-measuring set was taken up an 820ft. mast situated 32 wavelengths away from the aerial. It was realized that the measurements could only yield very approximate results since the effect of reflections from the steelwork was bound to modify the fields seriously. It was thought, however, that the tests might confirm the theoretical deductions that a concentrated lobe of energy was radiated at angles a few degrees from 90° to the vertical. The results supported this anticipation. The set was allowed to rest on a horizontal bracing and the receiving dipoles protruded a few inches from the side of the mast. Readings were taken at intervals of several feet as the set was carried up the mast. The resulting diagram, together with the theoretical diagram calculated for a separation of 32 wavelengths between aerial and mast, is given in Fig. 20. Having regard to the conditions under which the tests were made the agreement is as good as could be expected. Reception tests were made with America on several days, and a typical schedule of the results is shown in Table 6.

The average gain of the $H\frac{1}{2}$ aerial over the B1 aerial was 1·2 decibels when the input was corrected for equal power, and thus the angle of projection as deduced from Fig. 21 was 82°. Similar values for a test made on the 25th August, 1932, were −1·6 decibels and 81°. Pulse tests were also made alternately on B1 and $H\frac{1}{2}$ aerials over a period of several hours with the object of ascertaining whether appreciable difference in the observed patterns would be obtained. The result in all cases was practically the same, namely that no great difference in the pattern formed on the oscillograph was discernible when different types of transmitting aerials were used.

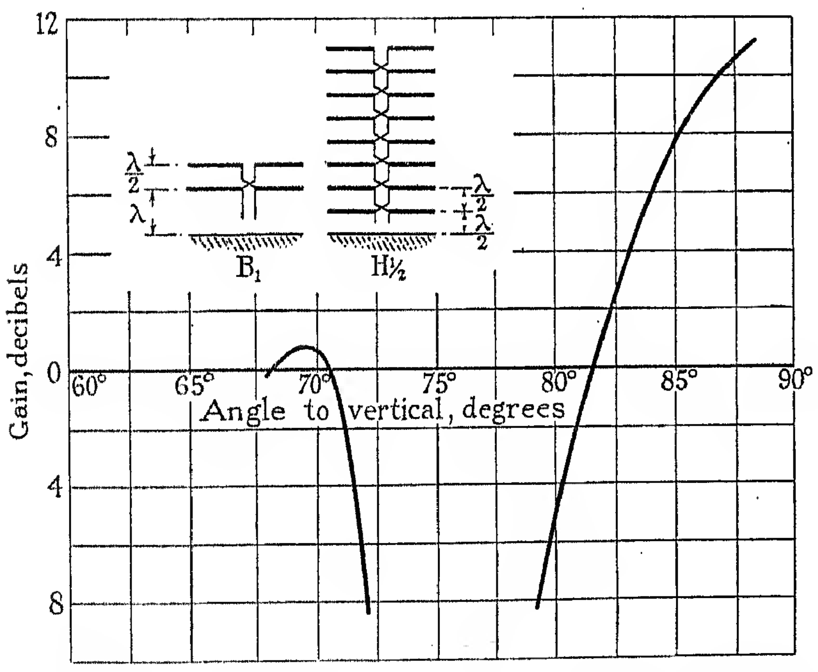



Fig. 21.—Calculated gain in field strength of horizontal radiators raised $\frac{1}{2}$ wavelength above earth, over a 2-tier system raised 1 wavelength above earth, for equal power radiated.

In November, 1932, a few tests were made using wavelengths of 20·78 m and 30·645 m, alternately changing from one wavelength to the other and using similar types of B aerials, fixed respectively at $\frac{1}{2}$ and $1\frac{1}{4}$ wavelengths above the earth. It was found that very little difference existed between the angles of projection in the two cases. The actual results obtained are shown in Table 7.

Table 7.

*Tests between Rugby and Holmdel, U.S.A., 10th November,* 1932.

Call signs GBW and GCW; wavelengths 20·78 and 30·645 m.

| Period of tests | Wavelength | Projected average angle to the vertical |
|---|---|---|
| G.M.T. | metres | degrees |
| 1200–1258 | 20·78 | 81·0 |
| 1310–1348 | 30·645 | 75·3 |
| 1420–1458 | 20·78 | 74·0 |
| 1530–1608 | 30·645 | 75·8 |

Similar tests between the same hours were also made on the 5th January, 1933. The average angles of projection on this date were 78° for the 20·78-metre wave and 77·5° for the 30·645-metre wave.

## Results of Teneriffe Tests.

Tests between Rugby and Teneriffe were made from the 19th to the 26th May, 1932, and on the 11th and the 18th December, 1932.

In the first tests made between Teneriffe and Rugby on 20·78 m, single-tier (A) types of horizontal aerials were used, one being fixed at $\frac{1}{4}$ wavelength above the earth and the second being hoisted in $\frac{1}{8}$-wavelength stages from $\frac{1}{4}$ to 1 wavelength above the earth. Owing to the needs of commercial traffic it was only possible to test between the hours of 0615 and 0830 G.M.T., and in consequence it required 5 days to complete a series of tests. Furthermore, during a part of the tests weak fields and considerable fading were experienced. The results, which are shown in Fig. 22, can therefore be only treated qualitatively, but they serve to illustrate the fact that there was a definite angle of projection of the energy that gave the highest field strength in Teneriffe. This angle was 71° to the vertical during the particular period of the tests, namely the 19th to the 26th May, 1932.

The tests on the 11th December, 1932, were conducted from 1330 to 1800 G.M.T., and those on the 18th December, 1932, took place between 1030 and 1610 G.M.T. The tests on the 11th December were ineffective after 1630 G.M.T., and the last part of the tests was therefore repeated on the 18th. During these tests the B type of aerial was used, the standard aerial being fixed with its lower member $\frac{1}{2}$ wavelength above the earth. The movable aerial was raised in $\frac{1}{4}$-wavelength stages to a height of $3\frac{1}{4}$ wavelengths above the earth, measured from the lower radiators. The shape of the curve of field strength shown in Fig. 23 is very similar to that obtained in May, except that the large increase in intensity was

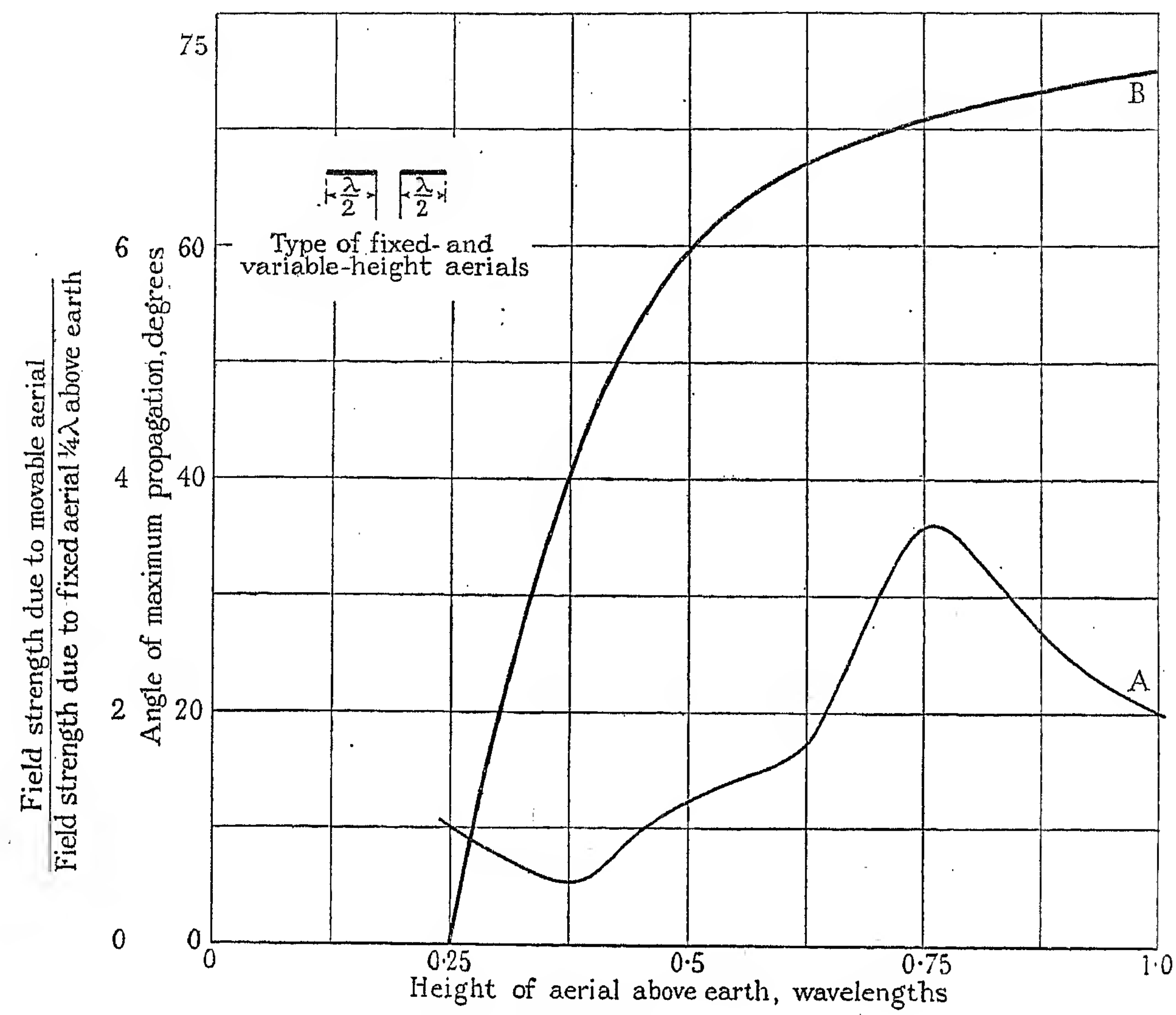



FIG. 22.

A. Field strength measured in Teneriffe, 19th to 26th May, 1932.
B. Calculated angle to the vertical of direction of maximum radiation. The following values were used in the calculations: $\kappa = 10$, $\sigma/f = 20$, $\lambda = 20 \cdot 7$ m.

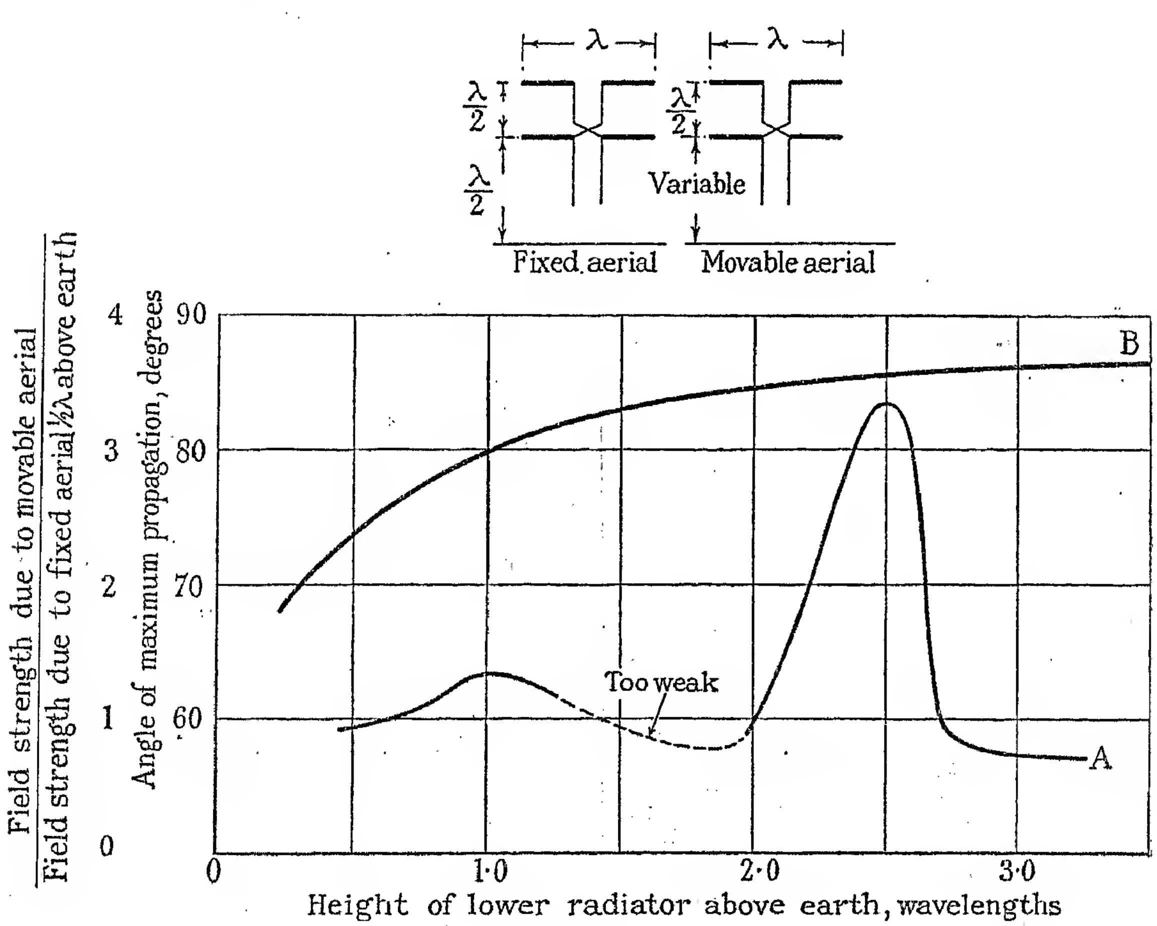



FIG. 23.

A. Field strength measured at Teneriffe, 11th and 18th December, 1932.
B. Calculated angle to vertical of direction of maximum radiation.

noticed when the angle to the vertical of the axis of the primary lobe of radiation was 85·5°. The significance of this result is discussed later in the paper.

### Results of German Tests.

The first tests to Germany were conducted in July, 1932, on a wavelength of 20·78 m. No definite conclusion could be drawn from the results of these tests. Violent fluctuations of field strength were experienced, and it was quite obvious that the wavelength used was not suitable for transmission over the comparatively short path to Berlin from Rugby. The second series of tests—on the 23rd and 24th July, 1932—was therefore made on a higher wavelength, 30·645 m.

A single-tier (A) aerial was raised in $\frac{1}{4}$-wavelength steps from $\frac{1}{4}$ to $1\frac{1}{2}$ wavelengths high, and the resulting field strengths were compared with the field due to a similar aerial fixed $\frac{1}{2}$ wavelength high. As before, the tests were conducted during the daylight period between the hours of 0800 and 1400 G.M.T. The results have been plotted in Fig. 24, where the curve showing the direction of maximum emitted radiation as the aerial was raised has also been drawn. It is at once apparent that the shape of the curve is very different from that obtained in either the American or the Teneriffe trials. Three maxima were obtained, two corresponding to angles of projection of the primary lobe of 60° and 74° to the vertical respectively. As the field was still rising when the aerial had reached the maximum height possible during these tests, the exact value of the third maximum and the corresponding angle of projection cannot definitely be stated. It is possible, however, that a maximum value of field equal to about 2·5 times the smallest of the three maxima would have been obtained at an angle of 82°. This matter is referred to later, in the general discussion on the results of the whole of the tests.

The third of the series of tests was made on a wavelength of 30·645 m on the 7th November, 1932, between 1200 and 1800 G.M.T., using a 2-tier (B) type of aerial raised from $\frac{1}{2}$ wavelength to $2\frac{1}{2}$ wavelengths above the earth. Thus a beam varying from a high angle of elevation to a low angle was projected. Unfortunately reception conditions were not very favourable, chiefly owing to interference from another station, but the tests showed quite definitely that the highest field strength was obtained when the lower radiator was $\frac{1}{2}$ wavelength above the earth, i.e. when the maximum energy was radiated at an angle of 74° to the vertical.

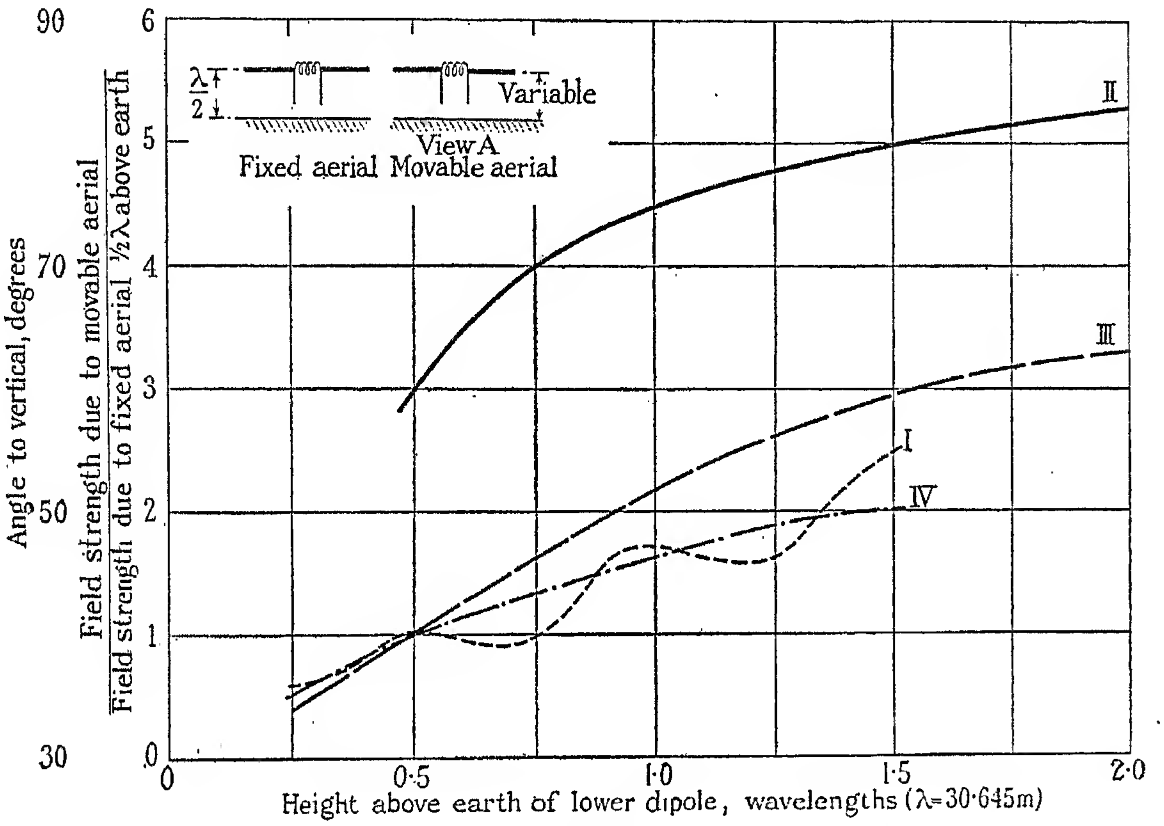



Fig. 24.

I. Strength of field measured at Berlin, 23rd and 24th July, 1932, from 0800 to 1400 G.M.T., due to aerial shown at A.
II. Angle to vertical of maximum radiation of primary loop.
III. Calculated strength of field due to aerial shown at A, at 85° to vertical.
IV. Calculated strength of field due to aerial shown at A, at 82° to vertical.

### Discussion on the Results of the Tests.

The propagation tests to America, Teneriffe, and Germany, were made primarily with the practical object of ascertaining whether particular angles of projection of a beam of energy resulted in improved field-strength values at the distant receiving station. This knowledge was required to enable the author to design arrays giving the maximum field strength per unit of cost. The tests have shown definitely that there is a particular angle of projection and that among other factors this angle depends upon the distance between the receiving and transmitting stations. A study of the various curves of Figs. 7, 8, 18, 22, 23, and 24, also provides some information concerning the propagation of the waves. Taking the German curve first (Fig. 24), the progressive rise in received field strength during the 23rd

and the 24th July, 1932, as the single-tier (A) aerial was raised is very marked. Curves have also been drawn in the same figure, giving the expected field strength as a ratio of that due to a fixed aerial ½ wavelength above the earth, assuming that reception was made of one ray only emitted at an angle of 85° or 82° to the vertical. It will be observed that the actual curve follows in general the 82° calculated curve, making peaks corresponding to heights of 0·5 and 0·95 wavelength above the earth. At these heights the directions of the axis of the primary lobe of radiation were 60° and 74° approximately. Reference to Fig. 25, which gives the skip distance for various angles of projection of a ray for various values of the effective height of the ionized layer, on the assumption of simple geometric laws, shows that a ray projected at an angle of 82° to the vertical would be received in one skip at Berlin (1 000 km from Rugby) provided the effective ionized reflecting layers were 95 km high. Moreover, it is significant that rays projected at angles of 74° and 60° would reach Berlin in two and three skips respectively if reflected from a layer 90 to 100 km high. The resultant of rays arriving by more than one path would be expected to account for the shape of the curve, and peaks would occur when the field strength due to the rays travelling by alternative paths reached a maximum value. The evidence obtained during this test therefore strongly supports the belief that the major portion of the energy received at Berlin was confined to the region below the E layer. To test this belief still further, the values of the penetration frequency of the E layer were obtained from the Radio Research Station, Slough. Weekly noon readings of this frequency were recorded, and the observations for July, 1932, and the early part of August, 1932, are given in Fig. 26. Although the value on the 24th July was not measured, the value by inference would be about 5 megacycles per sec.

It has been shown* that, ignoring the effect of the earth's magnetic field (which is small at the frequencies and angles of projection considered), if $f_c$ = critical frequency, $f$ = frequency of wave under consideration, $\theta_c$ = critical angle measured from the vertical, and $x$ = (height of apex of ray above earth)/(radius of earth),

Then $$f_c^2/f^2 \doteqdot \cos^2\theta_c + 2x\sin^2\theta_c$$

Thus, taking $f$ as 9 790 kilocycles per sec. ($\lambda = 30\cdot645$ m), and the height of the apex of the ray above the earth as 90 km, we find that $\theta_c = 61°$. It is therefore probable that only the energy radiated at a less angle to the vertical than about 60° would have penetrated the E layer on the particular day of the tests. The tests made on the 7th November, 1932, on a wavelength of 30·645 m, using a 2-tier (B) type for both the fixed and the movable aerial, showed very different conditions. The fact that the received field strength was greatest when the movable aerial was in its lowest position, namely ½ wavelength above the earth, can be regarded as evidence that the F layer was chiefly responsible for the bending of the rays back to earth. The angle of maximum radiation with the aerial at this height is 74°, but had the angle been decreased by lowering the aerial it is quite possible that still higher fields might have been obtained.

The critical frequencies during the tests varied from about $2\times10^6$ to $1\times10^6$ cycles per sec., giving a critical angle varying from 78·5° to 84° to the vertical. Thus rays projected at an angle to the vertical less than about 78·5° would be expected to penetrate the E layer.

The practical conclusion resulting from these tests is that, to provide the most effective reception, the design of the array for summer service must be different from that required for a winter service.

With regard to the Teneriffe test between the 19th and the 26th May, 1932, and on the 11th and the 18th December, 1932, on 20·78 m, here again summer and winter conditions were different, since the best angles of projection appeared to be respectively 71° and

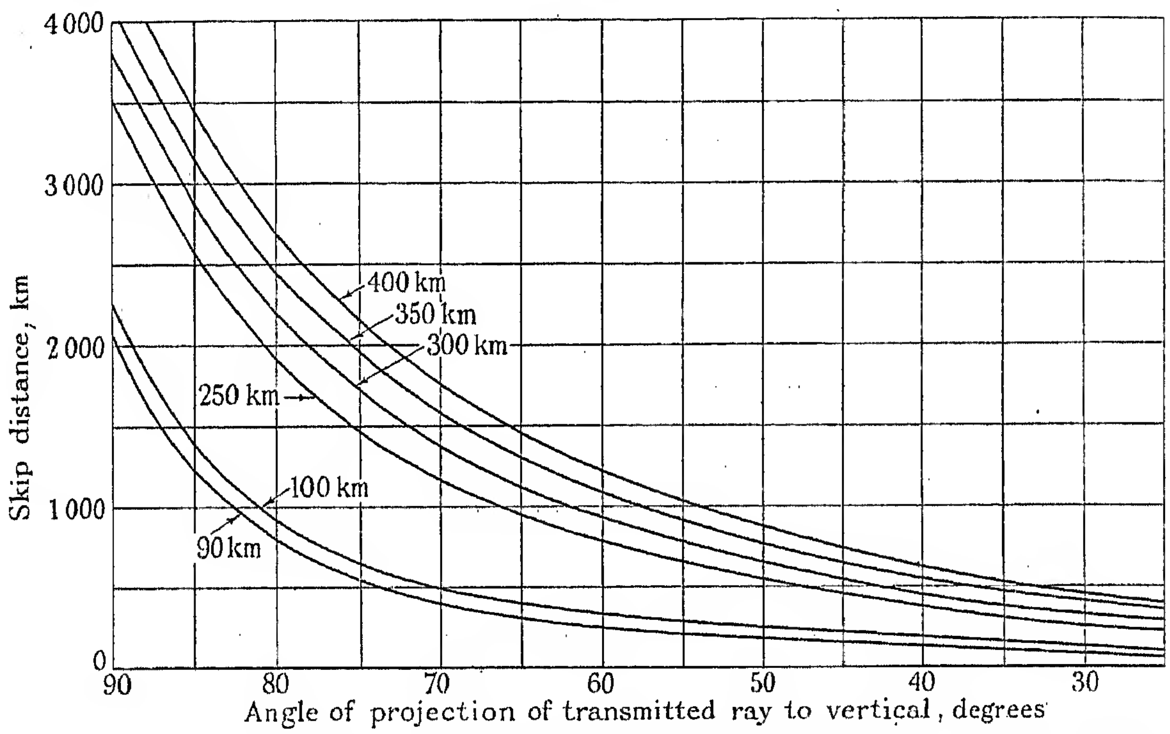



FIG. 25.—Relation between angle of projection and skip distance for various effective heights of the ionized layer.

* See Bibliography, (9).

85° to the vertical. Reference to Fig. 25 shows that these angles correspond to two and one skips respectively between Rugby and Teneriffe—a distance of 2 850 km—for effective heights of the F layer equal to 300 km and 285 km respectively. The absolute field strengths during the December tests were considerably higher than during the May tests, a fact which supports the conjecture that the ray suffered two reflections in May.

The results experienced during the Teneriffe experiments were contrary to those obtained during the Berlin tests; in the former, in summer the lower angle of projection was most successful, whilst in the latter the high-angle projection gave the best results.

There are, however, many features of dissimilarity between the two sets of tests. The wavelengths were different; the Rugby–Teneriffe route is almost north to south, whilst the Rugby–Berlin route is approximately west to east; and the distance of Teneriffe from Rugby is almost three times that of Berlin. This latter fact probably provides the key to the solution, since for propagation by the E layer at least two skips were necessary between Rugby and Teneriffe, whilst only one was required between Rugby and Berlin. There seems little doubt that on the Teneriffe route during the May tests the E layer effectively held down the rays projected at a high angle to the vertical, and that the double passage of the ray in a long almost horizontal path in the E layer resulted in greater attenuation than the alternative passage right through the E layer at an angle much greater than glancing incidence and a double passage via the F layer. This conclusion receives support from the following remarks by Eckersley:* "Geometrically, a glancing initial ray which fails to penetrate the E layer is possible, but if we make an approximate calculation of the attenuation of such a ray we find it is so large that it cannot be received even over moderate distance."

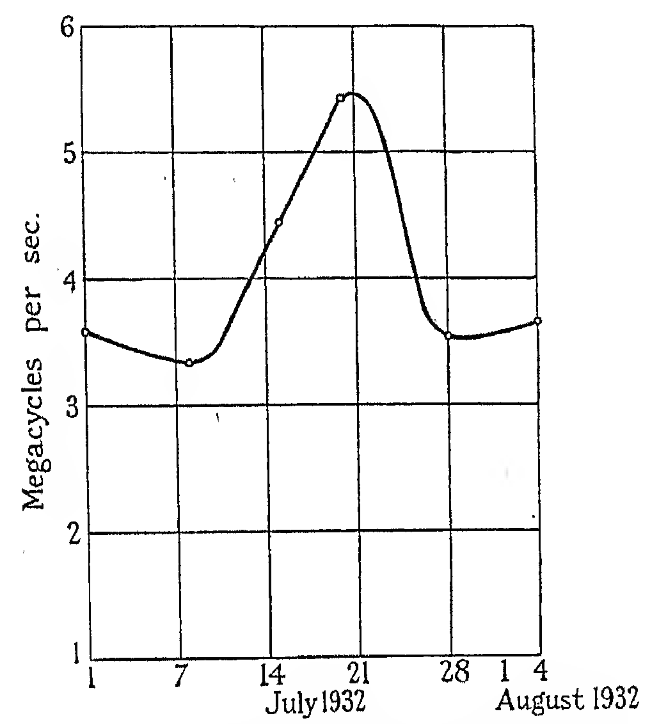



FIG. 26.—Critical frequency of penetration of the E layer, taken at Slough at noon.

It has already been stated that during the German tests in July, only rays having an angle to the vertical less than 60° could penetrate the E layer. Thus since at this time of the year the F layer would be expected to have an effective height appreciably less than 300 km, two reflections from this layer would be required to span the distance between Rugby and Berlin. It therefore seems probable that a ray, entering the E layer at an angle several degrees removed from glancing incidence and bent down from the E layer to reach Berlin in one skip, would be less attenuated than the ray which was twice reflected from the F layer. While the danger of generalizing from the few tests made is realized, on the practical side the indications point to the desirability—if not the necessity—of using different designs of arrays for summer and winter transmissions to achieve the best results.

The American tests were more comprehensive and exact than either the German or the Teneriffe tests, owing to close collaboration with the American Telephone and Telegraph Co. and the Bell Telephone Laboratories. In the U.S.A. tests it was usually possible to arrange to measure the downcoming angle at the receiving end at the same time as the angle of projection was measured at the transmitting station. It was thus possible to correct the received field so as to take account of the vertical characteristics of the receiving aerial. The correction was not a large one, however, as the variation in the average downcoming angle was only a few degrees. The first fact of note is that the best angle of projection on 20·78 m during daylight conditions appears to be about 78° to 80° to the vertical. As the distance between the transmitting and the receiving station is about 5 400 km, this angle suggests that propagation takes place in two skips from an ionized layer 460 to 400 km high. Since local measurements made at Slough gave heights of the F layer usually between 200 and 300 km, it could also be argued with a greater degree of probability that three skips were made, the height of the layer being between 275 and 230 km. With this height, two skips would require an angle of projection of approximately 85° to 86·5°, which values are usually too high to permit penetration of the E layer even in winter when the ionization is a minimum. As with the Teneriffe transmissions, propagation by repeated reflections between the E layer and earth would be expected to produce fields of negligible strength. Repeated tests made with the H½ aerial, giving maximum radiation between 85° and 86° and considerable radiation in directions making even greater angles to the vertical, failed to produce evidence of appreciable energy being received by a path starting from the transmitting aerial at angles around 86°; and the received fields, compared with those due to the standard transmitting aerial, were such as to support the view that the major portion of the received energy was projected from the transmitter at angles around 80°. It was a noteworthy fact that the downcoming angle was quite independent of the type of transmitting aerial used. For example, an H½ type of aerial emitting at 85° over 16 times the power radiated from B1-type aerial, produced no measurable difference in the average received downcoming angle. This result was observed to hold whatever the downcoming angle: the type of transmitting aerial used did not appear to modify the result.

* See Bibliography, (9).

Similar results were obtained when four B-type aerials fixed $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and $1\frac{1}{4}$ wavelengths above the earth respectively were used for transmitting purposes, thus providing further evidence in favour of a single preferential path.

With regard to the angle of reception in the U.S.A., generally on the 20·78-metre wave the average value varied a few degrees on both sides of 76°. There were, however, a few occasions when angles greater than 85° were registered. It was noticed on several occasions, particularly during the lighter months when the tests were made from noon, that there was a gradual fall of a few degrees in the value of the received angle of incidence as the tests proceeded. Remembering that noon is about 5 hours later in New York than in England, the explanation might be that during the early afternoon the intensity of ionization of the F layer along the transatlantic path was appreciably different. The result would be that in the early afternoon the F layer would not behave as a concentric shell of uniform height above the earth for the whole distance between Rugby and the U.S.A. Thus in the early afternoon the layer would in effect be tilted up at the American end, with the result that at the receiver the downcoming ray would make a greater angle with the vertical than the projected ray. Later in the afternoon, as the ionization in England decreased and that in the U.S.A. increased, the layer would be expected to become more uniform in its effective height above the earth and then the tendency would be for the angle of the incident ray at Holmdel to become less. Such has been found to be the case.

With regard to the 30·645-metre tests, sufficient evidence as yet is not available regarding the best angle of projection, although the tests in November showed definitely that the angle was about the same as with the 20·78-metre transmissions.

The whole of the practical conclusions can be summarized briefly by stating that there is a definite relationship between the value of the field strength at a receiver and the angle of projection of the ray at the transmitting aerial. This angle depends on the distance apart of the stations and on the ionization of the E and F layers, which varies with the time of day, the season, and the year. In the case of transmission on 20·78 m from Rugby to New York, for all-daylight conditions an angle of projection varying a few degrees on either side of 79° to the vertical was the best. The results prove the necessity whenever a new service is under consideration of making tests over a reasonable period to ascertain the best angle of projection, and of repeating these tests at regular intervals after the service has been inaugurated.

## Part 2. SUPPORTING STRUCTURES FOR ARRAYS

### Introduction.

The best angle of projection of a beam of radiation having been decided, the next question concerns the design of a suitable array and of the supporting structures. Design cannot be divorced from cost, and since usually the greatest cost of a complete aerial system lies in the structures, data are necessary to enable reasonably accurate computation to be carried out. It is therefore proposed in this section to refer briefly to the question of supporting structures. The principle factors to be considered in the design of towers are the top horizontal loading, the type of bracing, the strut formula to be used in calculations, and the torsion due to the pull of the aerial-supporting cable. The author has already considered these factors in some detail in a previous paper;* the conclusions at which he has arrived may be summarized in the following way.

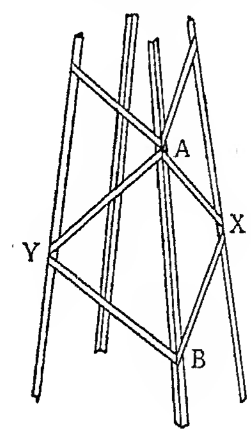
### Wind Load.

Records of wind velocities in the British Isles show that a speed of 110 m.p.h. at a height of 40 ft. above the earth has been experienced. This would necessitate a wind resistance of 42 lb. per sq. ft. of exposed surface of a tower built of equal angle-steel sections.

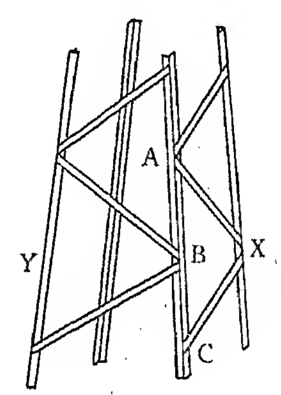
This value of wind velocity represents gust conditions of an intensity that seldom prevail in the British Isles, and usually in all but the most exposed position 30 lb. per sq. ft. will be ample. Since radio stations are usually built in open exposed positions, 40 lb. per sq. ft. has been adopted for the structures referred to in this paper. A screening factor of 1·8 is the maximum needed, and a less value can be employed at the top of the tower where the ratio of distance between faces of the tower to the width of section of angle iron is a minimum; a safe overall value to adopt is 1·6.

### Type of Bracing.

Since horizontal face bracing introduces the conception of a "fictitious" member parallel to the posts, which involves difficulties in calculating the stresses in posts and




(a)




(b)

Fig. 27.

bracings, horizontal face bracings should usually be avoided. Where single bracing is used, calculation of

* See Bibliography, (10).

the stresses is rendered more accurate by utilizing the arrangement shown in Fig. 27(*a*), in preference to that illustrated in Fig. 27(*b*). In the former the column AB is fixed at A and B in the two perpendicular planes ABX and ABY, whilst in the latter the fixation of the column is partial at A, B, and C. This fact introduces uncertainty in the column formula.

fact necessitates provision for torsional resistance in the design of the towers, since for maintenance purposes or in the event of breakdown it might be necessary to remove one of the curtains. This provision of a cross-arm increases the cost of each tower very appreciably, not only on account of the extra cost of the arm but also owing to the increased mass of the tower, which has to

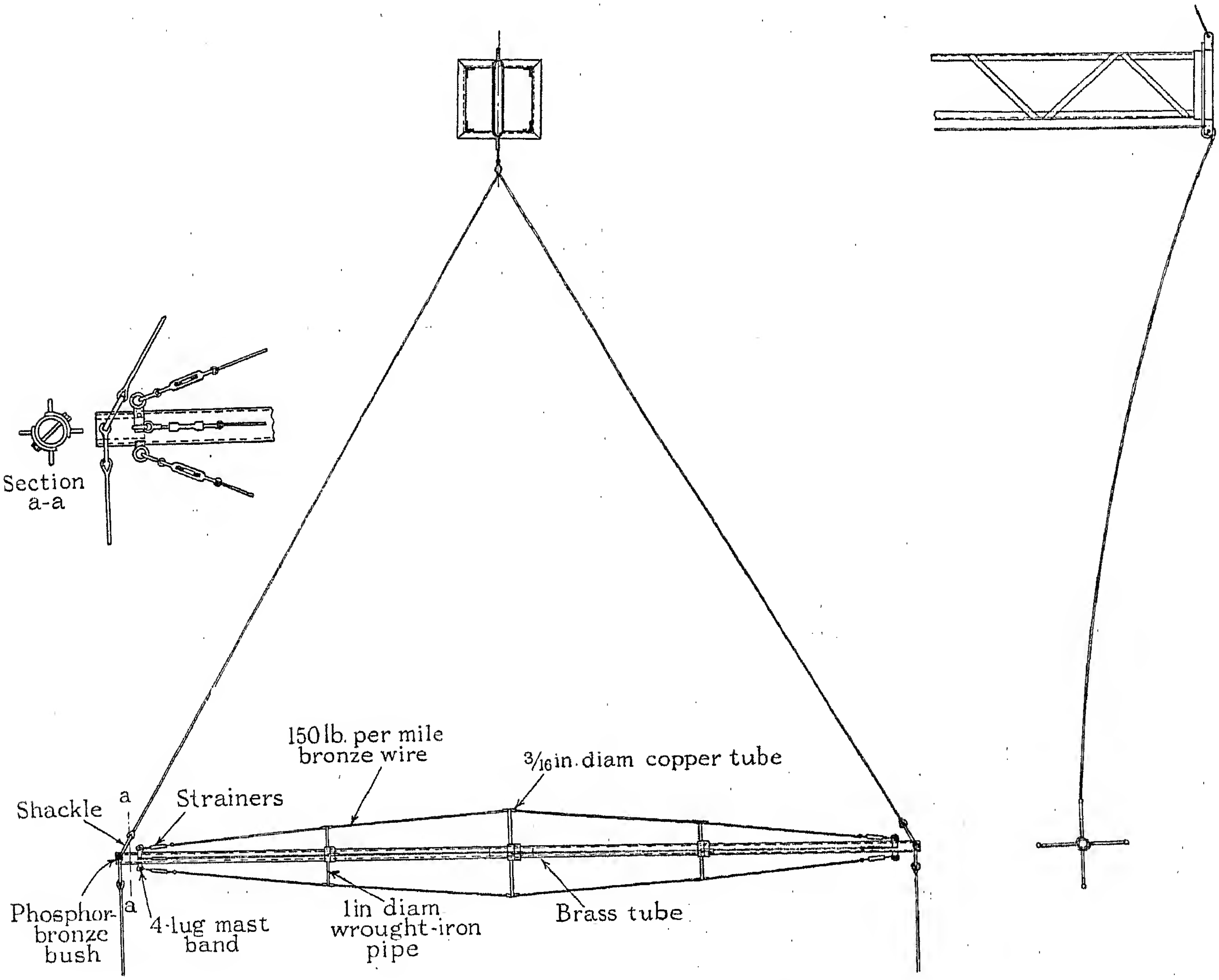



Fig. 28.—Spreader for short-wave arrays.

### Strut Formula.

A straight-line strut formula is the most practicable type; the actual formula used by the author is:—

$$\text{Permissible stress in lb. per sq. in.} = 18\,000 - \frac{80l}{r_g}$$

where $l$ is the effective length of the column, and $r_g$ is the minimum radius of gyration of the steel section. The effective length ($l$) can be equal to $\frac{2}{3}$ AB if the joints of the bracings at A and B are well bolted to the posts.

### Torsion.

Arrays usually have two curtains suspended about $\frac{1}{4}$ wavelength apart from the cross-arms of towers. This be designed to resist torsion and the extra top wind load. To remove this objection, the author has designed spreaders built up of central tubes and outer silicon-bronze tension wires. A typical design, and the method of suspending the spreader from a tower, are shown in Fig. 28. Spreaders of this type, tested to withstand a horizontal pull of 2 tons applied at the end of the spreader and at right angles to its length, are in use. The longest spreader as yet used is of 24 ft., a length suitable for an array designed for a wavelength of 30 m. An additional advantage of the spreaders is that they enable the whole array to be lowered in one operation, since they are suspended from one steel rope passing over a pulley at each tower.

### WEIGHT OF TOWERS.

The type of tower considered in this section, therefore, is the tapered structure without cross-arms. Such a structure, having a top horizontal load $P$ due to the aerial-supporting rope, will experience a bending moment $Py$ at a distance $y$ from the top, necessitating a post with a section of breadth $b_p$ to resist the resulting stress. Furthermore, the bracing must have a width $b_s$ to resist horizontal shearing forces due to P. The exposed surfaces offer resistance to wind, and in consequence the breadth of both bracing and posts must be increased to meet the additional stresses, assuming that the thickness is already a maximum. Each additional increment of breadth, by giving rise to an increment of wind resistance, produces increased loading, with the result that instead of the mass of the tower increasing almost linearly with height as would be the case if there were no wind load the increase is almost exponential. The exponential relation between height and mass is seen from the curves

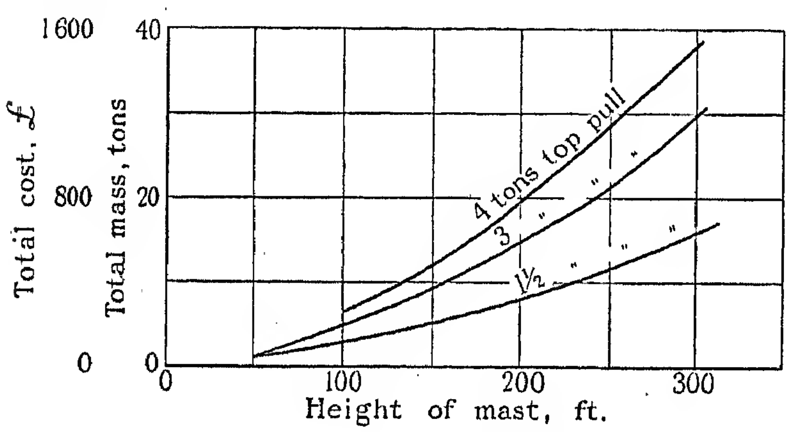



FIG. 29.—Total mass and cost of square self-supporting masts.

of Fig. 29, giving values taken from actual designs. The equation of the curves is approximately of the type

$$W = k_1 + Pe^{k_2 h}$$

where $W$ is the mass of the tower in tons, $P$ is the top horizontal pull in tons, $h$ is the height of the tower in feet, and $k_1$ and $k_2$ are constants depending on the wind load. In the same figure the cost of complete structures is given on the basis of £40 per ton weight of steel.

Two features of these curves which have an important bearing on the question of tower economics are: (1) The cost is almost directly proportional to the top horizontal load. (2) The cost increases rapidly with height. Any increase in array gain, obtained by increasing the number of radiators in the vertical plane or by raising the height of the arrays, must therefore be carefully balanced against the increased cost of towers.

## Part 3. TRANSMISSION LINES.

The choice of transmission lines lies between the open-wire and the concentric-tube type, and since the former are much less costly than the latter their adoption is justified provided their use is not attended by serious loss of efficiency. The term efficiency covers both loss of energy and, in the case of reception, degradation of signals due to undesired "pick-up" by the lines.

The author has already put forward the view* that the radiation losses from twin-wire transmission lines having equal antinodal values of current at corresponding points are not great, and has shown* that the greatest radiation losses per unit of length in twin-wire lines occur at the ends and that the radiation losses from long lines are not proportionally greater than those from short lines. It is proposed in this section to examine the question of radiation from twin lines in greater detail.

In the case of feeders to transmitting aerials, the total loss in the lines is usually of greater importance than the distribution of radiation from the lines. On the reception side, however, particularly where feeders from different aerials run fairly close to each other, the distribution of the field from the transmission lines is the more important. Even though the total energy radiated from a twin open line is small, there is a possibility that there may be narrow lobes of radiation of sufficient intensity to give rise to appreciable pick-up between feeders. Before proceeding to investigate this possibility, it is

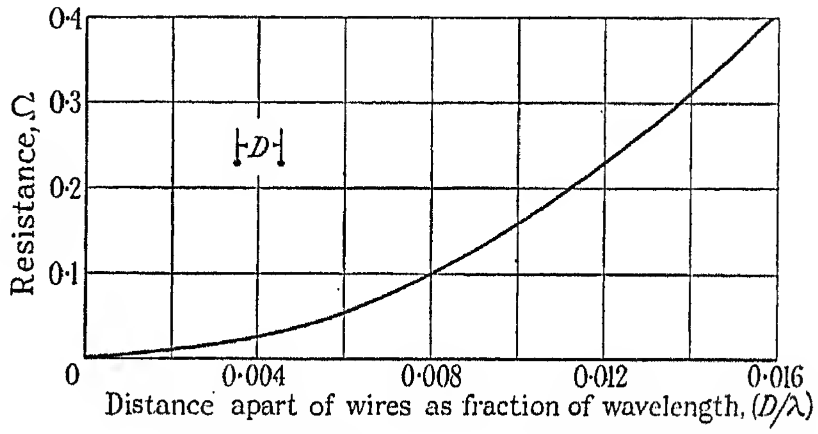



FIG. 30.—Radiation resistance of twin-wire transmission lines.

relevant to inquire into the effect of pick-up. Under normal conditions, the selectivity of the receiver will be quite sufficient to guard against interference between stations on adjacent allocated wavelengths. When fields on one wave are much stronger than those on the other, however, any additional energy transferred to the receiver of the weak signals via the feeders will increase the difficulty of reception.

Furthermore atmospherics, giving rise to all frequencies, will cause greater interference with the weak signal if the feeders have appreciable pick-up. In any large receiving station equipped with beam arrays having various orientations atmospheric disturbances from all directions may be received at good strength on one or more arrays. The transmission lines from the arrays converge on the receiving room and are thus in fairly close proximity at the receiver end. Unless the transmission lines are practically non-radiating, therefore, conditions will be extremely favourable for picking up atmospheric disturbances arriving from any direction, and the advantages of the directivity of the arrays will to a great extent be lost.

With regard to the total radiation losses from twin-wire transmission lines, the author has already shown† that, in the case of lines having standing waves, the loss is almost independent of length. In the case of lines carrying progressive waves of negligible attenuation,

* See Bibliography, (8).

* See Bibliography, (11). † *Ibid.*

Sterba and Feldman* have shown that the loss on twin lines separated by a small fraction of a wavelength is practically independent of length and is approximately equal to $160I^2(\pi D/\lambda)^2$ watts, where $I$ is the current in amps. and $D/\lambda$ is the separation of the wires, expressed as a fraction of a wavelength.

In Fig. 30 a curve has been plotted giving the value of the radiation loss for various distances between the wires, whilst in Fig. 31 curves have been drawn giving the ratio of the high-frequency ohmic resistance to the radiation resistance at a frequency of 18 750 kilocycles per sec.

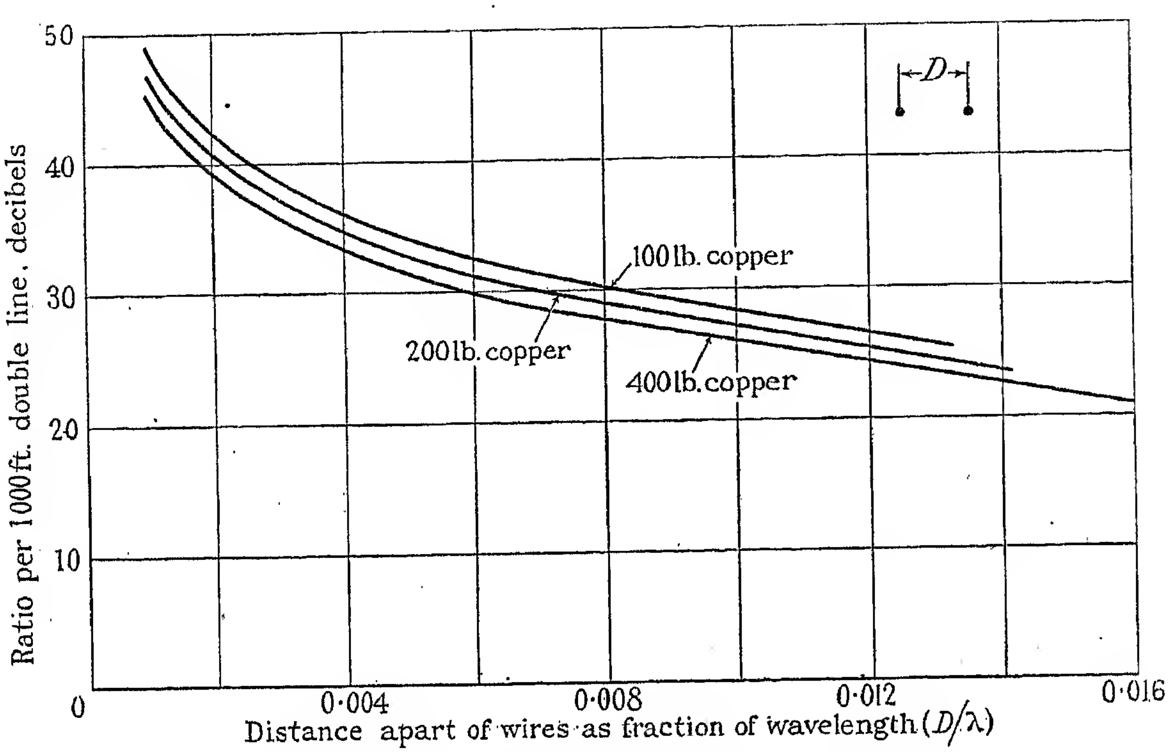



FIG. 31.—Ratio of high-frequency resistance to radiation resistance of twin-wire transmission lines ($\lambda = 16$ m).

The high-frequency ohmic resistance has been calculated from data given in the Bureau of Standards (U.S.A.) Circular No. 74. It is at once obvious that the total radiation loss in a twin line is inappreciable provided the distribution of currents in the two lines is similar but opposite in phase.

With regard to the distribution of radiation, consider a wire AL (Fig. 32*a*) of length $n\lambda$. Taking A as the origin and regarding the current as progressing from A to L, at a point P located a great distance $r_0$ from A the electric field intensity, $d\epsilon$, due to the current in a small length of wire, $dx$, will be proportional to

$$\frac{\delta}{\delta t}\cdot\frac{I_0 \sin\theta_1}{c^2 r_0}\sin\left[\omega\left(t-\frac{r_0}{c}\right)+\frac{2\pi x}{\lambda}(\cos\theta_1-1)\right]dx$$
$$=\frac{I_0\omega\sin\theta_1}{c^2 r_0}\cos\left[\omega\left(t-\frac{r_0}{c}\right)+\frac{2\pi x}{\lambda}(\cos\theta_1-1)\right]dx$$

where $I_0 \sin \omega t$ is the instantaneous value of the current at A, and $c$ is the velocity of propagation of electromagnetic waves in space, which is practically the same as the velocity of the current along the wire. Thus the electric field intensity due to a non-attenuated current in the whole wire is

$$\epsilon=\frac{I_0}{c^2 r_0}\omega\sin\theta_1\int_{x=0}^{x=n\lambda}\cos\left[\omega\left(t-\frac{r_0}{c}\right)+\frac{2\pi x}{\lambda}(\cos\theta_1-1)\right]dx$$
$$=\frac{2I_0}{cr_0}\cdot\frac{\sin\theta_1}{(\cos\theta_1-1)}\cdot\sin\left[n\pi(\cos\theta_1-1)\right]\cos\left[\omega\left(t-\frac{r_0}{c}\right)+n\pi(\cos\theta_1-1)\right]$$

As for the purposes of this discussion we are only concerned with the relative amplitudes of the electric field, the expression $\sin\theta_1\sin\left[n\pi(\cos\theta_1-1)\right]/(\cos\theta_1-1)$ need only be considered.

The multiplying factor to take account of the reflection from the earth can be obtained by regarding the reflected waves as being due to an image running parallel to the wire and separated therefrom by a distance $2h$ (Fig. 32*b*). This assumes a perfectly reflecting earth, which is approximately correct when, as in this case, the radiators are parallel to the earth. As the current in the image is reversed in sign, the multiplying factor $2\sin\left[(2h\pi/\lambda)\sin\theta_1\cos\beta\right]$ is obtained.

If now a second parallel wire (Fig. 32*b*), having the same height $h$ above the earth, is brought a distance $D$ from the first wire and carries current reversed in phase at the appropriate points, a third multiplying factor is obtained, namely $2\sin\left[(D\pi/\lambda)\sin\theta_1\sin\beta\right]$. The full expression for the amplitude of the field intensity thus becomes

$$\epsilon_2=\frac{2K\sin\theta_1\sin\left[n\pi(\cos\theta_1-1)\right]}{(\cos\theta_1-1)}\times 2\sin\left(\frac{2h\pi}{\lambda}\sin\theta_1\cos\beta\right)\times 2\sin\left(\frac{D\pi}{\lambda}\sin\theta_1\sin\beta\right)\quad(2)$$

* See Bibliography, (12).

The expression

$$\frac{2K \sin \theta_1 \sin [n\pi (\cos \theta_1 - 1)]}{(\cos \theta_1 - 1)}\cdot$$

gives the radiation characteristic of a single straight wire

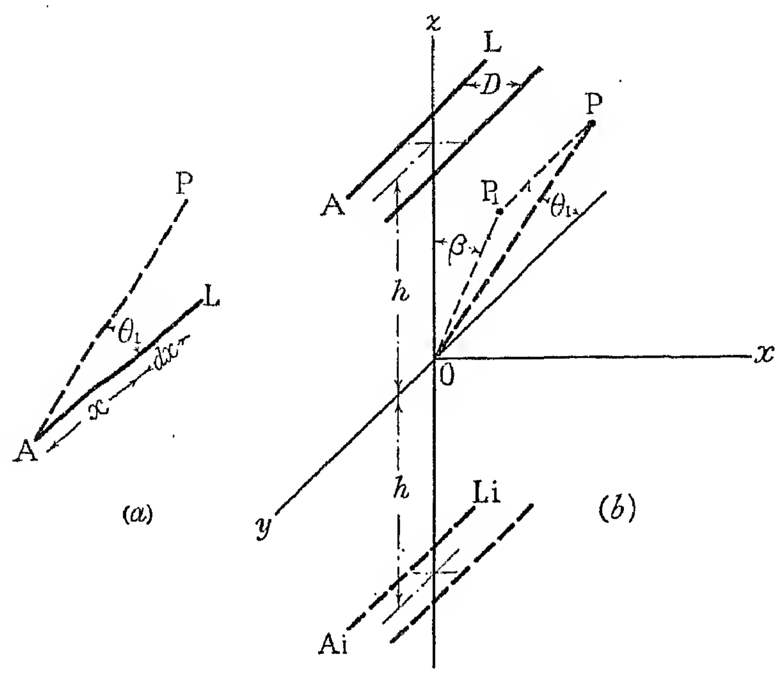



FIG. 32.

in space, assuming no reflections from other objects. As the length of the wire and therefore the value of $n$ increases, the angle $\theta_1$ made by the axis of the primary lobe of radiation decreases. For any particular value of $\theta_1$ between $\pi/2$ and 0, the expression is a maximum for the length of wire which makes $\sin [n\pi (\cos \theta_1 - 1)] = -1$. Now when $\theta_1$ is small

$$\frac{\sin \theta_1}{(\cos \theta_1 - 1)} = \frac{\sin \theta_1}{\sqrt{(1 - \sin^2 \theta_1)} - 1} \doteqdot \frac{-2}{\theta_1}$$

The limiting value of the maximum field intensity due to a wire of infinite length in space is therefore infinity. This, however, is a hypothetical case, since the reflection from the earth must be considered.

The radiation characteristic of a single wire running parallel to the earth is therefore more correctly represented by

$$\epsilon_1 = \frac{2K \sin \theta_1 . \sin [n\pi (\cos \theta_1 - 1)]}{(\cos \theta_1 - 1)} \times 2 \sin \left(\frac{2h\pi}{\lambda} \sin \theta_1 \cos \beta\right) \quad . \quad (3)$$

It is convenient to represent maximum radiation from transmission lines of various lengths in terms of the maximum radiation from a line $\frac{1}{2}$ wavelength long and $\frac{1}{4}$ wavelength high. This is roughly the maximum height of the transmission lines used in practice. Under these conditions the maximum radiation occurs in the plane where $\beta = 0$. Denoting by $\theta_{1m}$ the value of $\theta_1$ at which the expression (3) is a maximum, it is necessary to differentiate and equate to zero in order to find the value of this angle. This operation gives the following relationship for the maximum value of (3).

$$\sin \theta_{1m} \tan \theta_{1m} \left\{ n\pi \cot [n\pi (\cos \theta_{1m} - 1)] - \frac{1}{\cos \theta_{1m} - 1} \right\} - \frac{2h\pi}{\lambda} \sin \theta_{1m} \cos \beta \times \cot \left(\frac{2h\pi}{\lambda} \sin \theta_{1m} \cos \beta\right) = 1 \quad (4)$$

In the particular case of the half-wave radiator, the value of $\theta_{1m}$ for maximum radiation is 67° approximately, and the maximum field intensity is $4K \times 1\cdot 23$.

Fig. 33 shows the relationship between $\theta_{1m}$ and $n$ derived from expression (4), and also the maximum value of the field intensity for increasing values of $n$. This maximum intensity approaches $4\pi K$, the limiting value of expression (3) as $\theta_{1m}$ approaches zero. Thus

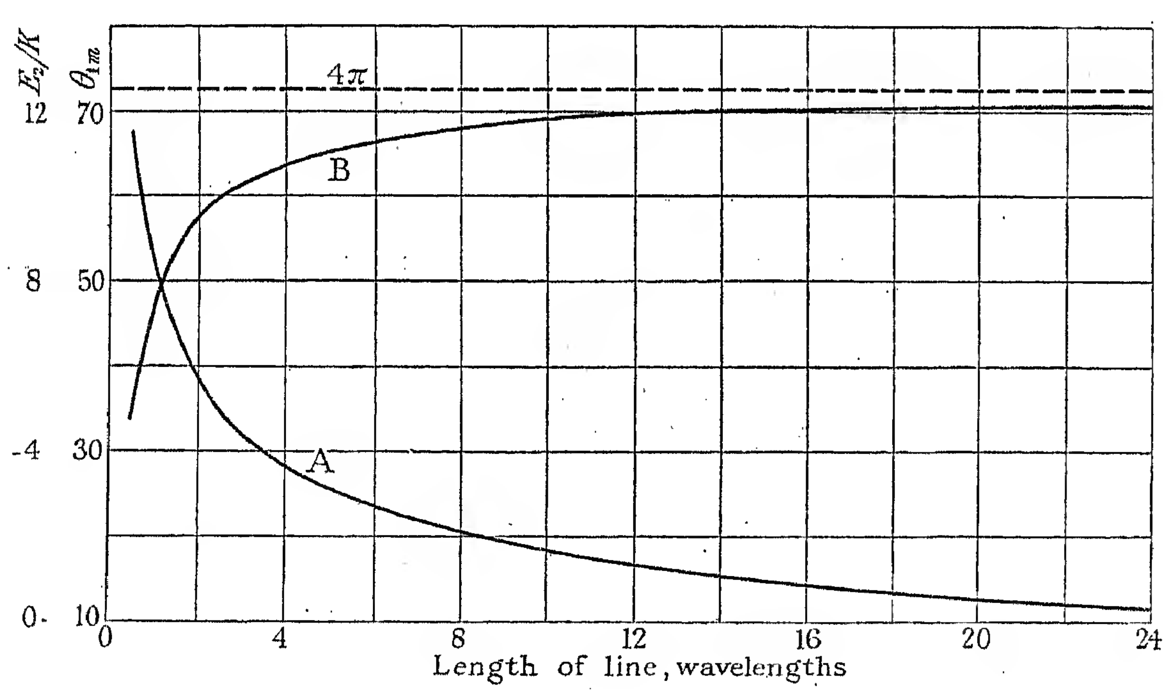



FIG. 33.

A. Relationship between angle $\theta_{1m}$ and length of a single transmission line ($N\lambda$) placed $\frac{1}{4}$ wavelength above perfectly reflecting earth.
B. Maximum value of field intensity of principal lobe of radiation due to single line referred to in A.

the field intensity of the principal lobe of radiation of a single infinitely long line, running parallel to the earth $\frac{1}{4}$ wavelength high and having unattenuated progressive currents, is $(\pi/1 \cdot 23)$ times the intensity due to a similar line of length $\frac{1}{2}\lambda$. Hitherto, a single line only has been considered. The effect of running a second parallel wire is to introduce the factor $2 \sin [(D\pi/\lambda) \sin \theta_1 \sin \beta]$, by which expression (3) must be multiplied.

This factor approaches zero as $\theta_1$ becomes vanishingly small, thus reducing to zero the intensity of the field, which for a single-wire line has been seen to have a limiting value $4\pi K$. Moreover, since the separation $D$ of the two wires is a small fraction of a wavelength, the factor increases in value as $\theta_1$ increases from 0° to 90°. Thus the greatest value of the field intensity due to a long twin-wire line is not that arising from the primary lobe but that which arises from some higher-order lobe. To ascertain the greatest value of the field, it is necessary to examine expression (2). Since $D\pi/\lambda$ is extremely small, the expression may be written thus:—

$$\frac{8KD\pi}{\lambda} \cdot \frac{\sin^2 \theta_1 \sin \beta}{\cos \theta_1 - 1} \cdot \sin \left(\frac{2h\pi}{\lambda} \sin \theta_1 \cos \beta\right) \times \sin [n\pi (\cos \theta_1 - 1)]$$

Since $\sin^2 \theta_1/(\cos \theta_1 - 1) = -(1 + \cos \theta_1)$,

this expression may be reduced to

$$\frac{-8KD\pi}{\lambda}(1 + \cos \theta_1) \, . \sin \beta \sin \left(\frac{2h\pi}{\lambda} \sin \theta_1 \cos \beta\right) \times \sin [n\pi (\cos \theta_1 - 1)] \quad . \quad . \quad (5)$$

The last factor in the expression gives the number of lobes of energy radiated from the twin-wire line for any value of $n$; the remaining factors enable the envelope of amplitudes of the field intensity due to the various lobes to be determined.

Several interesting deductions can be made from expression (5). Firstly, except for small values of $n$, the largest radiation occurs along directions such that $\theta_1$ lies between 50° and 60° approximately; secondly, whatever the value of $n$ the maximum field intensity at a distant point cannot exceed $\pm(8KD\pi/\lambda) \times 0 \cdot 88$; and thirdly, the pick-up in the case of lines used for reception purposes is

$$20 \log_{10} \left(\frac{8KD\pi}{\lambda} \times 0 \cdot 88 \times \frac{1 \cdot 23}{4K}\right), \text{ or } 20 \log_{10} (2 \cdot 165 D\pi/\lambda)$$

decibels compared with the standard half-wave line. Thus if $D/\lambda = 1/70$, which corresponds approximately to a separation of 9 in. between the wires of a twin line used for a wavelength of 16 m, the pick-up is − 20 decibels. It is important to notice that this value is only attained when the received ray arrives at a particular angle. It seems, therefore, that normally the maximum intensity of radiation or the pick-up of a twin-wire line, according to whether the line is used for transmitting or receiving purposes, is not great provided always that the line is accurately balanced.

## Tests.

### *Maximum Pick-up by Transmission Lines.*

The small intensity of the radiation or pick-up renders accurate tests on twin-wire lines very difficult. Moreover, since the lobes of radiation have a high angle of elevation, field tests are almost impossible. The observer therefore has to be satisfied with observations on distant stations, having different bearings around the receiving station. Unfortunately the horizontal half-wave aerial is not suitable as a standard of reference for measurement purposes, since its characteristics in a horizontal plane vary between a maximum and a zero value. A vertical aerial, however, has a circular polar diagram in a horizontal plane, and is thus more suitable as a standard. With the idea of obtaining some knowledge of the order of magnitude of the pick-up of a 2-wire transmission line, a vertical half-wave aerial was therefore used as a standard of reference. Tests made on distant stations using a half-wave vertical aerial and a horizontal half-wave aerial having their centres at approximately the same height above the earth, showed that on the average there was little difference between the signal strengths received by the two aerials provided the distant station was in a direction approximately at right angles to the horizontal aerial. The half-wave vertical aerial had its lower end 3 ft. from the earth, and terminated in a tuned circuit from which short balanced transposed transmission lines were taken to a receiver. The twin-wire transmission line under test consisted of 400lb.-per-mile wire built 18 ft. above the earth on porcelain insulators spaced 9 in. apart. The total length of line was 1 060 ft., and the distant end was terminated in a fine high-resistance wire having a resistance equal to the measured surge impedance of the line, namely 550 ohms. Several distant stations having a transmitting wavelength in the region of 30 m were observed, the stations being chosen to cover as large a range of bearings as possible. The method employed was to tune to a particular station and then to switch from the transmission line under test to the comparison vertical aerial. This process was repeated at intervals of 2 or 3 minutes and the receiver adjusted by means of the attenuator to give equal outputs. The average of many readings showed that the vertical aerial gave 18 decibels gain over the transmission line, no single reading showing a gain of less than 14 decibels. Although the tests were admittedly rough they served to show that the pick-up on a 2-wire balanced line is not appreciably greater than might be anticipated from theory.

### *Total Loss on Line.*

With regard to the total loss in a 2-wire line, tests were made by measuring the input and output on a measured length of transmission line consisting of a 1 200 ft. run of 400 lb. per mile copper wire spaced 9 in. apart. The termination consisted of a Striplite lamp giving a total resistance of approximately 550 Ω. The small portable transmitter already described was used to feed energy into the line, and thermo-couple ammeters were clamped at the adjacent maximum- and minimum-current points at both ends of the line. Particulars of the line are given in Fig. 34, which also shows the results of the tests made at various frequencies and the resistance loss calculated by means of the tables and formulæ given in Bureau of Standards Circular No. 74. For the purpose of comparison, tests were also made on a concentric copper-tube line 1 200 ft. long running

parallel to the open-wire line. The inner and outer tubes were connected at both ends to short lengths of twin-wire lines, which were matched to the impedance of the tube line by a quarter-wave matching line. Striplite was used to absorb the energy at one end, and current measurements were made at the adjacent maximum and minimum points in a short length of open wire at both ends. The construction of the tube line is shown in Fig. 34, together with the test results. The calculated loss of the tube line has been plotted, the values being obtained from the following formula due to A. Russell:—

$$R = \sqrt{(\rho\mu f)}\left(\frac{1}{a} + \frac{1}{b}\right) \times 10^{-9}$$

where $\rho$ = resistivity (1 700 electromagnetic units for copper); $\mu$ = magnetic permeability; $f$ = frequency, in cycles per sec.; $a$ = outer radius of inner conductor, in

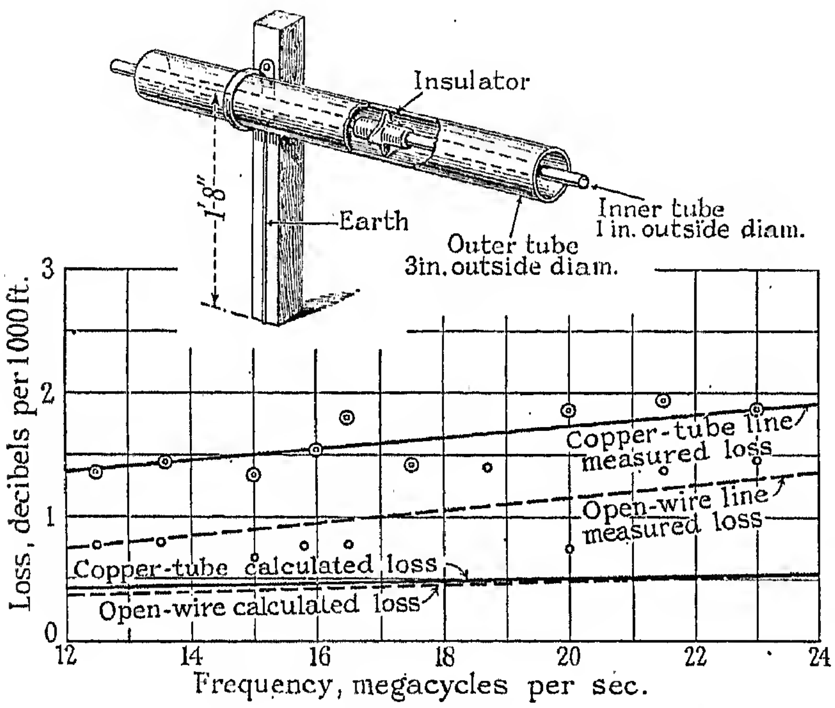



FIG. 34.—Power lost in transmission lines.

– – – – Open twin-wire line, supported from porcelain insulators fixed to wooden poles spaced 150 ft. apart. Weight of conductors 400 lb. per mile, spacing between conductors 9 in.
——— Concentric-tube line, inner tube supported on insulators 2½ ft. apart, outer tube supported on wooden stakes and earthed every 5 ft.

cm; $b$ = inner radius of outer conductor, in cm; and $R$ = equivalent resistance per cm, in ohms.

The curves illustrate that although the calculated values of the resistance losses of the open-wire and concentric-tube lines are almost the same there is an appreciable difference in the measured losses, the wire line being the more efficient. The fact that insulators are spaced every 150 ft. on the open line, and every 2½ ft. on the tube line, possibly accounts for this higher efficiency. The results of these and other tests show that overhead twin-wire lines can be used with reasonable efficiency, and that the pick-up, which might be undesirable in the case of a receiving station, need not be unduly large.

In practice, when several open-wire transmission lines enter a receiving station in close proximity to each other the " throw-in " or " cross-talk " can be reduced to very small limits by transposing the pairs within the building at distances of about 18 inches. Thirty twin lines entering the Baldock radio station have been erected in this manner.

## Part 4. RELATIONSHIP BETWEEN CAPITAL OUTLAY ON ARRAY SYSTEMS AND RESULTING GAIN IN FIELD STRENGTH.

### CALCULATION OF GAIN OF ARRAYS.

The problems of the best angle of propagation of waves in a vertical plane and of the relationship between height and cost of steel towers for supporting arrays having been investigated, and the fact that open-wire transmission lines can be efficiently used at both receiving and transmitting stations having been confirmed, the important question of the capital outlay required on aerial systems to produce various relative field strengths at distant places, and conversely to procure various degrees of augmentation of the received signal at the receiver, can be considered. The evidence presented in Part 1 seems sufficiently satisfactory to justify the conclusion that there is a preferential angle of projection, and that energy projected at other angles does not, on the average, greatly affect the values of the resultant field strength measured over a period of a few minutes. On this understanding it is possible to calculate the relative field strengths at a distant station due to aerials having different radiation characteristics.

The comparison is conveniently made with a fixed horizontal aerial, ½ wavelength above the earth, as a standard of reference. If now the preferential ray is projected at an angle $\beta$ and the resulting fields due to the standard aerial and the compared aerial are $\epsilon_{1\beta}$ and $\epsilon_{N\beta}$ respectively, the aerial gain in decibels will be represented by

$$20 \log_{10}\left[\frac{\epsilon_{N\beta}}{\epsilon_{1\beta}}\sqrt{\left(\frac{P_1}{P_N}\right)}\right]$$

where $P_1$ and $P_N$ are the power radiated by the standard and compared aerials respectively. Two methods are available for calculating the ratio $P_1/P_N$. In the first, the expression for the field intensity $\epsilon$, obtained for any point equidistant from the aerials, is substituted in the Poynting vector $(c/4\pi)\epsilon^2$. The integration which is then necessary to obtain the total power radiated is very laborious. A second and more convenient method is to calculate the radiation resistance of the radiators by methods suggested by Pistolkors, and since the two methods yield practically the same results the second will be used. Two approximations are made: (1) The earth is a perfect reflector; and (2) the radiation resistance is the same for individual radiators in an array. The first approximation has already been stated to be justified in the case of horizontal radiators, and the second introduces errors which are negligible as far as the present application is concerned.

If $R_1$ and $R_N$ are the respective resistances of the single standard half-wave radiator and of the individual half-wave elements of an array consisting of $N$ radiators, then it follows that if the current is the same at the middle

of each radiator the gain of the array in decibels is given by the expression

$$20 \log_{10} \left[ \frac{\epsilon_{N\beta}}{\epsilon_{1\beta}} \sqrt{\left(\frac{R_1}{R_N N}\right)} \right] \quad . \quad . \quad . \quad (6)$$

The value of the radiation resistance of any one radiator in an array is obtained by taking account of the effect of every other radiator upon it, in a manner explained by Pistolkors.* When the values of $\epsilon_{1\beta}$ and $\epsilon_{N\beta}$ (calculated as described in Part 1) and of $R_1$ and $R_N$ are known, it is then a simple matter to calculate the value of expression (6).

In the foregoing discussion the assumption has been made that the radiation resistance as obtained by the methods of Pistolkors gives accurate values. In order to ascertain to what extent this assumption is justified, tests were made on a single-tier, two half-wave (A) aerial and a 2-tier, two half-wave (B) aerial. The A aerial was fixed at a height of $\frac{1}{2}$ wavelength above earth, whilst the B aerial was raised in three stages—from $\frac{1}{2}$ to $\frac{3}{4}$, and finally to 1 wavelength, above the earth. The aerials were connected to a transmitter by a twin-wire transmission line, 100lb.-per-mile copper being used for aerial and lines. The input to the aerials was measured at adjacent maximum- and minimum-current points in the line near the aerials, and the aerial currents were measured at the point of maximum current, i.e. at the middle of each half-wave radiator. High-frequency ammeters of the thermo-couple type were employed for all current measurements. The currents in the individual radiators of the aerials in any particular location were practically the same. For example, the current in each of the four radiators of the B$\frac{3}{4}$ aerial did not differ by more than 4 per cent from the average value of the current in all the radiators. The results of the tests are summarized in Table 8. Having regard to the limitations imposed by the method of test, the agreement between the measured and calculated values of the resistance is considered to be reasonable. The errors introduced into the general deductions by the lack of agreement are almost negligible.

* See Bibliography, (14).

TABLE 8.

*Comparison between measured and calculated values of resistance of half-wave radiators ($\lambda = 20\cdot78$ m).*

| Type of aerial and number of $\frac{1}{2}$-wave radiators | Input | Sum of squares of aerial currents in radiators | Resistance of single half-wave radiators | | Measured resistance / Calculated resistance |
|---|---|---|---|---|---|
| | | | Total resistance, by measurement | Radiation resistance, by calculation | |
| | (a) | (b) | (c) = $\frac{(a)}{(b)} \times 1\,000$ | (d) | (e) = $\frac{(c)}{(d)}$ |
| | kW | (amps.)$^2$ | Ω | Ω | |
| A$\frac{1}{2}$, 2 | 3·46 | 34·2 | 101·0 | 86·0 | 1·17 |
| B$\frac{1}{2}$, 4 | 6·12 | 88·0 | 69·5 | 74·0 | 0·94 |
| B$\frac{3}{4}$, 4 | 3·9 | 84·0 | 70·0 | 76·0 | 0·92 |
| B1, 4 | 6·35 | 88·0 | 72·0 | 75·5 | 0·95 |

The maximum gain of an array is obtained when the axis of the primary lobe of radiation lies along the preferential direction of projection. If then in expression (6) for the gain of an array over the standard half-wave aerial the angle $\beta$ is such as to give this coincidence of axis and direction, and thus a maximum value of $\epsilon_{N\beta}$, the value of $\epsilon_{1\beta}$ for the standard half-wave aerial will usually be less than its maximum value. For example, if the preferential angle of projection is 79° to the vertical, then reference to Fig. 4 shows that if a 2-tier aerial is used when the height of the lower radiator is 1 wavelength above the earth the direction of maximum radiation will coincide with the preferential direction.

Reference to Fig. 3 shows, however, that the radiation at 79° from a single-tier aerial $\frac{1}{2}$ wavelength above the earth is only 1·1 (approximately) compared with 3·75 (approximately) in the previous case. Thus, by inserting these values in expression (4), the gain of a 2-tier aerial over a standard half-wave radiator is found to be

$$20 \log_{10} \frac{3\cdot75}{1\cdot1} \sqrt{\left(\frac{69\cdot3}{61 \times 2}\right)} = 8\cdot4 \text{ decibels}$$

Since for a given number of tiers the shape of the vertical polar diagram in a plane normal to the plane of an array of similar radiators is the same regardless of the span of the array, the gain due to the additional vertical lines of radiators can readily be obtained provided the average radiation resistance of the radiators in the arrays is known. For example, the average calculated values of the radiation resistance of a single half-wave radiator of a 4-tier array whose lowest member is 0·7 wavelength above the earth are 60 Ω and 80 Ω, according to whether the span of the array is $\frac{1}{2}$ wavelength or 8 half-wavelengths. The gain in decibels of the larger array containing 32 radiators, over the smaller array containing 4 radiators, is thus:—

$$20 \log_{10} \sqrt{\left(\frac{R_4}{R_{32}} \times \frac{32}{4}\right)} = 10 \log_{10} \left(\frac{60}{80} \times \frac{32}{4}\right) = 7\cdot8.$$

As the span of an array increases, the variation of the average resistance with span becomes very small, and

thus the gain in decibels of an array over a similar type of array having the same number of horizontal tiers is $10 \log_{10}$[(Span of wider array)/(Span of smaller array)]. Applying the foregoing principles, the values of the spans respectively are given in Figs. 35 and 36. With most arrays a reflector curtain is provided. Such a curtain is usually located at a distance of about $\frac{1}{4}$ wavelength from the main excited curtain, and at this

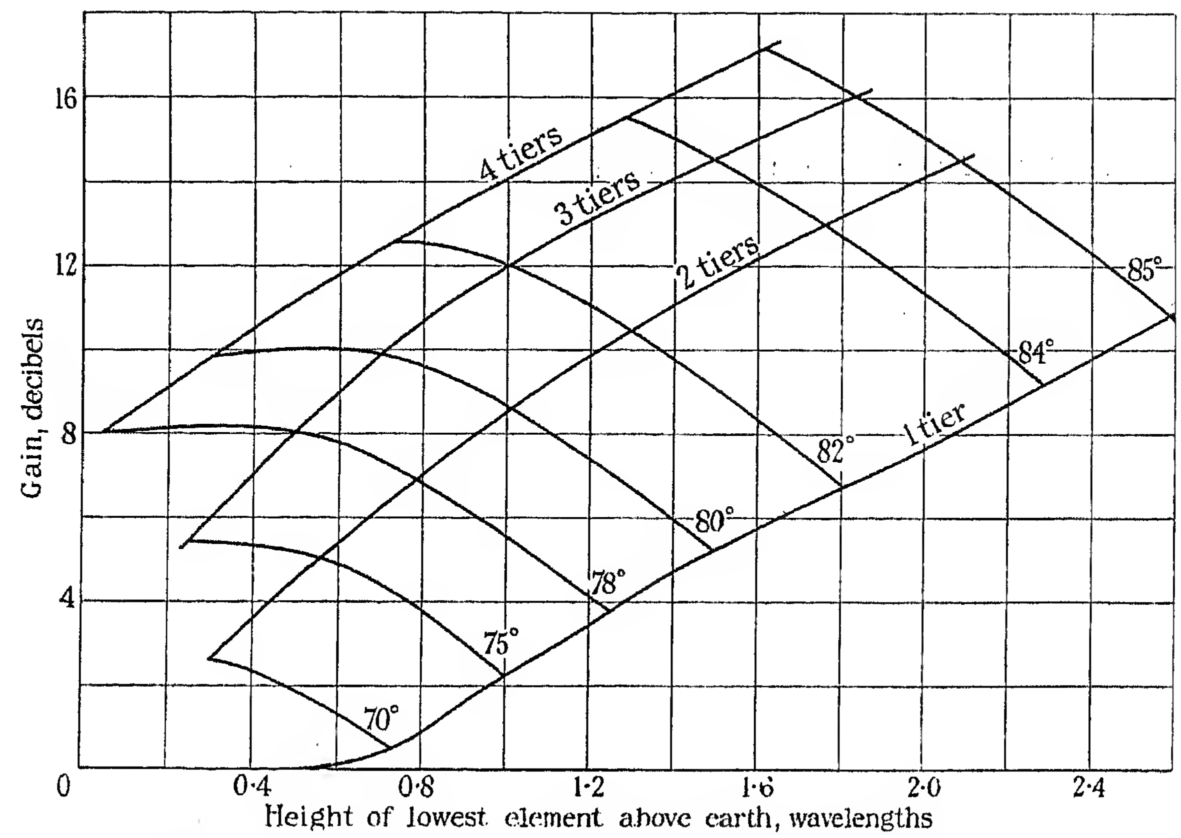



FIG. 35.—Gain of 1, 2, 3, and 4 tiers of half-wave horizontal radiators spaced vertically $\frac{1}{2}$ wavelength apart between adjacent radiators, over a single half-wave radiator fixed $\frac{1}{2}$ wavelength above earth. The heights of the various aerials are such as to give a maximum field strength in directions making the indicated angles to the vertical.

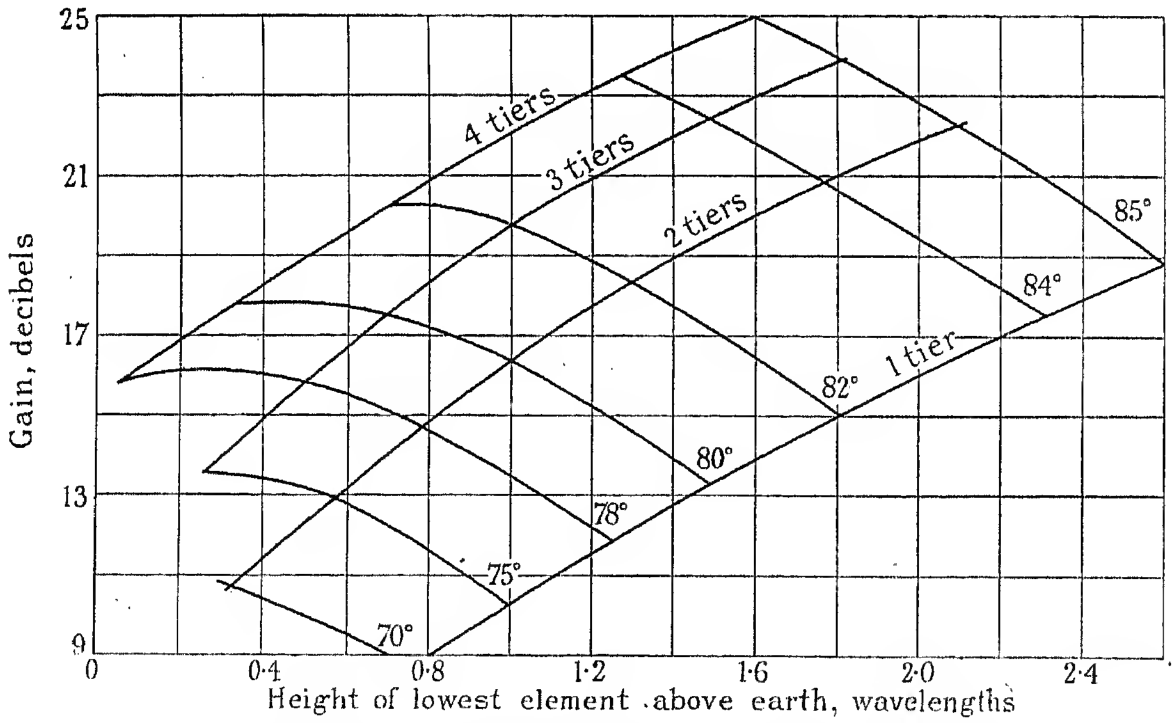



FIG. 36.—Gain of 1, 2, 3, and 4 tiers of half-wave horizontal radiators spaced vertically $\frac{1}{2}$ wavelength apart between adjacent radiators (each tier consisting of 8 such radiators), over a single half-wave radiator fixed $\frac{1}{2}$ wavelength above earth. The heights of the various aerials are such as to give a maximum field strength in directions making the indicated angles to the vertical.

maximum gain for various spans of arrays comprising 1, 2, 3, and 4 tiers, have been calculated for several angles of projection of the primary lobe of radiation in a vertical plane normal to the array. Typical curves of gain relating to arrays having $\frac{1}{2}$- and 4-wavelength short distance it hardly changes the shape of the vertical polar diagram in the desired plane of propagation at the angles of projection considered in this paper. Thus the addition of a reflector curtain will merely result in an increase of 3 decibels in the gain.

## Cost of Array Systems.

There now remains to be considered the cost of array systems designed to produce various gains. At this stage practical experience must aid theoretical deduction in arriving at the best means of producing certain gains at the lowest cost. A type of array system which combines simplicity of construction, economy of first cost, ease of maintenance, and efficiency of operation, is outlined in Fig. 37. It comprises horizontal half-wave radiators supported between steel lattice-work masts. The two curtains, one directly energized and the other inductively excited from it, are separated by approximately ¼ wave-length by means of the metal spreaders to which reference was made in Part 2. The dips and tensions in

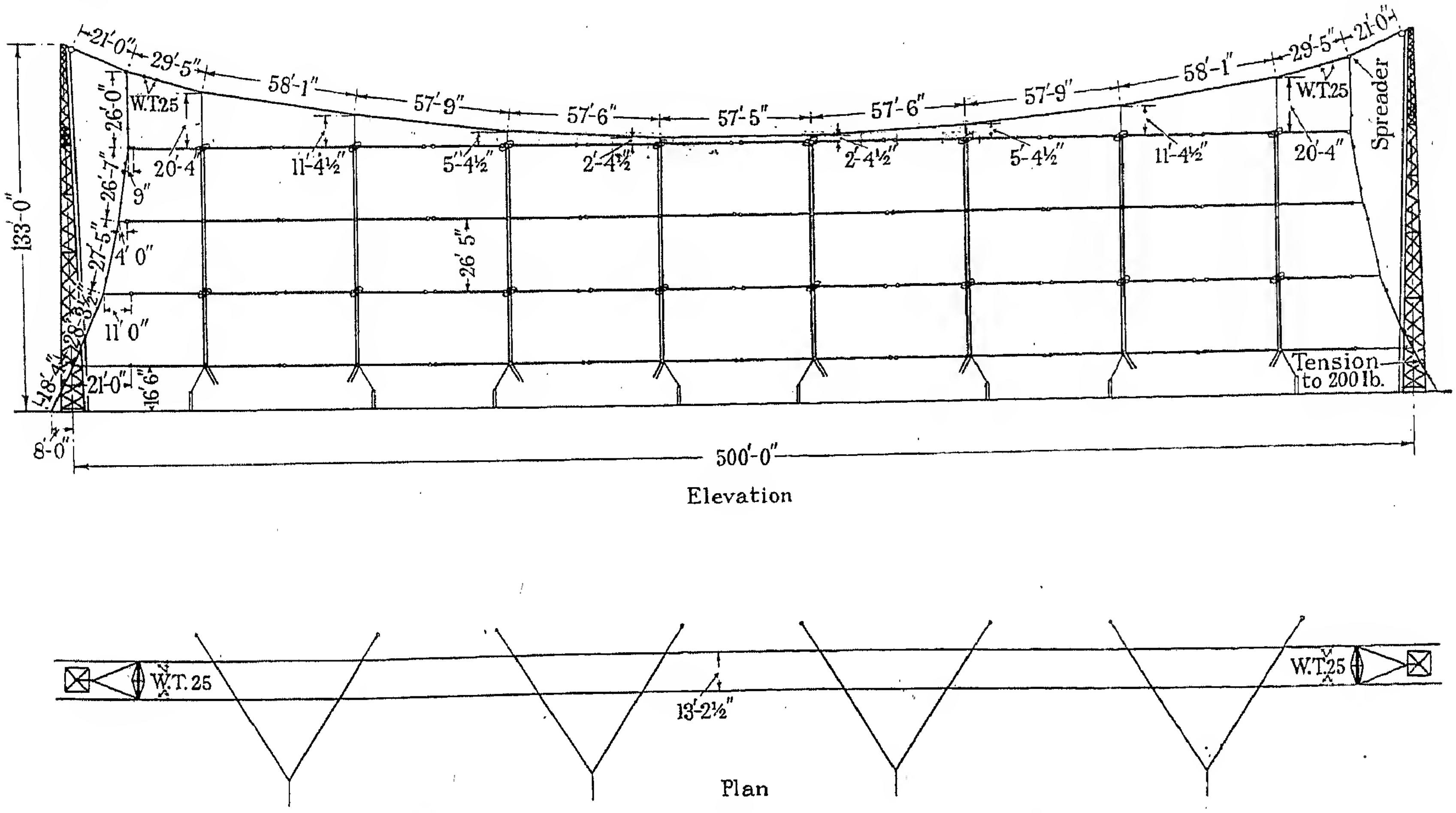



Fig. 37.—Typical horizontal-type array with reflector.

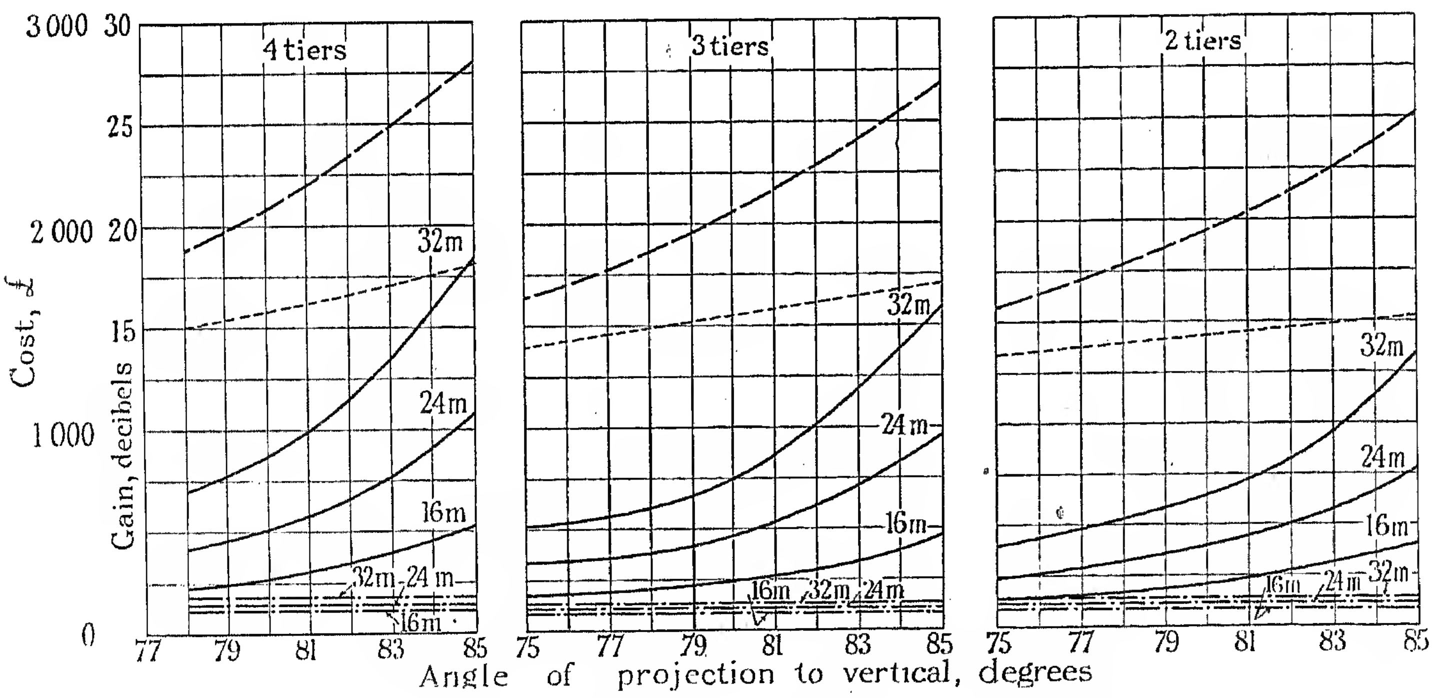



Fig. 38.—Cost of array installation to give maximum gains over a standard half-wave aerial for different angles of projection to the vertical for a 4-wavelength span array.

Cost of towers.
Cost of array and 1 000 ft. open-wire transmission line.
Maximum gain over actual standard half-wave aerial.
Maximum gain over hypothetical half-wave aerial.

supporting cables have been arranged so that a horizontal pull of 30 cwt. on the top of the masts, and a downthrust of 15 cwt., shall not be exceeded. These values have been chosen as a result of experience of a large number of designs. Sometimes a slight reduction in cost can be effected by reducing the dip and thus the height of the supporting cable; the extra weight of the additional tower is more than compensated by the reduced weight and cost of the remaining towers. The results of a very large number of calculations are shown in curve form in Figs. 38, 39, and 40. These give the estimated costs, excluding contingencies and overhead charges, of various

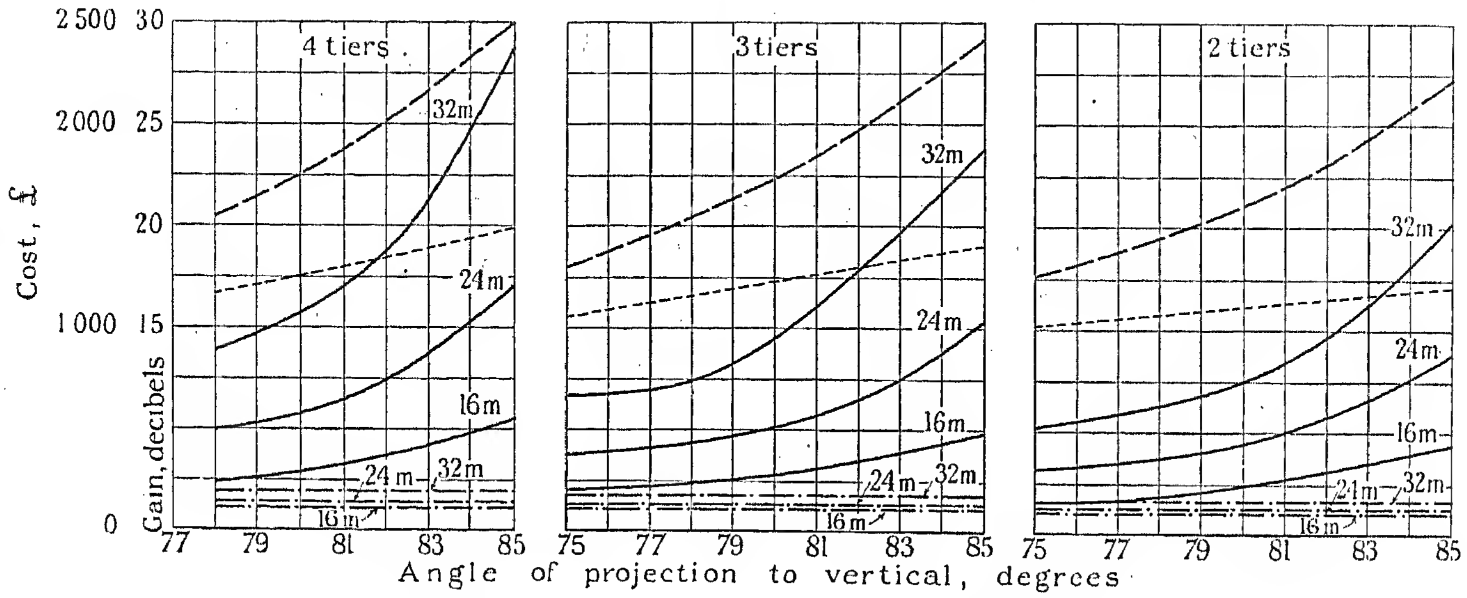



FIG. 39.—Cost of array installation to give maximum gains over a standard half-wave aerial for different angles of projection to the vertical for a 6-wavelength span array.

Cost of towers.
Cost of array and 1 000 ft. open-wire transmission line.
Maximum gain over actual standard half-wave aerial.
Maximum gain over hypothetical half-wave aerial.

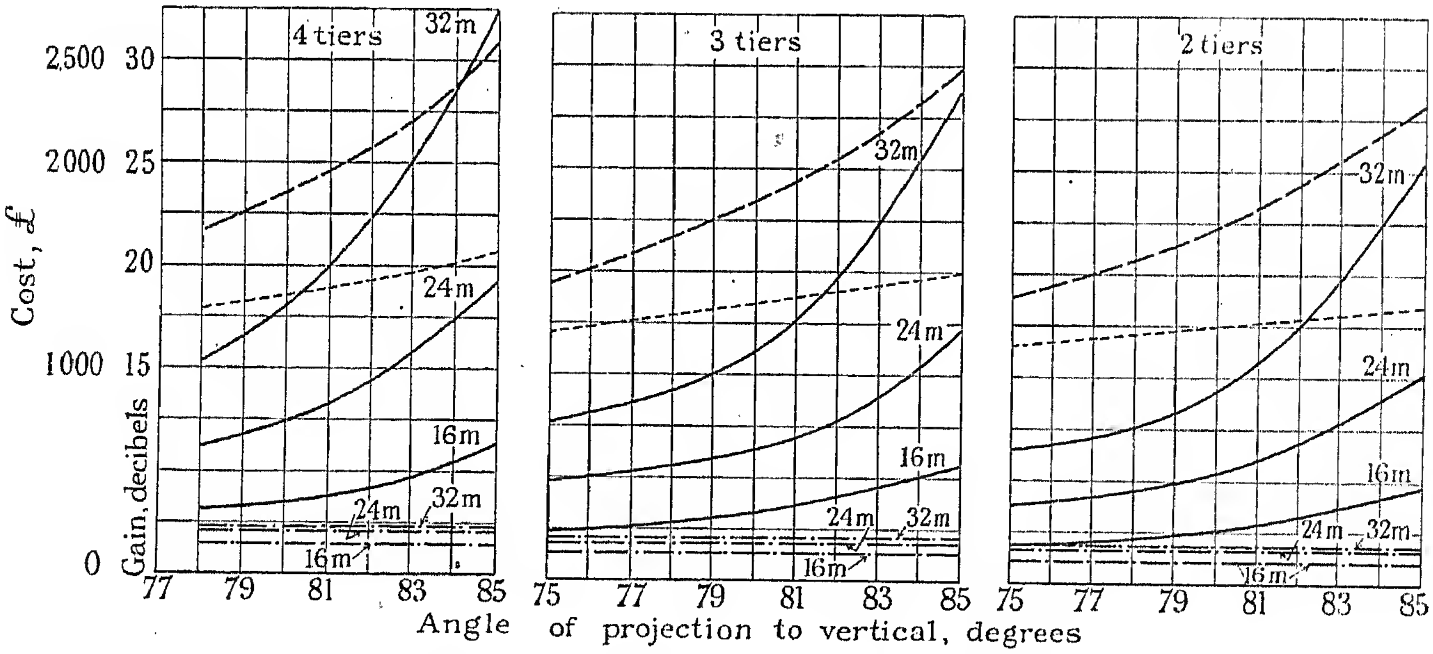



FIG. 40.—Cost of array installation to give maximum gains over a standard half-wave aerial for different angles of projection to the vertical for an 8-wavelength span array.

Cost of towers.
Cost of array and 1 000 ft. open-wire transmission line.
Maximum gain over actual standard half-wave aerial.
Maximum gain over hypothetical half-wave aerial.

supporting towers, but usually the economic advantage of reducing the height is more than offset by the increased cost due to the greater top pull on the towers.

In deciding the cost of array installations the number of towers to support the arrays has been chosen so as to produce the most economical results. Costs are sometimes reduced by erecting an additional tower for the purpose of diminishing the span and thus the dip of the array installations giving maximum gain over a half-wave horizontal aerial fixed ½ wavelength above the earth. In the same figures the curves of the array gains, obtained as already explained, are also given, an allowance of 3 decibels for reflector gain being added. In studying the gains it is necessary to bear in mind that the direction of maximum radiation from the standard aerial makes an angle of approximately 60° with the

vertical, and that at greater angles the radiated energy is less. This explains the increased gains of the arrays as the direction of maximum radiation makes greater angles with the vertical. The ideal standard aerial would be one that radiated uniformly in all directions in vertical planes of propagation. Such a standard is impossible of attainment. A useful standard would be a hypothetical aerial having the same radiation resistance and maximum radiation as the actual half-wave aerial, but whose maximum radiation could be directed at any angle with the vertical. With this hypothetical aerial as a standard of reference it would be possible to compare the maximum field strengths due to the various arrays with the field strength due to an aerial emitting constant radiation at the relevant angle of projection. Curves have been added to Figs. 38, 39, and 40, giving this comparison with the hypothetical aerial. By plotting in each case the cost of the array against the corresponding gain, derived from Figs. 38, 39, and 40, the curves of Fig. 41 have been obtained. These curves give the minimum cost only. For example, at an angle of projection of 80° a 4-tier 6-wavelength span array for a wavelength of 24 m costs £720 complete, whilst a 2-tier 8-wavelength array having the same approximate gain costs £670. The lower price has therefore been taken. The curves of Fig. 41 illustrate the following facts.

(1) The cost of array systems rises rapidly for a given gain as the wavelength increases.

(2) The cost of array systems designed for a particular wavelength rises rapidly as the gain increases; particularly in the case of arrays designed for the longer wavelengths.

(3) The cost of arrays designed for a particular wavelength increases rapidly for a given gain over the hypothetical aerial as the angle of projection approaches 90°. For example, the costs of complete array systems for a gain of 16 decibels over the hypothetical aerial for 24 m at angles of projection of 78°, 80°, and 85°, are £510, £540, and £980 respectively.

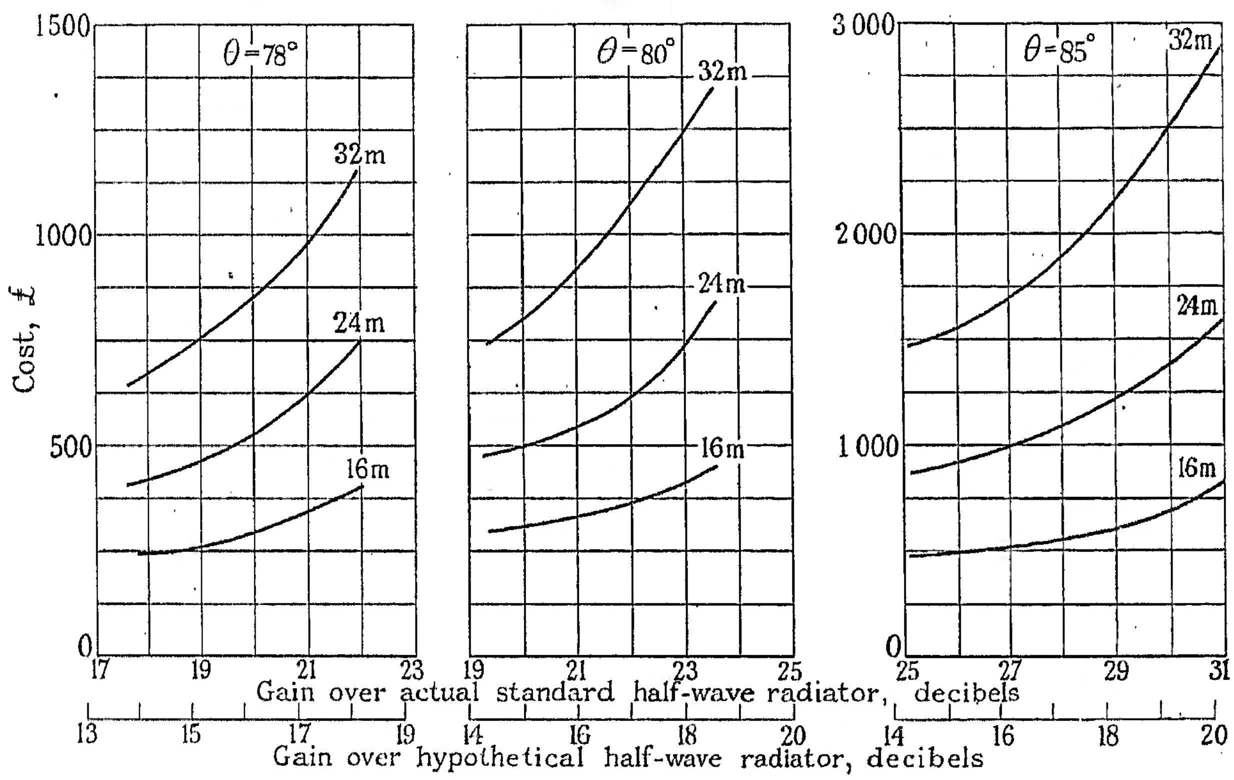



FIG. 41.—Maximum cost of horizontal-radiator type aerial array systems to give various gains over a single half-wave horizontal radiator fixed ½ wavelength above earth.

## CONCLUSIONS.

Briefly summarized, the main conclusions to be drawn from the investigations described in this paper are:—

(1) Before the design of an array system is undertaken, tests should be made over several months to decide upon the best angle of propagation in the vertical plane. If this varies appreciably with the seasons, as in the case of the Rugby–Berlin circuit, several arrays having various angles of projection and reception, or one array capable of having its angle varied, should be built.

(2) In the case of the Rugby–New York circuit, the angle of projection averaged over periods of a few minutes on the wavelength tested—20·78 m—varied very little over a yearly period, its average value being about 79° to the vertical.

(3) When the direction of maximum radiation from an array coincides with the preferential direction of projection the received field strength is a maximum. Increasing the height of the radiators raises the cost of structures and reduces the received field strength.

The thanks of the author are due to the Engineer-in-Chief of the British Post Office for facilities for carrying out the investigations and for permission to publish the results recorded in this paper. The collaboration of the various authorities mentioned in the paper, and in particular of Mr. H. T. Friis and Mr. R. K. Potter, is highly appreciated. Finally, sincere thanks are due to the author's colleagues, particularly to Mr. W. Bewick, Mr. J. L. Howard, Mr. H. G. Crook, Mr. J. A. Sheppard, and Mr. E. E. Brown, for their assistance; and to Mr. R. A.

Watson Watt, Mr. J. F. Herd, and the staff at Slough, for their unfailing courtesy in providing data relating to propagation phenomena.

## BIBLIOGRAPHY.

(1) A. Meissner and H. Rothe: " On the Determination of the Optimum Radiation Angle for Horizontal Antennæ," *Proceedings of the Institute of Radio Engineers*, 1929, vol. 17, p. 35.

(2) T. L. Eckersley: " Short-Wave Wireless Telegraphy," *Journal I.E.E.*, 1927, vol. 65, p. 601.

(3) N. J. O. Strutt: "Measurement of the Electrical Properties of the Earth between 20 and 2 × $10^7$ cycles per second," *Elektrische Nachrichten-Technik*, 1930, vol. 7, p. 387.

(4) J. A. Ratcliffe and F. W. G. White: " The Electrical Properties of Soil at Radio Frequencies," *Philosophical Magazine*, 1930, vol. 10, p. 667.

(5) M. Baumler, K. Kruger, N. Plenal, and W. Pfiter: " Radiation Measurements of a Short-Wave Directive Antenna at the Nauen High Power Radio Station," *Proceedings of the Institute of Radio Engineers*, 1931, vol. 19, p. 812.

(6) H. T. Friis and E. Bruce: " A Radio Field Strength Measuring System for Frequencies up to Forty Megacycles," *ibid.*, 1926, vol. 14, p. 507.

(7) R. K. Potter and H. T. Friis: " Some Effects of Topography and Ground on Short-Wave Reception," *ibid.*, 1932, vol. 20, p. 699.

(8) T. Walmsley: " Beam Arrays and Transmission Lines," *Journal I.E.E.*, 1931, vol. 69, p. 299.

(9) T. L. Eckersley: " Studies in Radio Transmission," *ibid.*, 1932, vol. 71, p. 405.

(10) T. Walmsley: " Principles of Radio Tower Design," *Post Office Electrical Engineers' Journal*, 1931, vol. 24, p. 231.

(11) T. Walmsley: " Distribution of Radio Resistance in Open Wire Radio Transmission Lines," *Philosophical Magazine*, 1931, vol. 12, p. 392.

(12) E. J. Sterba and C. B. Feldman: " Transmission Lines for Short Wave Radio System," *Proceedings of the Institute of Radio Engineers*, 1932, vol. 20, p. 163.

(13) J. S. McPetrie: " A Method of Determining the Effect of the Earth on the Radiation from Aerial Systems," *Journal I.E.E.*, 1932, vol. 70, p. 382.

(14) A. A. Pistolkors: " The Radiation Resistance of Beam Antennas," *Proceedings of the Institute of Radio Engineers*, 1929, vol. 17, p. 562.

Amongst other works consulted, the publications of Prof. E. V. Appleton, Dr. R. L. Smith-Rose, and Messrs. R. M. Wilmotte and J. S. McPetrie, have been of great assistance.

## Discussion before the Wireless Section, 3rd January, 1934.

**Col. A. S. Angwin:** The deductions made in the paper have been further applied to one or two particular cases. In connection with the American transmission services on wavelengths of the order of 20 m, an array has been modified to give an angle of elevation of approximately 10° and the anticipated gain has been produced, the field strength being much increased as a consequence of the new design. Further measurements have revealed the tendency, with increase of wavelength, to an increase in the optimum angle of elevation for the transmission. Tests on one of the Canadian transmitters utilizing a wavelength of the order of 60 m indicate that the angle is of the order of 20° to the horizontal, instead of 10° as for wavelengths of the order of 20 m. I am very interested in the author's slides showing the radiation characteristics of transmission lines, and I think that the value of the paper would be increased if one typical example of these were added to it. His summary of the economical application of his data is confined to the consideration of horizontal radiators; I assume that for assemblies of vertical radiators the polar diagram in the vertical plane necessarily has an angle of maximum radiation which is almost horizontal. I should be glad if the author would briefly indicate why he omits to consider vertical radiators and also arrays of the Chireix type, which are neither horizontal nor vertical.

**Mr. N. Ashbridge:** The paper is of great interest to the B.B.C. on account of its bearing upon the problems of the recently introduced Empire broadcasting service. This service is not merely concerned with maintaining communication between two fixed points; it has to provide simultaneous reception over a large area. The author lays stress on the point that there is an optimum angle for projecting the ray, and that this optimum angle varies with distance. This consideration immediately presents a difficulty, because the distances in any one zone, for any one transmission of the Empire service, vary enormously. Moreover, at the point of reception one cannot rely on an aerial being erected which receives at an optimum angle. In trying to provide a worthwhile service we are therefore seriously handicapped, unless we decide to have a very large number of transmitters for each particular zone and for each particular programme. Already for each zone we have to transmit on two different wavelengths, and if we are also to transmit with several arrays for different angles in order to provide for the differences in distance, the cost might become prohibitive. One of our great difficulties has been that of finding out the true facts of each problem. We have not been in the fortunate position of the author, who has had the help of experts at both ends, but have had to rely on a kind of mass observation. We find that it is not safe to judge the service we are giving in Australia by reports from one or two professional observers in that country, because it seems to vary considerably over a few hundred miles. The results of our mass observation seem to be far more useful, however, than one would have expected. We make use of printed forms on which reports can be made out by picked and careful observers, but we have no hope of being able to

obtain a number of measurements expressed in terms of field strengths. In view of the difficulties, I am surprised at the excellence of the results which are obtained. With regard to the types of arrays, we started with vertical dipoles raised only $\frac{1}{4}$ wavelength above earth. They had reflectors, and a varying number of elements was provided in order to give the necessary horizontal directivity. We soon came to the conclusion that we were not giving the best possible service, and we therefore tried the effect of $\frac{1}{2}$-wave horizontal dipoles at considerable heights above the earth. A scientific comparison of the two shows at once that elevated horizontal dipoles are infinitely superior to lower vertical dipoles. Whether this has anything to do with the angle at which the energy is projected I am not certain; it may be merely due to a greater efficiency of radiation. We intend to investigate the problem on much more scientific lines. I should like to mention that at the Pontoise station there is a French broadcasting short-wave transmitter which employs a Chireix aerial. Our mass observations show that Pontoise is giving a better service on short wavelengths than any other broadcasting station in the world, and this seems to suggest that the higher type of aerial is more likely to give an all-round good result than a lower one. Does the author think that the nature of the ground where the rays are re-reflected affects the result obtained? Most of his information is based on the Atlantic route, for which the intervening "earth" is all sea. I should like to know whether the results would have been different if they had been obtained on a nearly all-land route, such as between here and India. Does it matter very much if one of the areas where the energy is re-reflected from the earth comes in a large sandy tract like the Sahara, or, probably worse still, in mountainous regions? In the latter case the fact that the size of a mountain is very considerable compared with a wavelength may be of importance. With regard to the question of the best type of aerial to erect to get maximum efficiency at the lowest cost, in broadcasting this is complicated by the fact that the write-off on an aerial system may be small compared with programme costs. This fact does not affect the present Empire broadcasting station, however, because we are running it on a rather restricted budget. It is more applicable to an ordinary national broadcasting service.

**Mr. T. L. Eckersley:** The author has investigated in a very thorough manner the optimum angle of vertical projection of a beam aerial, and his results appear to agree well with those obtained by other workers. Although it is explicitly stated that the investigation is one which determines this optimum angle, and not essentially the ray angles, the analysis is based on the assumption of a single downcoming ray. While no doubt in a large number of cases examined by him on the 20·78 m wave one single ray was predominant, there is plenty of evidence to show that on longer waves there may be as many as four or five rays sharing the energy more or less equally. The evidence suggests that the wave chosen was short enough for the higher-angle rays to escape, leaving one predominant ray; so that there is general agreement between the observed height/field-strength curves and the curves calculated on the assumption of a single ray. Perhaps some of the anomalies in these curves are due to the existence of multiple rays. In spite of this doubt it is comforting to find good agreement between the author's results and those of other workers. Three methods have been employed for finding the ray angles. One is the method of raising the transmitting aerial, described by the author. In the second method, which has been applied recently by the Radio Research Board, the angle is deduced from the measurement of the phase difference of the e.m.f.'s induced in two spaced aerials. The third method uses the time-delay of facsimile pulses to give the required angles. The first two are based on the assumption of a single ray, and the last on the assumption that the time-delay of the first pulse is known. It is satisfactory, therefore, to record that the results obtained by the three methods are in accord, at least for transatlantic transmission. The predominant angle found by the author is 79° (11° vertical elevation), and the Radio Research Board have found predominant angles of 73°, rising to 79° in the summer months. Facsimile observations give 82° and 71° for the second- and third-order rays respectively reflected at a height of 340 km. Recently, Friis, Feldman, and Sharpless* have given results obtained by a combination of phase-difference and pulse methods, for a 33·28 m transatlantic transmission. They obtain, as an example, 72·5°, 70°, 64°, and 56°, the last two agreeing with the values of 63° and 56° determined by facsimile methods for the fourth- and fifth-order rays. The agreement is better on the higher-angle range. I am not sure that it is quite correct to express the feeder theory in the way adopted in the paper. The author states that the radiation losses from each element of line are not the same, the radiation being less at the middle of the line and greater at the ends. I am not sure that it is permissible to divide the line into parts and speak of the radiation resistance of each part, because this depends on the current/voltage distribution of all the rest. If, however, he merely means that the energy radiated by the line is not proportional to its length, I am in agreement with him. In this connection I would point out that there is no outward radiation from an infinite line so long as the phase velocity along the line is less than that of light; but if the phase velocity along the line is greater than that of light there is a component of outward radiation. Phasing condensers in series with the line would produce such an effect, and the pick-up radiation from such an open line might become important.

**Mr. J. F. Herd:** The part of the paper which is of particular interest to us at the Radio Research Station, Slough, is that dealing with long-distance propagation, particularly the information given on pages 561 and 562. I am sure the author will agree that our knowledge of the conditions is still inadequate to permit very definite conclusions to be drawn regarding the trajectory and the number of reflections concerned in transatlantic propagation. Measurements of the downcoming angles of these signals are in systematic progress at Slough in co-operation with the Post Office, using the spaced-dipole method already referred to by Mr. Eckersley. The apparent tilt of the layer mentioned on page 562 is

* *Proceedings of the Institute of Radio Engineers*, 1934, vol. 22, p. 47.

confirmed by our measurements in the opposite transatlantic direction. The lack of any important influence of the form of the transmitting aerial on the downcoming angle at the receiver is illustrated by measurements made within the past few days, at the suggestion of Mr. H. L. Kirke, on the American stations Rocky Point WAJ (22·26 m) and Lawrenceville WMA (22·4 m). The former is a telegraph station providing keyed-morse emissions, the latter a telephony station having a continuous-wave carrier. These stations gave comparable field strengths at Slough, and Fig. A shows that the downcoming angles of the signals from the two stations were practically identical, despite the fact that different types of transmitting-aerial systems are believed to be in use at the two stations. The difference in the early part of the period shown in Fig. A is effectively within the limits of experimental error, while later the two curves are still more closely identical. Mr. Eckersley has referred to our measurements by means of short-

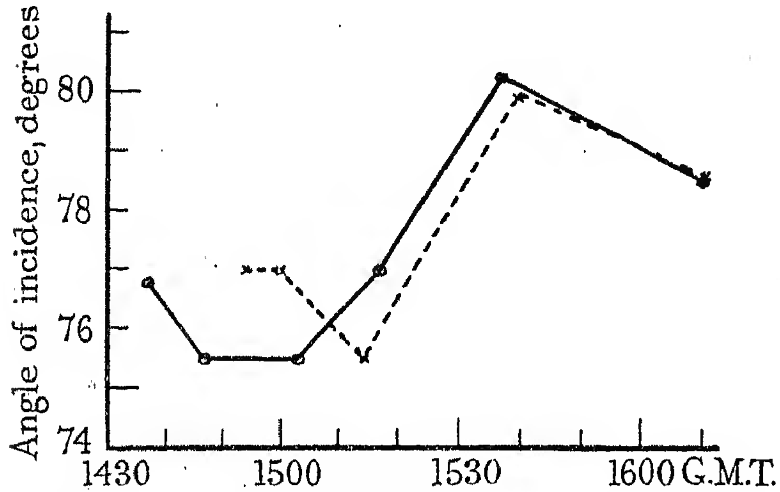



FIG. A.—Comparison of angles of incidence of WAJ (Rocky Point, 22·26m) and WMA (Lawrenceville, 22·4m), 1st January, 1934.

—O——O— WAJ
- -X- - -X- - WMA

duration pulse signals, and to the possibility of there being more than one downcoming ray. A programme of observations on such pulses from Lawrenceville has been in progress for some time and is giving us considerable information as to the number of rays present and their respective strengths. As the subject calls for considerable work at different times of day and at different seasons, as well as on the different wavelengths employed at the various times, a comprehensive report will not be available for some time.

**Mr. H. L. Kirke:** The author brings to light two important factors which have not been described previously. These are: (1) the proof that the vertical polar diagrams agree substantially with the observed signals, and (2) the fact that the changes in transmitting angle do not affect the downcoming angle. All the aerials which he describes have radiators spaced $\frac{1}{2}$-wavelength between the conductors and presumably have the same phase of current in each radiator. Has the author considered the question of using other spacings or other phases? Whereas the actual angles which have been deduced from the measured field-strength ratios in the calculated polar diagrams vary between $71\frac{1}{2}°$ and $81°$, it would appear from Table 5 that if the difference in signal strength is a sufficiently accurate guide for the deduction of the angle and the actual path of the wave from the transmitter, then it could also be used to show that the downcoming angle at the receiver is affected by the transmitting angle. If in Table 5 instead of averaging the deduced value of optimum transmitted angle one takes the average of each particular aerial at different times, one finds that for the 2-tier aerial $\frac{1}{4}$ wavelength above the ground the average angle for the four tests was 74·5°, that for $\frac{3}{4}$ wavelength high was 76·4°, and that for $1\frac{1}{4}$ wavelengths high was 79·5°. While these figures do not agree with the other evidence that has been produced to the effect that the average angle is not affected by the maximum radiation angle of the transmitter, nevertheless it does look as if something of that sort is happening. Alternatively it could be deduced from Table 5 that for the time period 1324 to 1400 G.M.T. the path taken was at a very high angle to the vertical for the $\frac{3}{4}$- and $1\frac{1}{4}$-wavelength conditions as compared with the other conditions. I am very interested in the question of vertical aerials, because of the results that we have obtained by comparing high horizontal aerials with low vertical aerials and a high vertical aerial with a high horizontal aerial. In the latter case the result was greatly in favour of the horizontal aerial, probably owing to the fact that our feeding conditions to the vertical aerial were not satisfactory and that there was a possibility of coupling of the vertical aerial to the mast from which it was supported. I should like to know the author's opinion as to the difference between the vertical and the horizontal aerial; whether it is a question of greater earth losses in one case than in the other, or of the different polar diagrams of the two types. I was interested in Mr. Herd's remarks, in view of the measurements that were made by the Radio Research Board at Slough in the summer of 1933 between Lawrenceville and Drummondville. In this case the difference in angle was very great, but the difference in bearing was only about 7°. The difference in bearing could affect the results on account of the closer proximity of the one path to the magnetic pole than the other. Despite the fact that the two aerials on which Mr. Herd made measurements were entirely different, the downcoming angles were apparently the same.

**Mr. J. S. McPetrie:** I am interested particularly in that part of the paper dealing with reflection at the earth. The author, following Wilmotte and myself, denotes the complex reflection coefficient by the expression $(K - jK_1)$. The negative sign in this expression necessitates the two terms $d_1$ and $d_2$ given on page 545 having opposite signs. This point is not at first obvious, and certain workers have assumed both $d_1$ and $d_2$ to be positive with the result that they obtain an error of $\pi$ in the computed phase of the reflected wave. If the reflection coefficient were represented by $(K + jK_1)$, however, this confusion would not exist, as both $d_1$ and $d_2$ would then be positive quantities. Fig. 2 shows the vertical section of the polar diagrams of a $\frac{1}{2}$-wave aerial for two sets of earth constants. Throughout the paper the author is dealing with large angles of incidence. For such angles, in the region of 80°, would there be much difference between the computed curves and that

obtained when the earth is assumed to be a perfect reflector? If there is no great alteration in the polar diagram for such large angles of incidence, the earth could have been assumed throughout the paper to be a perfect reflector without affecting the author's deductions and with much simplification in the analysis. The earth's approximation to a perfect reflector, of course, would not apply for cases in which the angles of incidence are small. Wilmotte* has shown that maximum directivity is obtained from a given aerial array when it has greatest spread in the plane perpendicular to that in which the beam is required. The author has proved conclusively that for long-distance communication this direction is almost horizontal. In this case the aerial array should lie approximately in the vertical plane perpendicular to the direction of the receiver. The arrays designed by the author lie in the vertical plane, and should therefore conform very nearly with the ideal design for distant transmission.

**Mr. H. Dewhurst:** We at Farnborough have experimented for some years with simple arrays. We are more particularly interested in reception over a band of frequencies, and have been somewhat intrigued with the possibilities of long tilted wire antennæ displaying aperiodic characteristics. The horizontal diamond antenna due to Bruce is of this type, and it combines small cost with ease of erection, balancing, and maintenance. The omission from the paper of comparative data dealing with these antennæ is somewhat disappointing. It would appear that for moderate gains the diamond antenna compares very favourably in regard to cost with those mentioned by the author. Turning to Fig. 41, I should prefer the hypothetical standard in the matter of gain to the one in which the standard aerial gives a lobe at an angle which is not necessarily the one required. The angle that the lobe makes is a function of height of aerial above the earth. I think that one should define the obtainable gain in such a way as to make the effect of the earth substantially the same on both the array and the comparison standard antenna. The effect of proximity of the earth on the radiated field strength is obtained by multiplying the characteristic of either in free space by the same factor. I imagine that for a moderate gain—of the order of 12 decibels—giving a vertical downcoming angle of 78° on 20 m, one could erect a Bruce diamond array for considerably less than £400, the cost of the corresponding array of the type mentioned in the paper. By raising or lowering the two ends of the long major axis of these horizontal diamond arrays it is possible to vary the vertical directivity diagram by a few degrees without substantially altering the horizontal directivity diagram. Supposing, for example, that the array has been erected so as to give a vertical angle of 75°, and the two ends are then raised or lowered by 1° or 2°; if the required station comes in at much greater strength it is almost only a question of looking at the array to obtain a more accurate estimate of the downcoming angle.

**Mr. A. J. Gill:** I do not agree with the adjective "sensitive" as applied to the field-strength measuring set mentioned in Figs. 15 and 16. Regarded as a local field-strength measuring set for use in a transmitting station it is fairly sensitive, but its sensitivity does not compare with that of one for measuring fields of a few microvolts. As regards the measurements at Slough on the height of the Heaviside layer, the point at which the reflection occurs will be 900 km from Slough. Does the author think the height of the Heaviside layer would be the same at the point of reflection as it is at Slough? With regard to the author's deductions as to the most favourable angle of transmission, one cannot fail to be impressed with the care which has been exercised in checking and allowing for every possible variable which may influence the correctness of the results. If, however, there are factors which have not been taken into account or to which wrong values have been assigned, the usefulness of the experiments to the radio engineer as far as they determine the arrangement of antenna elements to give optimum signals is not diminished by any possible inaccuracies in assumptions necessary to the computation of the angles of transmission.

**Mr. L. B. Turner:** The first and almost the last time I met the author was at Abu Zabal, Egypt, where he was erecting a large single antenna supported on many masts, to work from a Poulsen arc at a wavelength of thousands of metres. How times have changed in the world of wireless communication is illustrated by the fact that he now erects sets of 100 antennæ and upwards, supported between two masts and working on wavelengths of a few metres.

**Mr. C. E. Strong:** An outstanding feature of the paper is the comparatively small changes in the field obtained by the variations in height mentioned by the author. The main question is, how can the information which he has obtained be applied to reduce selective fading? If we could get a sufficiently sharp beam in the vertical plane, and if it also turned out that the angles of the two paths or two groups of paths in the case of wide band transmissions (assuming there were only two) arriving at the receiving station were sufficiently separated to enable only one of them to be picked up, we could immediately reduce selective fading by having an antenna at the receiving station whose vertical angle could be varied.

**Mr. C. S. Franklin** (*communicated*): The best angle of elevation for the beam has always been a controversial subject, particularly in regard to the limits between which it may vary. I have specified a very large number of arrays during the past 10 years, and have established a practice of projecting the beam some 8° above horizontal for distant services, increasing to about 18° for nearer services. The paper appears to support this practice fairly well. I should like to have seen in the paper more information regarding the evidence obtained by the cathode-ray oscillograph. With regard to the comparison of open twin transmission lines and the concentric-tube type, it is interesting to note how satisfactory it is claimed the open twin line can be. It would have been more interesting still if a few figures had been given showing the amount of pick-up obtained when by chance the line is unbalanced, and how critical the balance has to be. I was responsible for the introduction and design of the concentric-tube feeders for the Imperial beam stations. The open twin type of feeder, although it could be more efficient and cheaper, was

* *Journal I.E.E.*, 1928, vol. 66, p. 955.

not then adopted because experience showed that the balancing of the lines was critical and would not remain constant. The reliability of the concentric-tube feeder was much greater. Other organizations besides the Marconi Co. have since adopted the concentric-tube feeder, and I believe it still remains the most reliable type.

**Prof. J. Hollingworth** (*communicated*): I am very interested in the values obtained by the author for the angles of incidence of long-distance transmissions, as I have always thought that these angles were smaller than some experimenters had anticipated. This belief has been confirmed in many cases by my own observations. While the author's results are quite definite on the practical side as regards the best angle of projection for which a transmitting system should be designed, they raise one or two points in the theory of propagation which call for some comment. Referring first to Table 5, in test 5 the individual values of the angle in successive observations varied from 69° to 80·5°. I should like to ask whether this variation is to be regarded as instrumental or as real (i.e. propagational). If instrumental, it seems that the mean value obtained, 76·9°, must be subject to an appreciable factor of error; if propagational, that considerable changes must have been taking place in the layer. Now although this mean angle may fit in with a definite number of steps it is difficult to make this same number of steps cover individual readings without assuming either large variations in the layer height in short periods, or a rather more complex mode of propagation. Of course, in work of this nature the "possible" heights of the layer (i.e. those lying within the extreme limits which have been directly observed at various times) permit of so much latitude that a "possible" explanation is almost always obtainable. I feel that such angle measurements should always be accompanied by a simultaneous observation of the layer, and I am hoping to start some experiments on these lines shortly. I fully agree with the author that tests such as the German ones over a distance of about 1 000 km are so limited by what may be called geometrical factors that the mode of propagation may be quite different from that which holds over longer distances. I shall therefore confine myself to the American and Teneriffe tests, and I should like to put forward an alternative suggestion. In each case the lower angles which arise are assumed in the paper to be due to one skip more than the higher ones. Now if a ray only just passes through the E layer it will emerge from the top nearly horizontal, and will only regain its original direction if the ionization between the layers falls to zero. There is, however, considerable evidence that this is not the case, and that the ionization in this region is approximately the same as that at the top of the E layer. If this is so, the propagation with the low angle of incidence proceeds with the same number of skips as the others, but does not become parallel to the earth's surface until it has emerged from the E layer. Practically, of course, this makes very little difference, and in the case of waves of about 20 m the possible amount of bending by the E layer is small; but my experiments on 50 m at a distance of 2 500 km have definitely shown that an angle of incidence of 30° is possible in such cases. The importance lies in the fact that if propagation is of this form it is the intensity of ionization of the E layer rather than that of the F which is the critical factor in determining whether a transmission can "get through." The few experiments I have made show that abnormal angles of incidence are associated with abnormal values of the E layer, not of the F; but the results are at present too few to be conclusive. As regards the paragraph on page 562 as to the tilt of the layer affecting the receiving angle, I find this difficult to follow as the only point on the path at which the phenomenon could produce any effect is the actual spot at which the ray strikes the layer. I rather attribute it to the fact that during these hours the ionization of the E layer is steadily increasing, so that a ray reaching its upper surface at nearly grazing incidence would suffer an increasing amount of bending and consequently the received angle of incidence would decrease.

**Prof. L. S. Palmer** (*communicated*): The author's extensive series of experimental measurements by which the most suitable angle of elevation of the axis of the main lobe of radiation was measured and by which the factors controlling this angle were determined, is of particular value to designers of short-wave beam stations; but his discussion of the economic aspect would have been of greater value had a more economical form of antenna array been used for the experiments. The author has considered how to make his particular aerial array radiate most effectively; would it not have been better to have considered first the most economical design of aerial array with which to experiment? I do not think that this has been done. From the cross-connections shown in Fig. 12 it appears that the author has considered the phasing of the currents in the several dipoles and has so designed his tier that the e.m.f.'s in the antennæ due to the direct input from the transmitter are correctly phased for radiation perpendicular to the plane containing the antennæ. These "direct" e.m.f.'s are not, however, the only ones which produce the current in the aerials. The "indirect" e.m.f.'s due to re-radiation from adjacent dipoles are of such magnitude that their effects cannot be neglected compared with those of the "direct" e.m.f.'s. The phase difference between the e.m.f. in one dipole and the indirect e.m.f. produced by it in an adjacent dipole depends on the ratio of the distance between the dipoles to the length of the wave, and when this ratio is 0·5 (the value adopted by the author) the direct and indirect e.m.f.'s in any one dipole do not reinforce each other. Consequently the apparent admittance of the dipole and the efficiency of the tier of dipoles are not as great as they might be. The same question is involved in the discussion (pages 570 and 571) on the position of the reflectors behind the antennæ. I notice that on the former page the distance is referred to as "about $\frac{1}{4}$ wavelength," and on the latter page as "approximately $\frac{1}{4}$ wavelength." This seems to suggest that the author suspects some disagreement with the old theory, which required the distance to be exactly $\frac{1}{4}$ wavelength. In view of the distance employed, it is not surprising that the gain arising from the use of the reflector curtain is only 3 decibels. Dr. Smith-Rose raised this question in December, 1930, in the course of the discussion on the

author's earlier paper.* The reply then given was that "when it was ascertained that a 12 : 1 ratio of front to back field strength was obtained with a quarter of a wavelength spacing, it was considered of little economic importance to proceed further with the matter." These two questions, namely the spacing between the antennæ and the spacing between the antenna plane and the reflector curtain, have been much discussed by independent workers in Germany,† America,‡ England,§ and Japan,‖ and all these investigators have come to the conclusion that the distances used in this work are not the best either in theory or in practice. I should therefore be interested to know whether the author has any theoretical or experimental support for retaining the spacings of half a wavelength between the antenna and a quarter of a wavelength between the antenna plane and the reflector curtain. This point does not affect the value of the present paper. It merely indicates that when the best form of array is suitably operated after the manner suggested by the author, the resulting gains should be considerably greater than those indicated in Figs. 35 and 36, with a consequent decrease in the necessary power input to produce the required field strength at any given point.

**Dr. T. Walmsley** (*in reply*): Several speakers have suggested that the investigations covered by the paper could have been extended in several respects. I do not deny that extensions might be desirable, but I would point out that a lengthy period has already been devoted to the investigations and that the paper is already very long.

With regard to Col. Angwin's remarks about the slides showing the radiation characteristics of transmission lines, the subject matter of these slides is contained in the paper, the difference being that the verbal and the written method of presentation were not the same. I hope to deal more thoroughly with this matter at some later date in another publication. Concerning the optimum angle of radiation from vertical radiators, as far as has been ascertained it appears that this is very much the same as when horizontal radiators are used. Col. Angwin's assumption that the angle of maximum radiation is almost horizontal is, in general, correct.

Mr. Ashbridge dwells upon the particular conditions applicable to the Empire broadcasting service and stresses the difficulties of obtaining the most economical angle of projection or reception at places located at different distances from the transmitter. His remarks about the greater effectiveness of horizontal dipoles as compared with vertical dipoles are interesting and there is little doubt that the extra effectiveness of the former is due to their having, firstly, better vertical characteristics for his particular purpose, and secondly, greater radiation efficiency. Mr. Ashbridge expresses surprise with regard to the Pontoise station. I have no information regarding dimensions of the aerial. I agree that in general for long-distance transmissions high horizontal aerials give the best results, but it is pointed out in the paper that there is an optimum height. Mr. Ashbridge inquires what is the effect of the intervening earth upon the value of the received field. In my view, the effect is quite overshadowed by other considerations. It is well-known, for example, that east-to-west transmissions to America are not so consistent as north-to-south transmissions to South Africa, although in the former case the sea intervenes and in the latter the ground path is almost entirely earth.

Mr. Eckersley holds the view that unless there is one predominant ray, the method of raising aerials gives inaccurate results. It must be emphasized that the primary purpose of the tests was to obtain data for the design of commercial arrays and to ascertain to what extent height above ground was desirable. In showing that there is an optimum height dependent upon distance, conditions of the ionosphere, and other factors, the primary object of my investigations has been achieved. Regarding the question of optimum angle of propagation, I agree that the ideal condition for accuracy in the inferences is that one predominant ray shall be received. I hold the view, however, that the method of raising aerials by successive stages, and plotting the resultant measured field against the calculated curve, as described in the paper, can yield reasonably accurate results. The method enables a continuous curve to be plotted and so reduces the chance of faulty deduction due to the inaccuracy of individual observations. In effect, the method is equivalent to testing with a very large number of aerials having different vertical radiation characteristics. Thus if the received energy is due to several rays entering the transmitting aerial at different elevations separated by wide angular difference, there will be an averaging effect, since the ratio of the strength of individual signals will vary widely between the lowest and the highest position of the aerial. As already explained in the paper, however, when transmitting aerials having very different vertical radiation characteristics were compared by alternately energizing them with pulses, no great difference in the received pattern could be appreciated. The inference can therefore be made that since during pulse tests this wide variation was not observed as a concomitant of the type of transmitting aerial, the greatest portion of the energy that reached the receiving station was emitted within the confines of a very small angle at the transmitting aerial. Thus Mr. Eckersley's apparent assumption, that when four or five rays are received these have their counterpart at the transmission aerial emitted at elevations having considerable angular differences, does not appear to be justified by the evidence. Mr. Eckersley quotes the results of tests by Friis and his colleagues on the downcoming angles of individual rays during pulse tests. Four rays separated by 16·5° between the lowest and the highest were observed during one of the tests. These results do not give a complete picture, since frequently the angular separation between individual pulses is only 1° or 2°. Recent tests made at Slough of American pulse emissions confirm the fact that individual received rays of com-

* "Beam Arrays and Transmission Lines," *Journal I.E.E.*, 1931, vol. 69, p. 299.

† Gothe: "Wire Reflectors," Telefunken Laboratory Report, October, 1928; W. W. Tatarinoff: *Zeitschrift für Hochfrequenztechnik*, 1926, vol. 28, p. 117.

‡ A. A. Pistolkors: "The Radiation Resistance of Beam Antennas," *Proceedings of the Institute of Radio Engineers*, 1929, vol. 17, p. 562; C. R. Englund and A. B. Crawford: "The Mutual Impedance Between Adjacent Antennas," *ibid.*, 1929, vol. 17, p. 1277.

§ R. M. Wilmotte and J. S. McPetrie: "A Theoretical Investigation of the Phase Relations in Beam Systems," *Journal I.E.E.*, 1928, vol. 66, p. 949; L. S. Palmer and L. L. K. Honeyball: "The Action of a Reflecting Antenna," *ibid.*, 1929, vol. 67, p. 1045.

‖ K. Tani: "Theory of the Complex Antenna," Report of Radio Research in Japan, 1933, vol. 3, p. 19.

parable magnitude have usually a very small angular separation. It is interesting to note that the Rugby aerial, the emissions of which Friss was observing, has a main vertical lobe of radiation 16·5° wide measured about 7 decibels down from the maximum value, and that its axis makes an angle of 76° to the vertical. The other lobes are of small amplitude. As regards transmission lines, Mr. Eckersley refers to my statement that the radiation losses from each element of line are not the same, the radiation being less in the middle and greater at the ends. This statement is dealt with elsewhere* and does not form part of the arguments put forward in the paper.

I agree with Mr. Herd that our knowledge of the mechanism of long-distance propagation is inadequate, and I am of the opinion that many of the present ideas on the subject will be modified as the result of further investigations. The observations which are at present being conducted at Slough should throw additional light on propagation problems. Mr. Herd is wise in deferring a comprehensive report upon this subject until more work has been done.

Mr. Kirke raises the question of using other spacings and phases in radiators. Regarding the spacing, the $\frac{1}{2}$-wavelength is a convenient value for feeding purposes, but some tests have been made with other spacings. The results are such as would theoretically be expected and in general give no practical advantage. Tests have also been made by reversing the phase of radiators in arrays. These tests disclosed nothing which could not have been anticipated on theoretical grounds. Mr. Kirke draws attention to the fact that according to Table 5 the deduced angle of projection was highest when the 2-tier aerial was $1\frac{1}{4}$ wavelengths above ground and lowest when the aerial was $\frac{1}{4}$ wavelength high. My view on this matter is that the measurements made when the aerial was $1\frac{1}{4}$ wavelengths high give the more reliable results. The reasons for this view will be understood from an examination of Fig. 9. For the higher angles of projection, e.g. between 74° and 84°, the slope of the curve when $h = \frac{5}{4}\lambda$ is much steeper than for either the $h = \frac{3}{4}\lambda$ or the $h = \frac{1}{4}\lambda$ curves. Thus, for a given inaccuracy in the value of the received field, the error in the deduced angle would be least when the aerial was $1\frac{1}{4}$ wavelengths above earth. When tests were made with two of the same type of 2-tier aerials arranged so that one was fixed at 1 wavelength above ground and the other was raised, no regular increase in the angle as the aerials were raised higher was observed. Examination of Fig. 18 will reveal the fact that the angle was the same when the aerial was 1 wavelength high as when it was $3\frac{1}{4}$ wavelengths above ground. Mr. Kirke asks my opinion concerning the difference in results when vertical and horizontal aerials are used. In general the vertical aerial has less radiation efficiency than the horizontal, but the radiation characteristics in the vertical plane will usually be the predominant factor in long-distance transmission. The kind of polarization does not appear to have a marked influence.

Mr. McPetrie kindly pointed out to me, before the paper was printed, that the signs for $K$ and $d_2$ (page 545) should be negative. The modification he now proposes would remove the confusion of having $d_1$ and $d_2$ opposite in sign. He rightly points out that large angles of incidence are dealt with throughout the paper and asks whether there would have been much difference in the results had the earth been assumed to be a perfect reflector. The reply is, of course, well known to the questioner—there would have been no appreciable difference. But might not the angles of incidence have been small? In this case the earth constants, as Mr. McPetrie admits, would have affected the deductions.

Mr. Dewhurst's enthusiasm for a certain type of array seems to have led him into error regarding the deductions to be made from Fig. 41. An array for 20 metres is not mentioned; interpolation between the 24 m and 16 m curves shows that the cost would work out at about £300 for a gain of 17·5 decibels above a standard half-wave aerial. For a 12-decibel gain array on 20 metres, wooden poles would be used, and the cost would be a small fraction of that given by Mr. Dewhurst. When the required gain of an array for 20-metre working increases from values of the order of 12 decibels to values around 17 decibels, higher structures are required and poles can no longer be used. There is thus a sharp increase in the price at this stage. Mr. Dewhurst appears to have great faith in the beneficial effect of raising the tails of his aerial. I suggest that something more than looking at the array would be required to reach reliable conclusions.

Mr. Gill objects to the word "sensitive" as applied to the field-strength measuring set. The word must be taken in conjunction with the type and purpose of the apparatus to which it is applied. It cannot compare, of course, with the much larger and heavier sets used for measurement of very small fields, but in relation to the sets used previously for plotting polar diagrams of the field strength several wavelengths from arrays it can fairly be called sensitive. The term "height" as applied to the ionized layer is apt to give wrong impressions. I have used it in the geometric sense as the equivalent height of a sharply reflecting surface. This simple picture is not actually realized in practice. The degree of penetration of a ray as a function of the angle of incidence enters into the question raised by Mr. Gill, but at a distance of 900 miles approximately west of Slough the equivalent height would be expected to be about the same as above Slough.

Mr. Turner dwells upon the great changes that have taken place in radio practice. I well remember the late nights he spent at Abu Zabal, trying to elucidate reception phenomena, and I have a lasting recollection of building him in the desert an aerial 3 miles long which had to be patrolled to prevent the depredations of marauders!

Mr. Strong states that the outstanding feature of the paper is the comparatively small changes in field strength obtained. This is not entirely correct. In the German tests (Fig. 24), for example, the field increased by 12 decibels when the aerial was raised from $\frac{1}{4}$ wavelength to $1\frac{1}{2}$ wavelengths above earth.

With regard to the possibility of using a sharp beam to reduce fading, one of the objects of the tests was to investigate this possibility. A sharp beam might help, and there is some evidence of this when such beams are

* T. WALMSLEY: *Philosophical Magazine*, 1931, vol. 12, p. 392.

used for reception. On the transmission side, for long-distance services it is not at all certain that a sharp beam in the vertical plane reduces the ratio of the strength of the received multiple rays, but more investigation of this matter is required.

I am interested to learn that Mr. Franklin's experience regarding the angle of projection of beams lends support to my own conclusions. Mr. Franklin quite correctly appreciates the necessity of a good balance on open-wire lines for reduction of the pick-up. The out-of-balance depends upon the type of array used and the insulation of the lines and aerial. The question is chiefly one of economics—whether it is preferable to re-line up occasionally or to install the more expensive concentric-tube lines. Regarding the pick-up of an open-wire line, a balanced 4-wire line in square formation, having the diagonal wires cross-connected, is superior to a 2-wire line having the same order of spacing between the wires.

Prof. Hollingworth raises the question of the cause of the variability of the angles shown in test 5, Table 5, and asks whether this variation is to be regarded as instrumental or as propagational. The answer is—both. During these particular tests on the 13th and 14th August, 1932, the phenomena of tilting of the ionosphere, referred to on page 562, was experienced. Each day, when the tests started at noon, the angle of incidence at the American side of the radio channel was about 85°. This angle decreased towards the end of the tests to about 79°. The difficulty of obtaining an exact value of the incident field at the higher angle, using an open aerial, will probably account for the fact that the estimated transmitting angles on the 14th August, 1933, showed much greater variation during the first half of the test period than during the second half. Other remarks upon this matter have been made in answer to Mr. Kirke. Prof. Hollingworth's ideas regarding the methods of propagation of waves serve to confirm my views, formed as a result of many pulse tests, that a simple explanation fitting all the facts has not yet been evolved.

Prof. Palmer doubts whether the type of array chosen is the most economical; I selected this particular type, after experience of many others, because it combined simplicity and ease of maintenance with relatively low capital cost. There are occasions when other types are preferable, but for ordinary practical purposes it is difficult to improve upon the type chosen. I fail to understand Prof. Palmer's arguments regarding the efficiency of the tiers of dipoles. If the ordinary use of the term is intended, namely the ratio of energy radiated to energy supplied, it is not clear how the reinforcement of the direct e.m.f. by the indirect will improve matters. The efficiency of the horizontal type of array (using the word in the above sense) is probably well over 90 per cent and can hardly be increased. If, however, Prof. Palmer is thinking of efficiency in the sense of the greatest gain in the direction of maximum radiation for a given input, I agree that half-wave spacing may not give the best results. With a commercial array, however, the advantages of half-wave spacings are considerable and the extra gain made possible by any other spacing is very small. I have proved this fact to my own satisfaction, both theoretically and practically. Regarding the position of the reflecting curtain, personal investigations have satisfied me that there is little to be gained by increasing the distance from the directly energized curtain more than $\frac{1}{4}$ wavelength. The words "about" and "approximately" were used because the distance is not critical.

# MEASUREMENT OF THE ANGLE OF INCIDENCE AT THE GROUND OF DOWNCOMING SHORT WAVES FROM THE IONOSPHERE.*

By A. F. WILKINS, M.Sc., Graduate.

[From the National Physical Laboratory.]

(*Paper first received* 19*th January, and in final form* 2*nd March*, 1934.)

SUMMARY.

The paper describes a method of measuring the angle of incidence of downcoming short waves in which the phase difference between the e.m.f.'s in two similar horizontal aerials at the same height above the ground is determined from the trace on the fluorescent screen of a cathode-ray oscillograph, to the deflecting plates of which are applied the two aerial e.m.f.'s after similar amplification by receivers of the type developed at the Radio Research Station for cathode-ray direction-finding.

As the main object in view is the measurement of the downcoming angles of 20-metre waves from Lawrenceville, New York, working on the radio-telephone circuit to London, the aerial system used is designed for most efficient operation on this wavelength: but it is found quite practicable to use it on wavelengths up to 64 metres and also for signals from directions not widely divergent from that of the straight line at right angles to the aerials.

The results obtained show that, over the period January–April, 1933, one main ray accompanied by other and smaller-amplitude rays is, in general, received at Slough from the 20-metre Lawrenceville stations during their normal working period. The average angle of incidence of this main ray is 72° (measured to the normal to the ground).

Throughout the first four months of the year, the angle of incidence remained fairly constant over the working period, but, from about the beginning of April, 1933, the angle of incidence of the one main ray which was still present began to grow throughout the day. At the commencement of transmission, at noon G.M.T., the angle of incidence is of the same order as that obtaining throughout the day in the "winter" months. The angle increases gradually until values of 80° to 85° are obtained towards sunset. A drop in average field strength of the transmissions has also been noted since April.

The deduction that one main ray accompanied by smaller-amplitude rays is generally present in the downcoming radiation from Lawrenceville has been confirmed by the preliminary results of some short-duration 20-metre pulse transmissions from that station.

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## (1) INTRODUCTION.

It is now customary in short-wave point-to-point radio communication to employ special antenna arrays both at the transmitter and at the receiver. Their use at the transmitter enables the required field strength to be produced at the receiver with the expenditure of less power, while their use at the receiver, by increasing the voltage applied to the grid of the first valve of the receiver, enables the necessary output to be obtained with less amplification and consequently better signal/noise ratio. For the most efficient operation, arrays for transmission should be oriented so that the radiation is a maximum along the great-circle path joining the two points between which communication is to be effected; while, in the case of receiving arrays, signal pick-up should be greatest in this same direction. The transmitting array should also be arranged so that the angle of elevation of the axis of the radiated beam is the most favourable for reception at the distant point. For a receiving array the signal pick-up should be a maximum at the angle of incidence of the downcoming waves. It was to facilitate the design of the most efficient array for the reception of the 20-metre telephone signals from Lawrenceville, New York, that the apparatus described in this paper was erected.

Previous measurements of the downcoming angles of short-wave signals have been made by Friis,† Eckersley,‡ and Hollingworth.§ For measuring the angle in the case of 16-metre signals from Rugby, as received in the United States, Friis used two short vertical aerials spaced one-third of a wavelength apart in the great-circle direction of the transmitter, each aerial being connected to a separate receiver. A third aerial, connected to a local source for beating with the incoming signal, was arranged on a straight line passing through the two aerials. After rectification and amplification, the beat-frequency e.m.f.'s from the two receivers, whose phase difference was simply related to the angle of incidence, were applied to the plates of a cathode-ray oscillograph. On account of the small spacing between aerials and the way in which the beating signal was fed into the latter, the phase difference between the e.m.f.'s applied to the oscillograph was small for large angles of incidence, and the resulting cathode-ray trace was a thin ellipse. The method was not very sensitive to change in angle of incidence when this angle was large.

The method used by Hollingworth was a development of that used by Appleton and Barnett‖ and by Smith-Rose and Barfield,¶ and consisted in recording simul-

† See Reference (1). ‡ *Ibid.*, (2). § *Ibid.*, (3).
‖ *Ibid.*, (4). ¶ *Ibid.*, (5).

taneously the variations of received signal strength on a vertical aerial and a loop aerial with its plane in the plane of propagation. It can be shown* that the ratio of the e.m.f. in the vertical aerial to that in the loop is proportional to the sine of the angle of incidence of the downcoming wave. This method of measurement is thus not very sensitive for large angles of incidence, of the order of those to be expected in long-distance transmission.

In another scheme developed by Hollingworth† the e.m.f.'s from the loop aerial and the vertical aerial were separately amplified and then rectified. The vertical-aerial e.m.f. is $\sin\theta$ times the loop e.m.f. ($\theta$ is the angle of incidence of the downcoming ray), so that if the loop e.m.f. is reduced to $\sin\theta$ times the e.m.f. of the vertical aerial, and if, moreover, the rectifiers have identical characteristics, then the rectified currents are equal in amplitude and may be connected in opposition to a galvanometer with no resulting deflection. If the ratio of the amplifier gains is known, the angle of incidence can be determined. This method, while sound in the case of one downcoming ray, breaks down in the presence of several rays as the effect of any one of them cannot be completely removed, and interpretation of the records therefore becomes difficult.

Another development of the loop-aerial and vertical-aerial method was also used for downcoming-angle measurements by Eckersley.‡

The present method is similar to that of Friis, but is more sensitive to changes of angle in the case of large angles of incidence. It consists essentially in measuring the phase difference between the e.m.f.'s produced in two similar and parallel horizontal aerials arranged with their centres preferably along the great circle joining the transmitter and receiver and their axes at right angles to this direction. The method of making this measurement will be described later.

## (2) Theoretical Considerations.

The principle of the method is illustrated in Fig. 1, where A and B are parallel horizontal aerials under the

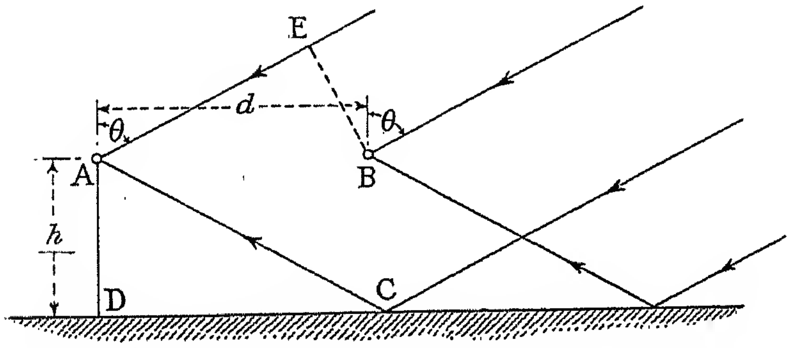


Fig. 1.

influence of a wave incident at angle $\theta$ in the direction E A, which is perpendicular to the aerials. The e.m.f. produced in each aerial is the vector sum of the e.m.f. produced by the direct wave (E A in the case of aerial A) and the e.m.f. produced by the wave reflected from the ground (e.g. C A). No matter what the effect of the ground on the resultant e.m.f. in the aerials may be, it is clear that in a homogeneous wave-field this effect will be similar for both, and the resultant e.m.f.'s will be of equal amplitude but will differ in phase by an angle $\phi = (2\pi/\lambda)\mathrm{AE} = (2\pi d/\lambda)\sin\theta$, where $d$ is the spacing and $\lambda$ is the wavelength.

If the direction of the transmitting station makes an angle $\psi$ with the line joining the centres of the two aerials, the phase difference between the e.m.f.'s becomes $(2\pi d/\lambda)\sin\theta\cos\psi$. For values of nearly 90° it is evident that large errors in the deduced value of $\theta$ will be caused by small errors in the measurement of $\psi$, and such an aerial system should not, therefore, be used in circumstances of wide divergence from the "broadside" condition.

Apart from $\psi$ considerations, the criterion of sensitivity of the method is the rate of change of phase difference between the aerial e.m.f.'s with angle of

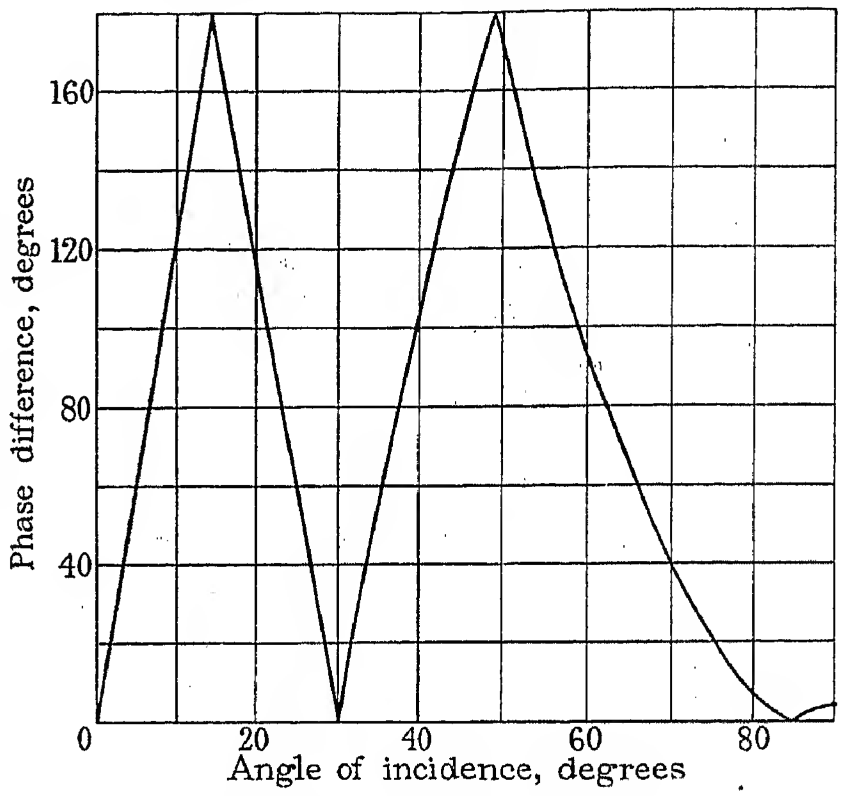


Fig. 2.—Lawrenceville (WMF), wavelength 20·73 m.

incidence, i.e. $d\phi/d\theta$, which is equal to $(2\pi d/\lambda)\cos\theta$. For large angles of incidence the sensitivity can be raised by increasing the aerial spacing $d$, and, in the actual aerial system used, $d\phi/d\theta$ is about 4° per degree of angle of incidence in the region $\theta = 70°$ on 20 metres with a spacing of about 2 wavelengths (see Fig. 2).

The sensitivity cannot, however, be increased indefinitely by increase of spacing, as a limit is set by non-homogeneity of field round the aerial system (diversity effects). Although, on account of the number of variables to be considered, it is not easy to deduce from the behaviour of the apparatus to be described that a non-homogeneous field is of frequent occurrence, it is felt that 2 wavelengths is about the maximum safe spacing to adopt.

## (3) Experimental Arrangements.

### (a) *Aerial System.*

As the measurements were to be performed in the first instance on wavelengths of some 20 metres, the system was designed for maximum efficiency on this wavelength.

* See Reference (5). † *Ibid.*, (3). ‡ *Ibid.*, (2).

The height of each aerial above ground was fixed at $\frac{1}{2}$ wavelength, a convenient height from the erection point of view, and such also as to ensure substantial "pick-up" from waves of large angle of incidence. The distance apart is 2 wavelengths, a value chosen so as to give a reasonable sensitivity to small changes in angle of

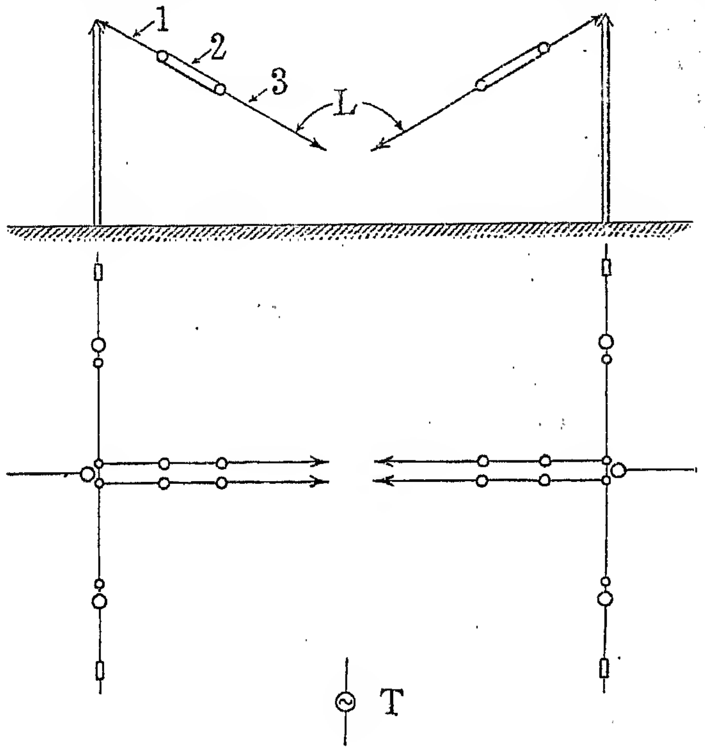


FIG. 3.

incidence and also to minimize the mutual interaction of the aerials. The calculated relationship of phase difference between the aerial e.m.f.'s and angle of incidence $\theta$ in the case of Lawrenceville WMF (20·73 metres) for the actual spacing of the aerials (41·6 metres) is given in Fig. 2.

The lay-out of the aerial system is shown in Fig. 3. The aerials are 1 wavelength (20 metres) long, their impedances measured at the position of connection to the transmission lines being $6\,000\underline{/0°}$ ohms approximately. Quarter-wave impedance-matching lines, sections 1 and 2 in Fig. 3, are used for matching the aerial impedance to that of the main run (section 3) of transmission line, whose characteristic impedance is $600\underline{/0°}$ ohms.

If $Z_s$ and $Z_r$ are the sending- and receiving-end impedances respectively of a transmission line $\frac{1}{4}$ wavelength long whose characteristic impedance is $Z_0$, then it is well known that $Z_0^2 = Z_s Z_r$. Connecting a $\frac{1}{4}$-wavelength of line (section 1, Fig. 3) of 600 ohms characteristic impedance to the aerials causes the impedance as measured at the junction of sections 1 and 2 to become $600^2/6\,000 = 60\underline{/0°}$ ohms. The matching of this 60-ohm portion of the system to the 600-ohm lines (section 3) is then carried out by another quarter-wave line (section 2) whose characteristic impedance must be $\sqrt{(60 \times 600)} = 190$ ohms; actually each conductor of the line consists of two lengths of 0·08 in. diameter copper wire in parallel, as shown in Fig. 4. The ends of the main transmission line are brought into a hut situated at the

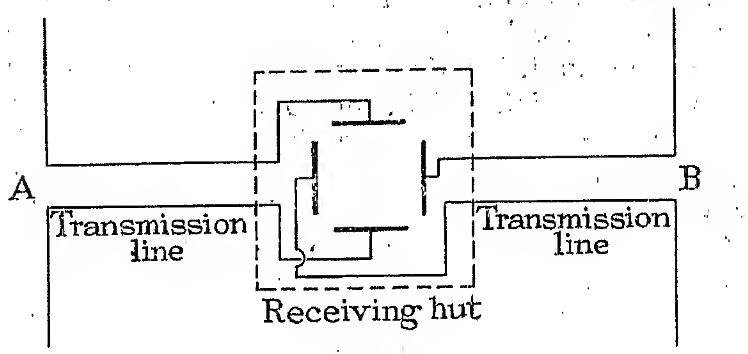


FIG. 5.

centre of the aerial system and are there connected to the receiving apparatus.

(b) *Receiving Equipment.*

From the foregoing outline it will be apparent that the essential principle of the method is the determination of the phase difference between the e.m.f.'s in aerials A and B of Fig. 1. This determination is effected by means of the cathode-ray oscillograph using the method illustrated in its simplest form in Fig. 5. The e.m.f.'s applied to the plates of the oscillograph will be of equal amplitude but will have a phase difference $(2\pi d/\lambda) \sin \theta$, and will therefore give rise to a trace on the fluorescent screen of the oscillograph which will be an ellipse with major axis lying along a line making an angle of 45° with the axes of the oscillograph. The phase difference is then the angle subtended by the minor axis at the end of the major axis of this ellipse.*

Considerable amplification has to be applied to the aerial e.m.f.'s before a suitable deflection of the electron beam can be obtained, and for this purpose amplifiers were used of a type developed at the Radio Research

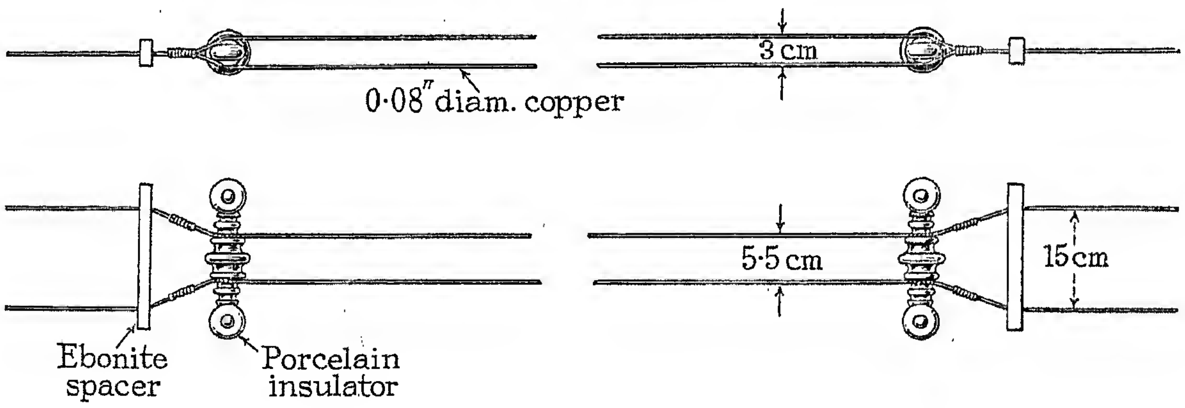


FIG. 4.

* See Reference (6).

Station in connection with the use of the cathode-ray oscillograph as a radio direction-finder. The principle of the amplifying scheme may be understood by reference to the schematic diagram of Fig. 6.*

The e.m.f.'s at the receiving ends of the transmission lines are similarly amplified by two stages of radio-frequency amplification before being applied to anode-bend detectors. The grids of these detector valves are supplied with heterodyne e.m.f.'s of equal amplitude and phase through symmetrical connections from a common oscillator. From well-known principles, if the signal-frequency e.m.f.'s have a certain phase difference, then the beat-frequency components of current in the anode circuits of the two detectors will have this same phase difference and their amplitude ratio will be the same as that of the original radio-frequency e.m.f.'s. The beat-frequency e.m.f.'s from the detectors are then amplified some 90 decibels before application to the deflecting plates of the cathode-ray oscillograph. If the gain and the phase rotation are the same in each amplifier, then the pattern obtained on the oscillograph screen will

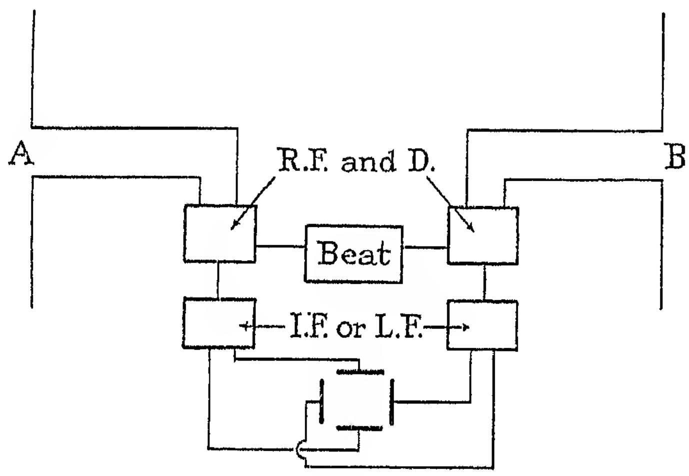


FIG. 6.

give a true measure of the phase difference between the aerial e.m.f.'s.

Originally a beat frequency of 2·5 kilocycles per sec. was employed and 3-stage amplifiers working at this frequency preceded the oscillograph. The band width of the amplifiers, measured 6 decibels down on the response curve, was 250 cycles per sec., and this fact, in addition to frequency drift of the beating oscillator, made it impossible to maintain the apparatus in correct adjustment for long periods. The narrow band-width also made it difficult to obtain results on the Lawrenceville transmitters when these were working with "warbling" carrier, i.e. with the carrier frequency varying rhythmically some 200 cycles per sec. above and below the nominal frequency.

The present amplifiers† consist of three stages using screened-grid valves and working at 100 kilocycles per sec., the amplified potentials then being passed on to the oscillograph through neutralized-triode stages capable of handling large potential-swings without overloading. These amplifiers give an overall gain of some 90 decibels and have a band width of 10 kilocycles. Their use has enabled the apparatus to be operated for periods of hours without requiring frequent adjustment; it has also enabled reliable results to be obtained on "warbling" signals, as the change of amplitude of the oscillograph trace with change of carrier frequency is now quite small.

Provided that the input e.m.f. from an external signal is the same to each receiver and that the two receivers behave in exactly the same manner as regards phase and amplification, the resulting trace on the oscillograph screen should be a straight line inclined at an angle of 45° to the oscillograph axes. In adjusting the receivers to obtain this condition the anodes of corresponding valves in each receiver are connected together. If, then, a 45° straight line is not obtained on the screen, it shows that the sections of the receiver between the anodes and the oscillograph are not matched, and adjustments must accordingly be made. This process is carried out throughout the receivers, and finally the transmission-line ends are connected together. If a 45° straight line is obtained, the apparatus is correctly adjusted and may be used for determining the downcoming angle of the signal by removing the common connection on the transmission lines, provided only that the aerial and transmission-line systems are similar. Separate tests were made to establish this fact.

(c) *Tests on Aerial and Transmission-Line System.*

(i) A measurement was made to check the equality of pick-up on the system due to electric force parallel to the aerials, and also of the attenuation due to the lines. This was carried out by receiving the horizontally polarized wave produced by a low-power horizontal-aerial transmitter set up directly under the centre of each aerial in turn, the vertical distance between transmitting and receiving aerials being the same in each case. The transmitting aerial was set parallel to the receiving aerial and about 2 yards above the surface of the ground, so as to reduce the cancelling effect on the field of the transmitting-aerial image in the ground. On connecting the aerials in turn to one of the receivers it was verified that the deflections of the cathode-ray spot were the same in both cases.

(ii) A vertical-aerial transmitter was set up at a position on the line joining the centres of the two receiving aerials and at a distance of about 100 yards from the aerials. Under such conditions there should be no e.m.f. produced in either receiving aerial. Actually it was found that, on carrying the transmitter in a circle round one of the receiving aerials, there was a sharp drop of pick-up with the transmitter in the broadside-on position to the aerial. The pick-up was then about 5 per cent of that observed in the end-on position, and much less than that obtained from the same transmitter with horizontal aerial set-up at the broadside-on position. This spurious pick-up may be attributed to pick-up on the transmission lines and direct pick-up on the receivers.

The horizontal-aerial transmitter was placed at a point T (Fig. 3) along the line of geometrical symmetry of the system. With the receivers correctly adjusted as already described, it was found that the resulting trace on the oscillograph was a thin ellipse whose major axis was inclined at 45° to the oscillograph axes, showing that the aerial e.m.f.'s were of equal amplitude but had about 4° phase displacement. This phase error in the system

* For a fuller description of the principle of the present scheme see Reference (7).

† For a full description of these amplifiers, and also the narrow-band low-frequency type, see Reference (7).

will give rise to about 1° of error in the deduced angle of incidence in the region over which the system is used.

### (4) Experimental Results.

Systematic observations of the three Lawrenceville telephone stations working in the 20-metre region (WMN, 20·59 metres; WMF, 20·73 metres; WMA, 22·4 metres) and also of the Montreal telephone station (CGA, 22·58 metres) were made over the period January–June, 1933.

For recording, the ellipse on the oscillograph screen was photographed on bromide paper (H and D 250) by means of a simple form of camera, satisfactory pictures being obtained with an accelerating voltage of 1 000 volts on the anode of the oscillograph. Fig. 7(*a*) shows such observations made under average reception conditions on Lawrenceville (WMF), while Fig. 7(*b*) shows records obtained under similar conditions on Montreal (CGA).

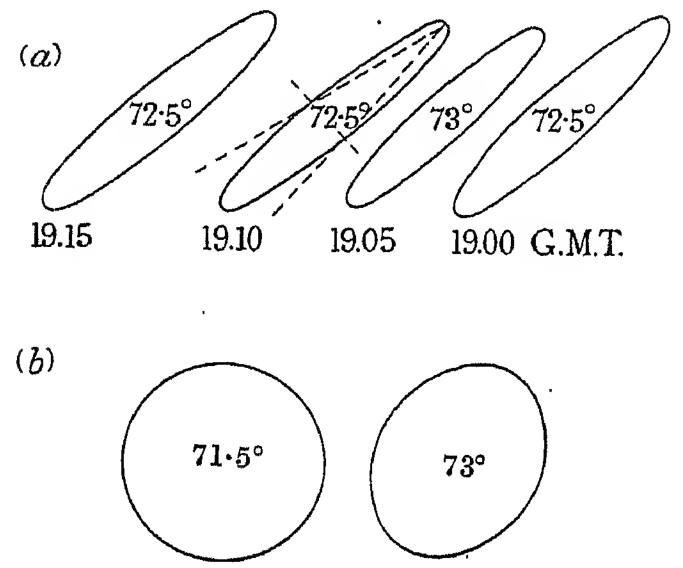


Fig. 7.
(*a*) Lawrenceville (WMF).
(*b*) Montreal (CGA).

The major axes of these ellipses all lie approximately along a line making an angle of 45° with the oscillograph axes; the phase difference between the e.m.f.'s applied to the deflecting plates of the oscillograph, and therefore the phase difference between the aerial e.m.f.'s, is given, as has been stated, by the angle subtended by the minor axis at the end of the major axis, as shown in Fig. 7. For WMF the mean phase angle is 22°, corresponding to possible angles of incidence of 75°, 32°, 28°, or 2°; and for CGA it is 83·5°, corresponding to angles of incidence of 73°, 42°, 25°, and 70°.* For the case of WMF, and calling the aerial nearer the transmitter the west aerial, it may be said that the e.m.f. in this leads that in the east aerial by $(360\, d/\lambda) \sin\theta$ degrees. Now if the ray were arriving at an angle of incidence of either 75° or 28°, the phase difference would be either 698° or 338°, i.e. the e.m.f. in the west aerial would lag on that in the east by 22°. For angles of incidence of 32° or 2° the e.m.f. in the west aerial would lead that in the east by 22°. The phase relationship of the e.m.f.'s applied to the oscillograph plates is determined by observing the direction of rotation of the cathode-ray spot, either stroboscopically or by noting the variation of eccentricity of the ellipse, as the capacitance of one of the tuning condensers in the radio-frequency amplifiers is altered in a known manner. For WMF it is found that the west aerial e.m.f. lags in phase on that of the east, thus suggesting angles of incidence of 75° or 28°.

Although there seemed to be little doubt that the larger angle was the actual value obtaining, a final decision was supplied by a test with the loop-aerial and vertical-aerial method described briefly in Section (1). Apparatus for making an aural comparison of the e.m.f.'s produced in a loop aerial and in a vertical aerial (by rapidly switching each aerial in turn on to a receiver) was set up at a position well clear of any disturbing effect likely to be produced by the aerial system described in the paper. After the e.m.f.'s produced in both aerials by a locally-generated 20-metre ground wave had been equalized, it was found that the Lawrenceville signals were roughly of equal amplitude on both aerials, thus suggesting a large angle of incidence. If 28° had been the correct angle of incidence, the loop e.m.f. would have been about twice as great as the vertical-aerial e.m.f., and a marked change of signal intensity would have been noted on switching over from one aerial to the other.

It may be noted that Eckersley* found that the angle of incidence of American stations received in England rarely fell below 70°.

From what is known about the reversibility of the ray-track in long-distance propagation,† it seems likely that the angle of incidence of the received ray is approximately the same as the angle of elevation (measured to the normal to the ground) of the main beam from the transmitter. It is interesting, therefore, to note that Walmsley,‡ by varying the angle of radiation in the vertical plane of the main beam from a 20-metre transmitter at Rugby, found that the strongest signals were received in America for an angle of radiation of 80° to the vertical.

During the first 4 months (January–April) the behaviour of the cathode-ray traces was very similar from day to day. During magnetically quiet days and when the signal was at its maximum value during fading, the trace was an ellipse whose major axis rocked about 5° on either side of a line making an angle of 45° with the oscillograph axes. The fact that the mean angle of inclination of the major axis is 45° and that the eccentricity of the ellipses changes slightly suggests that, for these stations, there is one main ray incident on the aerial system, the rocking of the ellipse being probably due to the influence of other and much weaker rays of varying phase with respect to the main ray. The presence of these secondary rays is more readily noticeable when the main ray fades to a value comparable with that of the secondary rays; the ellipse then becomes more agitated and exhibits rapid changes of eccentricity.

On several occasions during the period January–April the average signal intensity of these American signals became very low and the behaviour of the ellipse was comparable to that observed on quiet days at the times when the main ray had faded to a low level. Under

* In calculating these angles of incidence for Montreal the difference between the direction of the transmitter and the direction of the line joining the centres of the aerials was neglected, as it produced only a small error in the deduced value of the angle of incidence.

* See Reference (2). † *Ibid.*, p. 624. ‡ See reference (8).

such conditions no angle-of-incidence measurements were possible. It was always found that this behaviour coincided with a magnetic storm.

Towards the end of April there was a marked change in the diurnal variation both of angle of incidence and of field strength of all the transmissions. Whereas throughout the "winter" months of January to April the angle of incidence remained fairly constant over the transmitting period, as is shown in the typical case of Fig. 8, from the end of April there was a definite increase in angle during the day. Examples of this for one of the Lawrenceville stations and also for Montreal are shown in Fig. 9, where it will be seen that at the commencement of transmission in the morning (at a time when the mean density of ionization in the ionosphere over the path of propagation is low) the angle of incidence is nearly at the "winter" value. As the density increases, the waves will be returned from a lower level in the ionosphere and the angle of incidence will consequently

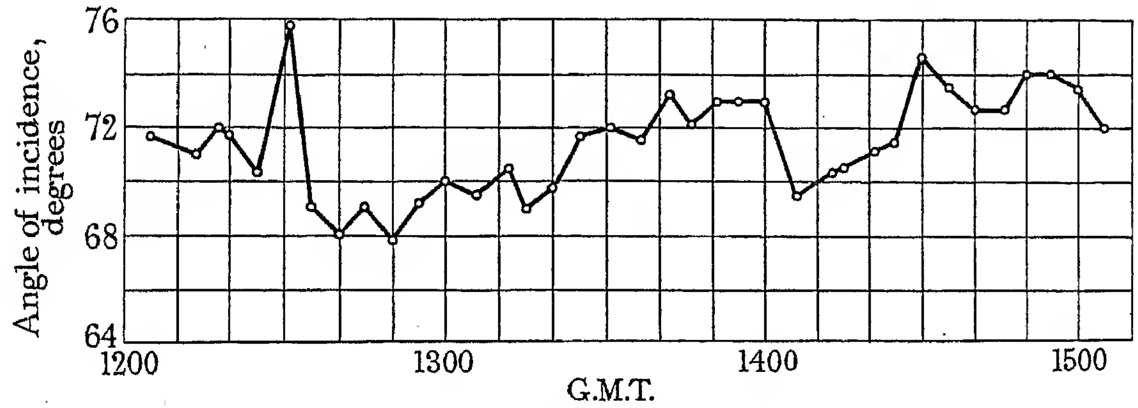


Fig. 8.—Lawrenceville (WMF), 10th January, 1933.

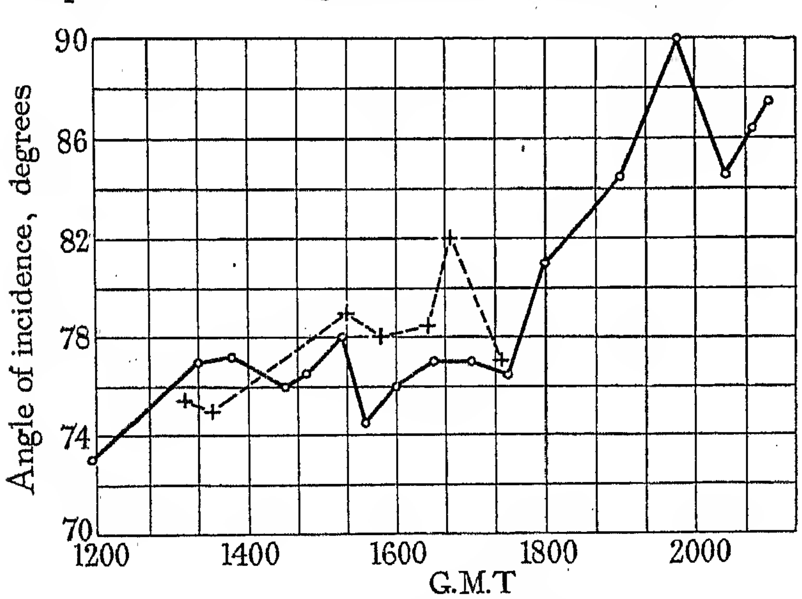


Fig. 9.—Results obtained 25th May, 1933.
—o—o— Montreal (CGA), 22·58 m.
- - + - - - + - - Lawrenceville (WMN), 20·59 m.

become larger. This is probably the explanation of the rise of angle actually observed. The small variation in angle observed during winter (see Fig. 8) may be explained as being the result of the fact that the diurnal variation of density of ionization is smaller for winter than for summer.

In addition to this change in angle of incidence, there was a reduction of mean field strength. The average daily field strength for the Lawrenceville stations during January–April, 1933, was some 30 microvolts per metre,* and the variation from this value was not considerable except shortly after sunset at the receiving end and during magnetic storms, when reductions in intensity were noted. After the middle of April a fall in strength to a value round about 10 to 15 microvolts per metre was observed. No systematic variation of field strength was noticed; sometimes there was a rise of strength commencing roughly at ground sunset at Slough and closely followed by a sharp fall until signals became "uncommercial." At other times the strength varied in an erratic manner throughout the day. The fall in strength during "summer" months is no doubt due to increased absorption consequent on greater density of ionization in the E region of the ionosphere. It is most likely that the normal variation of field strength of these transmissions, in which the waves are probably reflected from the F region, is for the rising tendency to be exhibited during the evening at Slough in summer owing to the diminution of absorption in the E region as the ions start to recombine. The erratic variations observed on other occasions are probably due to some secondary effect.

The deduction (based on the prevalence of ellipses whose major axes are inclined at 45° to the oscillograph axes and whose eccentricity is constant) that there is one downcoming ray whose amplitude is great in comparison with other rays was confirmed by the preliminary results of some short-duration pulse transmissions from Lawrenceville on a wavelength of 20·73 metres. These transmissions, which consisted of periods of continuous-wave (C.W.) transmission followed by pulses, were received on the same aerial system and on one of the receivers already described, a circular time-base being used to obtain separation of the echoes in the case of the pulse transmission. Towards the end of one of the preliminary C.W. transmissions the cathode-ray trace was an ellipse whose major axis was very steady in inclination and eccentricity and whose size was varying relatively slowly (amplitude fading). At the commencement of pulses, immediately after making this observation, the echo group was found to consist of a number of small echoes at the beginning of the group, followed by an echo of about 10 times their amplitude, this echo being followed at the end of the group by one and some-

* This is the field strength deduced from measurements made on one of the aerials used for the angle-of-incidence measurement, and is therefore the horizontal component of electric force at right angles to the direction of propagation and at a height of 10 metres above ground-level.

*

times two echoes of amplitude small in comparison with the main echo but somewhat larger than those occurring at the beginning.

On another occasion the ellipse produced by the carrier wave was noted to be changing rapidly in eccentricity and was rotating considerably; it was apparent that there were two or more rays of comparable amplitude incident on the aerial system. This deduction was substantiated by the echo group obtained on the commencement of pulses when, as before, several small echoes were present at the beginning of the group and these were followed by two or three echoes of roughly equal amplitude.

When receiving the normal telephony transmissions from Lawrenceville, the presence of more than one ray of considerable strength renders angle-of-incidence measurements impossible, but on pulse transmissions it becomes possible to measure the angle of incidence of all rays of sufficient amplitude to give a measurable deflection on the oscillograph. Adjustment of the two receiving sets in the manner described will result in each echo appearing as an ellipse on the oscillograph screen, each ellipse having its major axis inclined at 45° to the axes of the oscillograph. The application of a time-base will then separate the echoes (which now appear as envelopes of ellipses, owing to the drawing-out effect of the time-base) and enable the angle of incidence of each to be measured. Experiments with this measurement as object are at present being carried out.*

* Confirmation of the angle-of-incidence values given here has recently been provided by T. Walmsley in a paper entitled "An Investigation into the Factors controlling the Economic Design of Beam Arrays" (see page 543), which was read before the Wireless Section of the Institution after the present paper had been written.

Results in good agreement have also been given by Friis, Feldman, and Sharpless (*Proceedings of the Institute of Radio Engineers*, 1934, vol. 22, p. 47).

(5) ACKNOWLEDGMENTS.

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The author's thanks are due to Mr. R. A. Watson Watt, Superintendent of the Radio Department of the National Physical Laboratory; to Messrs. J. F. Herd and L. H. Bainbridge-Bell, also of that Department, for suggestions and advice in the course of the work; and to Mr. E. C. Slow, for assistance in the work of observation and reduction of records. Thanks are also due to Mr. T. Walmsley, of the Engineer-in-Chief's Department of the Post Office, for his interest and help in the course of the work; and to that Department itself for the erection of the aerial system.

REFERENCES.

(1) H. T. FRIIS: *Proceedings of the Institute of Radio Engineers*, 1928, vol. 16, p. 658.
(2) T. L. ECKERSLEY: *Journal I.E.E.*, 1927, vol. 65, p. 607.
(3) J. HOLLINGWORTH: *ibid.*, 1933, vol. 72, p. 236.
(4) E. V. APPLETON and M. A. F. BARNETT: *Proceedings of the Royal Society*, A, 1925, vol. 109, p. 621.
(5) R. L. SMITH-ROSE and R. H. BARFIELD: *ibid.*, 1926, vol. 110, p. 588.
(6) J. A. FLEMING: *Journal I.E.E.*, 1925, vol. 63, p. 1045.
(7) R. A. WATSON WATT, J. F. HERD, and L. H. BAINBRIDGE-BELL: "The Cathode-Ray Oscillograph in Radio Research" (H.M. Stationery Office), p. 140.
(8) T. WALMSLEY: *Nature*, 1932, vol. 130, p. 814.

# THE MEASUREMENT OF THE GRID-ANODE CAPACITANCE OF SCREEN-GRID VALVES.*

By T. Iorwerth Jones, M.Sc.

[From the National Physical Laboratory.]

(*Paper first received* 13*th September*, 1933, *and in final form* 6*th January*, 1934.)

Summary.

Two methods of measurement of the grid-anode capacitance of screen-grid valves are described.

In the first the working value of this capacitance is deduced from measurements of the change in the input capacitance of the valve upon reducing the anode load from a known value to zero. It is necessary to know the amplification factor of the stage. A description of the bridge employed for its measurement is given in Appendix 1.

The second method measures the grid-anode admittance with the filament cold. The result is obtained in terms of the ratio of the readings of two voltmeters and the settings of a variable air condenser covering a range of capacitance over which it can be calibrated directly.

## Introduction.

The introduction of the screen-grid valve has considerably increased the stability of the radio-frequency amplifier and the level of possible amplification. This improvement is attributable to the minuteness of the coupling between the input and output circuits of stages provided with screen-grid valves, and is associated with the small value of the grid-anode capacitance in such valves. From time to time, methods† of measuring this small capacitance have been described, but generally they have required special apparatus, and in particular a micrometer condenser (for the calibration of which the investigator has relied upon computation). These limitations have been avoided in the methods to be described in this paper.

The two methods are complementary. The first provides the means of measuring the grid-anode capacitance under working conditions. It measures directly the input capacitance of a screen-grid valve stage with a suitable load in the anode circuit. This quantity is important in itself, since it represents the load thrown by the stage on the stage immediately preceding it and determines among other factors the stability of the amplifier; but for our present purpose the fact that it consists of the product of the corresponding value of the effective amplification of the stage and of the grid-anode capacitance is utilized to deduce the latter from it.

The second method is designed to measure the grid-anode capacitance with the filament cold. Virtually it measures the grid-anode admittance of valves; but, except in cases of low insulation or of exceptionally low grid-anode capacitance, the admittance is determined almost entirely by the capacitance. Although the method is limited in this way in its application to derive an exact value of the grid-anode capacitance, it may be turned to good account as a routine test for the detection of valves which are defective as regards radio-frequency insulation. This is regarded as an important point. Such restrictions do not arise in the first method, which may be trusted to give the working value of the capacitance, while the second gives the value of the electrostatic capacitance between the electrodes when the insulation is good, and discriminates between valves of similar design with good and bad insulation.

* The Papers Committee invite written communications, for consideration with a view to publication, on papers published in the *Journal* without being read at a meeting. Communications (except those from abroad) should reach the Secretary of the Institution not later than one month after publication of the paper to which they relate.

† See References (1), (2), and (3).

## First Method. Grid-Anode Capacitance Under Working Conditions.

A few years ago the method of deducing the grid-anode capacitance of screen-grid valves from measurements of input impedance was suggested by the late Dr. D. W. Dye. It was investigated in the first instance by his colleague, Mr. J. E. P. Vigoureux, and later by the present author. Brief accounts of the early developments were given in the Annual Report of the National Physical Laboratory for 1931. In the meantime the properties of screen-grid valves have been considerably modified, involving a progressive lowering of the capacitance values and the raising of the amplification factors. The method described in those reports presented certain difficulties, and a few minor modifications became necessary to cope with the changed conditions. The nature of the complications which arose will be made clear at a later stage in this paper.

### *Theory of Method.*

The method makes use of the deduction made by Miller,† Nichols,‡ and Hartshorn,§ that, owing to the effect of the capacitance ($C_{ag}$) between the control-grid and the anode, the input impedance of thermionic valves varies with the load in the anode circuit. The expression for the effective input capacitance $C_g$ given by Hartshorn is

$$C_g = C_{fg} + C_{ag} + \mu C_{ag} - \mu R_a C_{ag} \frac{(R + R_a) - X \tan \delta_{ag}}{(R + R_a)^2 + X^2} \quad . \quad (1)$$

where $R$ and $X$ are the two components of the external anode impedance, and $\delta_{ag}$ is the loss angle of the anode-grid capacitance.

† See Reference (4). ‡ *Ibid.*, (5). § *Ibid.*, (6).

When the load consists of a pure resistance, a tuned circuit, or a choke in resonance, having an effective resistance $R$, this reduces to the form

$$C_g = C_{fg} + C_{ag} + [\mu R/(R + R_a)]C_{ag} \quad . \quad . \quad (2)$$

the third term of which consists of the grid-anode

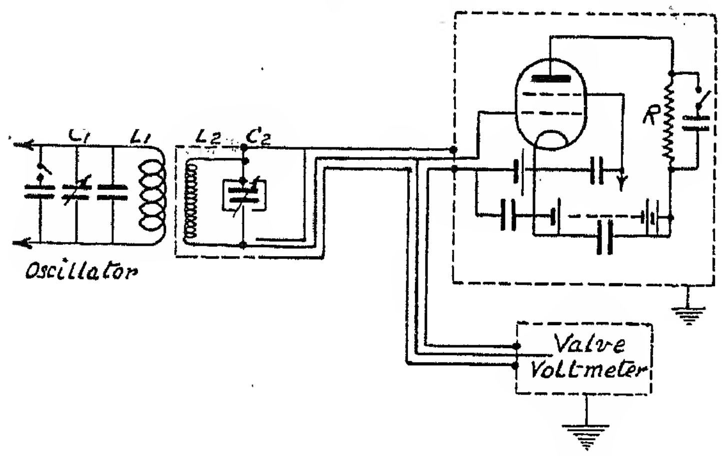


FIG. 1.—Circuit for input capacitance measurements.

capacitance multiplied by the effective amplification of the stage.

It follows, therefore, that if, as in the earlier experiments, a choke forms the load in the anode circuit, and if the frequency be varied in the region of resonance of the choke, the effective capacitance $C_g$ will follow a curve given by equation (1), and pass through a maximum value at the resonant frequency of the choke, given

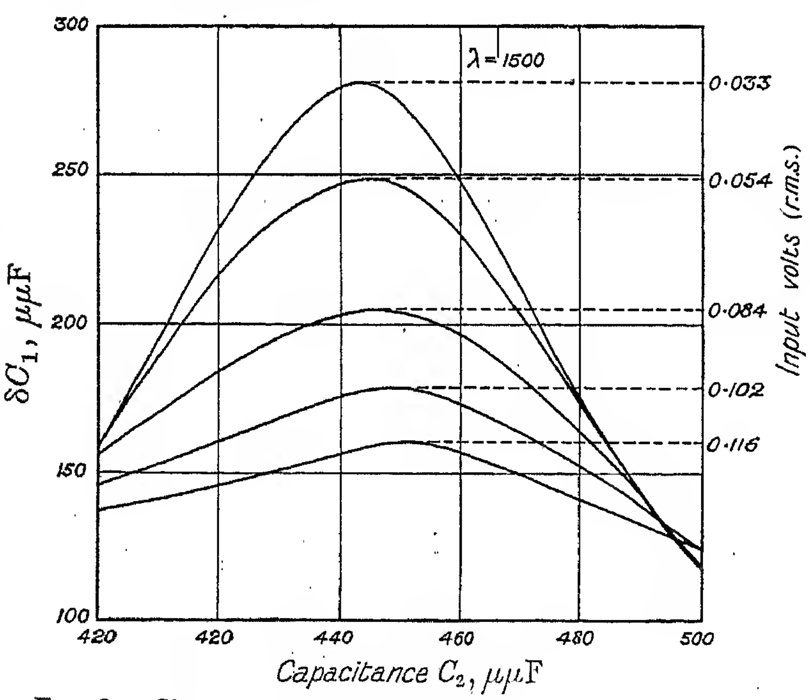


FIG. 2.—Change of capacitance with frequency for five input voltages.

by equation (2). If the choke is then short-circuited, the external resistance falls to a negligible value. The input capacitance falls as a result by an amount given by

$$\delta C_{max.} = [\mu R/(R + R_a)]C_{ag} \quad . \quad . \quad . \quad (3)$$

When it is borne in mind that modern screen-grid valves frequently possess voltage factors exceeding 1 000, the use of chokes or tuned circuits with low decrements implies a high overall amplification and consequently grave risks of overloading, even when only moderate input amplitudes are employed. Owing to the lower limit set to these amplitudes by the sensitivity of the resonance detectors, some overloading was inevitable. A direct result was the apparent dependence of the input capacitance and the amplification of the stage upon the amplitude of the input voltage. These effects are illustrated in Figs. 2 and 3. Steps had therefore to be taken to reduce the anode load in order to keep the output voltage within reasonable limits, while still retaining the main features of the method.

A fixed resistance of the vacuum type was therefore substituted for the choke, and the frequency was kept at a sufficiently low value to render the phase angle of

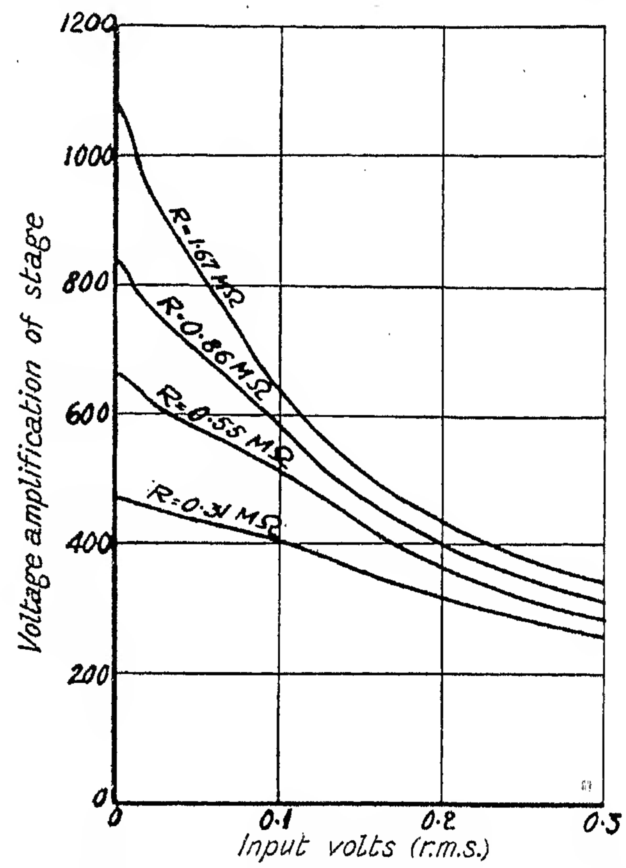


FIG. 3.—Voltage amplification of a tuned stage against input voltage.

$R$ = effective resistance of anode circuit.
$e_a$ = 200 volts.
$e_{sg}$ = 80 volts.
$e_g$ = − 1·5 volts.

the resistance of minor importance. Care was taken in mounting the resistance to keep the capacitance across its ends as low as possible.* It may therefore be taken to correspond with sufficient accuracy to the case given by equation (3), where the suffix "max." may be omitted. Thus

$$\delta C = [\mu R/(R + R_a)]C_{ag} \quad . \quad . \quad . \quad . \quad (4)$$

*Equipment and Procedure.*

The arrangement adopted for putting these principles into service is depicted in Fig. 1. Two tuned circuits are indicated; the first is formed of a coil of inductance $L_2$ and the total capacitance $C_2$ of the input circuit of the screen-grid valve. It is coupled to the second ($L_1$, $C_1$), which controls the frequency of the radio-frequency oscillator. The removal of the effective resistance $R$ from the anode circuit of the valve will alter

* See Appendix 2.

the effective input capacitance $C_2$ by an amount $\delta C_2$, given by equation (4). This may be done either by short-circuiting the resistance or by reducing the anode impedance to a negligible quantity by connecting a condenser of large capacitance across it. The method utilizes the second alternative, since it does not require readjustment of the anode-battery tapping. The two procedures were found to lead to the same result. In practice the factor $\mu R/(R + R_a)$ rarely exceeds 100, and this value should be borne in mind when choosing the anode resistance. In such a case an inter-electrode capacitance of $0 \cdot 002\,\mu\mu$F would give rise to a change ($\delta C_2$) of $0 \cdot 2\,\mu\mu$F in input capacitance, and would require an adjustment of that amount in $C_2$ if resonance was to be re-established upon short-circuiting the resistance. This cannot normally be observed on ordinary condensers to the necessary accuracy. By appropriate choice of the inductances $L_1$, $L_2$, however, capacitance adjustments of a larger order may be made by operating on the frequency of the oscillator. The necessary change $\delta C_1$ required in the oscillator capacitance is related to the actual change in input capacitance ($\delta C_2$) by the equation

$$\delta C_2 = (L_1/L_2)\delta C_1$$

Also

$$\delta C_1 = \frac{L_1}{L_2}\mu\frac{R}{R + R_a}C_{ag} \quad . \quad . \quad . \quad . \quad (5)$$

By making $L_1/L_2 = 100$, say, a further magnification of 100 is easily realized, and the observed change may amount to 20 $\mu\mu$F even in a difficult case.

The voltage amplification of the stage is determined on an amplification bridge at telephonic frequencies. The factors $L_1/L_2$ and $\mu R/(R + R_a)$ are then known, and $C_{ag}$ is immediately deducible from the observed change, $\delta C_1$.

For the capacitance measurements, thorough screening of the anode circuit of the valve is necessary. The valve is therefore mounted with the cap projecting into a metal box connected to the cathode, in which the resistance unit is centrally located. Condensers are provided across the battery sections connected to the grid, the screen-grid, and the anode.

The input circuit was connected across the grid-filament circuit of a valve voltmeter, which was effectively screened. The galvanometer associated with the voltmeter gave a full-scale deflection for an input voltage of $0 \cdot 15$ volt (r.m.s.). The circuit of the voltmeter calls for no special comment. Some difficulty was found in keeping the voltage input from the oscillator into the coil $L_2$ below the above amount, owing to the preponderance of capacitive coupling. The capacitive coupling between the coils $L_1$ and $L_2$ was eliminated by surrounding the coil $L_2$ by a wire cage and by employing concentric systems of leads with the outer casings connected to the screen box. These precautions also rendered the conditions independent of the movements of the observer.

The capacitance $C_2$ consisted entirely of the input capacitances of the two valves and the capacitance of the insulated lead to the outer casing of the concentric system, and did not exceed in the aggregate 100 $\mu\mu$F. An accuracy of $0 \cdot 0001\,\mu\mu$F in $C_{ag}$ therefore required an accuracy of 1 in 20 000 in the reproduction of the resonant frequencies of the input circuit and the setting of the oscillator capacitance to within 1 $\mu\mu$F. This order of precision was attained in the following way.

The tuning capacitance, $C_1$, of the oscillator was divided into three sections, a fixed portion and a variable portion (both always in circuit) and a fixed portion which could be connected or disconnected at will from a distance by means of a tapping key. The latter formed such a proportion of the total as brought the deflection of the voltmeter at its inclusion in, or its exclusion from, the circuit, down on to the steep sides of the resonance curve of the input circuit. In an ideal case, when the deflection of the voltmeter remains unchanged upon depressing the key, it may be assumed that the resonant frequency lies midway between the corresponding two frequencies of the oscillator, and the capacitance setting corresponding to it differs by a constant amount from the setting of the variable condenser which is always

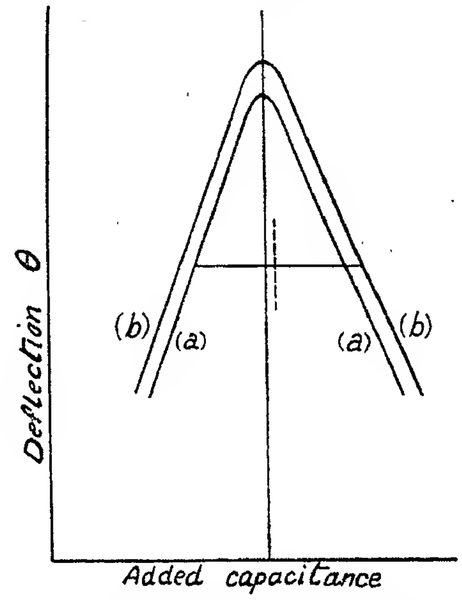


FIG. 4.—Effect of amplitude variation on resonance location.

in circuit. If this condenser is set to re-establish equality of departure from resonance in the above manner, first with the anode load equal to $R$ and then with the anode load practically zero, the change of reading will correspond to $\delta C_1$ in the formula.

This is only strictly true if certain conditions have been observed. It demands similarity in the shapes of the resonance curves corresponding to the two observations. This condition is easily satisfied by the resistance load, but a radio-frequency choke gave an unsymmetrical resonance curve when in circuit and a symmetrical one when short-circuited.

The main source of trouble arises out of the fact that the inductive coupling to the voltmeter circuit is already small, and the inclusion of an additional capacitance in the oscillator circuit is liable to increase that coupling and invalidate the comparison of the deflections on either side of resonance. If curve ($a$), Fig. 4, represents the deflections of the voltmeter with added capacitance in the normal case, and curve ($b$) those when the fixed capacitance is connected, the maxima corresponding to resonance will occur at the same settings effectively, but the two equal deflections will occur one on ($a$) and one on ($b$), so that their midpoint does not correspond

with the true resonance setting. The effect will be worse the smaller the normal coupling from the oscillator. It can be brought to a minimum by reducing the loop formed by the leads to the tapping key and confining the residual loop to a plane in which the mutual inductance between it and the voltmeter circuit is negligible. The neglect of this factor leads to trouble when the damping is considerably different in the two cases met with in these measurements.

No difficulty was obtained in securing constancy of frequency in the oscillator or steadiness of deflection in the voltmeter sufficient to enable satisfactory determinations of $C_{ag}$ to be made which could be repeated to $0 \cdot 0001\ \mu\mu$F.

The voltage amplification of the screen-grid valve stage was determined by means of a low-frequency amplification bridge. A brief description of the apparatus is given in Appendix 2. The measurements should be made with approximately the same input to the valve as was used for the radio-frequency measurements. A method which measures change of phase as well as magnitude of amplification, while not essential, is useful. The value of $\mu$ measured should refer specifically to the fundamental frequency, and to that alone. In this respect total voltage-output measurements are unsatisfactory when wave distortion is present.

This method of measuring grid-anode capacitances was used for values extending from $0 \cdot 002$ to $0 \cdot 1\ \mu\mu$F. In the latter case $R$ was given a sufficiently low value to ensure that $\delta C_1$ was of a convenient order of magnitude.

## Second Method. Grid-Anode Admittance with Filament Cold.

In this method the ratio of the direct grid-anode admittance $Y_{ag}$ of a screen-grid valve, to that of a known capacitance $Y$, is obtained in terms of the readings of two voltmeters; the one giving the voltage $V$ across the two admittances arranged in series, and the other the voltage $v$ across the larger capacitance $C$ alone. With perfect shielding, the following relation holds:—

$$Y_{ag} = Yv/(V - v) \doteqdot Y(v/V) \quad . \quad . \quad . \quad (6)$$

which reduces, when the grid-anode conductance is low in comparison with the suspectance, to the form

$$C_{ag} = Cv/V \quad . \quad . \quad . \quad . \quad . \quad . \quad (6a)$$

In practice, screening may not be sufficiently perfect to justify ignoring a small extraneous capacitance $C_e$ (Fig. 5) in parallel with $C_{ag}$; and the capacitance $C$, so far as it includes the input capacitance $C_i$ of the voltmeter V V, may not be assumed to be known. The procedure adopted has to restrict the measurement to the direct grid-anode admittance and provide for the determination of $C_i$ and for the elimination of the capacitance $C_e$ from the result.

### *Theory and Procedure.*

The equivalent circuit is depicted in Fig. 5. The filament and the screen-grid are connected permanently to the screen, so that the capacitances of the grid and of the anode to these electrodes are thrown into the small capacitances $C_{gs}$ and $C_{as}$ respectively, shown in the diagram. $C_{gs}$ is therefore thrown across the source and $C_{as}$ across the condenser C. In the circuit as shown, with the anode of the valve connected to the insulated terminal of the condenser C and of the valve voltmeter V V, the following relation holds, where $V_n$ is the reading of the electrostatic voltmeter E V, and $v_n$ that of the valve voltmeter.

$$v_1/V_1 = (C_{ag} + C_e)/(C + C_i + C_{as}) \quad . \quad . \quad (7)$$

If the anode be disconnected from the condenser C and be connected directly to the screen,

$$v_2/V_2 = C_e/(C + C_i) \quad . \quad . \quad . \quad . \quad (8)$$

Equation (8) may be used to eliminate $C_e$ from equation (7) and lead to a separation of $C_{ag}$. The appearance of $C_{as}$ in the denominator of (7) complicates the situation slightly. The procedure adopted is to observe and plot a series of values of $(V/v)$ for a sequence of settings of the variable air condenser C.

A straight line is obtained giving an intercept on the capacitance axis corresponding to the values of the

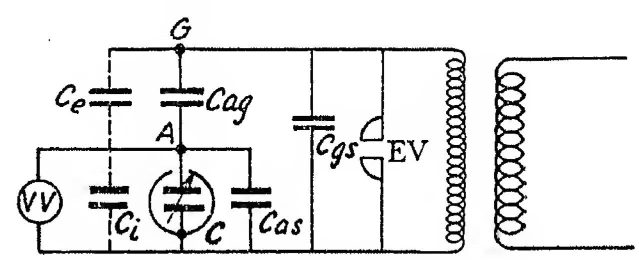

Fig. 5.—Equivalent circuit diagram.

quantities $(C_i + C_{as})$ and $C_i$ respectively, for the two sets of observations. The values of these terms in the denominators having been determined, the value of $(V/v)$ on each straight line corresponding to a convenient common value $(C')$ of capacitance in the denominator is read off.

Then
$$v_1/V_2 = (C_{ag} + C_e)/C'$$
$$v_2/V_2 = C_e/C'$$

whence
$$C_{ag} = C'\left[(v_1/V_1) - (v_2/V_2)\right]$$

In practice, $C_i$ is normally of the order of 30 $\mu\mu$F and $C_{as}$ is approximately 10 $\mu\mu$F. $C'$ therefore never differs greatly from 50 $\mu\mu$F. The variable condenser, C, covered a range from 12 to 65 $\mu\mu$F. With values of $V$ in the neighbourhood of 450 volts, convenient deflections are obtained on the valve voltmeter previously described even in the most difficult cases encountered. The ratio $C'/C_{ag}$ may easily reach a figure of 20 000.

Fig. 6 indicates the general arrangement of the apparatus. By the choice of suitable coils and close coupling to the oscillator, a fair step-up in voltage was obtained. The frequency of the oscillator was finally adjusted to bring the reading of the electrostatic voltmeter to a magnitude of the right order in relation to the valve-voltmeter deflection.

Owing to the square-law scale of the low-reading voltmeter, it was found necessary to augment the small extraneous capacitance $C_e$ in order to render the readings $v_2$ capable of accurate observation. A short length of

wire, connected to the insulated terminal of V V and rigidly supported, served this purpose.

The method requires a calibration of the valve voltmeter prior to use, and the accuracy attainable depends on the accuracy with which this calibration may be carried out.

Results obtained on valves possessing very low grid-anode capacitances, of the order of $0 \cdot 002\,\mu\mu$F, by the two methods described in the present paper, agreed so closely that the discrepancy could be explained by the conductance due to an effective shunt resistance of the order of 300 megohms at 250 kilocycles per sec. It is considered that this is a probable value for a good valve. Much lower values have been measured in inferior valves.

The ability of the method to discriminate readily between the average valve and those in which the radio-frequency insulation is low, is considered to be a distinct advantage of the method.

ACKNOWLEDGMENTS.

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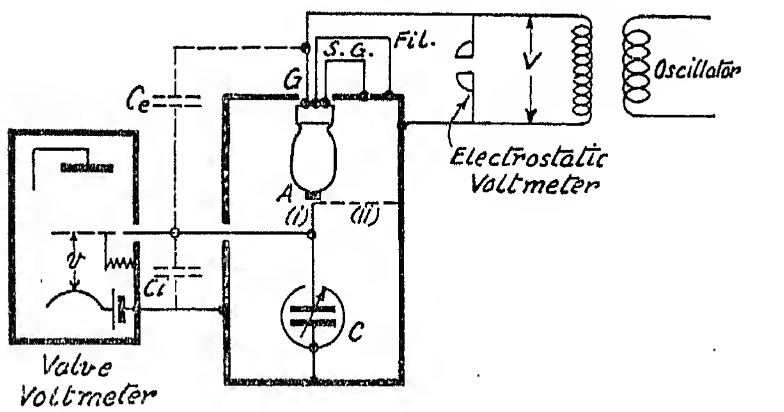



FIG. 6.—Actual circuit diagram.

mission of the Department of Scientific and Industrial Research.

The author desires to express his indebtedness to the late Dr. D. W. Dye, F.R.S., and to Dr. L. Hartshorn, for useful advice in the course of the work.

REFERENCES.

(1) A. W. HULL and N. H. WILLIAMS: "Characteristics of Shielded-Grid Pliotrons," *Physical Review*, 1926, vol. 27, p. 432.

(2) N. R. BLIGH: "Measurements of the Grid-Anode Capacitance of Screen-Grid Valves," *Experimental Wireless*, 1929, vol. 6, p. 299.

(3) T. H. KINMAN: "Measuring the Inter-Electrode Capacitance of Screen-Grid Valves," *Wireless World*, 1929, vol. 25, p. 610.

(4) J. M. MILLER: "Dependence of the Input Impedance of a Three-Electrode Vacuum Tube upon the Load in the Plate Circuit," *Bulletin of the Bureau of Standards*, 1919, vol. 15, p. 367.

(5) H. W. NICHOLS: "The Audion as a Circuit Element," *Physical Review*, 1919, vol. 13, p. 404.

(6) L. HARTSHORN: "The Input Impedances of Thermionic Valves at Low Frequencies," *Proceedings of the Physical Society*, 1927, vol. 39, p. 108.

(7) C. G. GARTON: *Journal I.E.E.*, 1926, vol. 64, pp. 274, 1085.

(8) H. DIAMOND and J. S. WEBB: "The Testing of Audio-Frequency Transformer-Coupled Amplifiers," *Proceedings of the Institute of Radio Engineers*, 1927, vol. 15, p. 767.

## APPENDIX I.

### THE MEASUREMENT OF THE VOLTAGE AMPLIFICATION OF A LOW-FREQUENCY AMPLIFIER.

The bridge which was used for the measurements of the voltage amplification of a single stage is illustrated

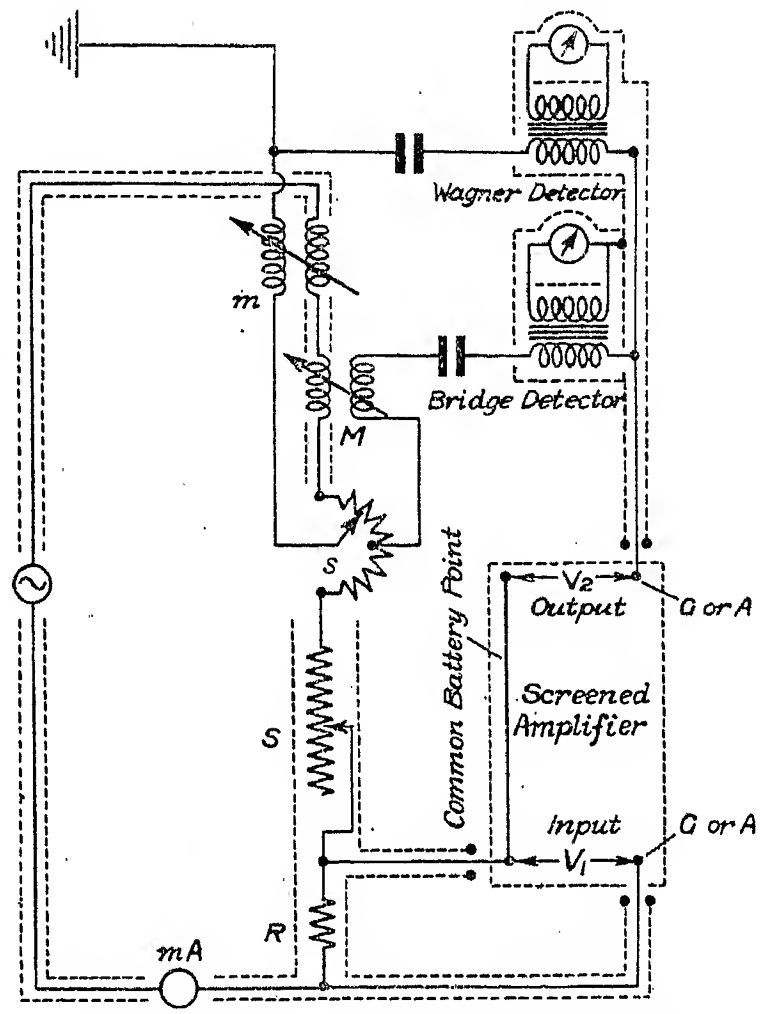



FIG. 7.—Stage-amplification measuring bridge.

in Fig. 7. It is identical with the bridge originally developed at the National Physical Laboratory by the author for the testing of audio-frequency transformers and low-frequency amplifiers. It has the advantage that it measures the phase ($\phi$) of the output voltage as well as the ratio ($v_2/v_1$). It also possesses the merit that the measurement is made for the fundamental frequency under observation, without regard to the harmonic frequencies. In cases where distortion is serious the value deduced from the total output may be very misleading.

The arrangement consists of a potentiometer provided with an auxiliary earthing device. Of the similar potentiometer bridges which have been described, one* is without a Wagner earthing system and the other† adopts an arrangement which differs from that described here. An earthing arrangement should be employed, and this should obviously aim at bringing the detector to earth potential and at minimizing the effects of

* See Reference (7). † *Ibid.*, (8).

capacitances to earth. The author's scheme is the only one which possesses these merits.

The main circuit consists of resistances, R, S, and s, together with the primaries of two mutual inductances, m and M. The input circuit of the amplifier is connected across R, which is of sufficiently low impedance in comparison to permit neglecting the current division. If the current in the primary circuit be $I$, the input voltage $v_1$ is then given by $RI$. The output voltage is balanced in the secondary circuit of the potentiometer by the voltage induced in the secondary winding of the mutual inductance M and the voltage drop in the resistance S, which includes half the resistance of the slide s. The output voltage $v_2$ is therefore equal to $(jM\omega + S)I$ when the input and output voltages are in opposition as shown in Fig. 7, and $(jM\omega + S + R)I$ when they are in phase, in which case the common point has to be transferred to the opposite side of the resistance $R$.

Thus $\quad v_2/v_1 = \sqrt{(M^2\omega^2 + S^2)}/R$

or $\quad \sqrt{[M^2\omega^2 + (S + R)^2]}/R$

And $\quad \tan\phi = M\omega/S, \quad \text{or} \quad M\omega/(S + R)$

The amplifier is enclosed within its own screen, which is connected to the common point of the input and output circuits. This is normally the common battery point. The leads of the bridge and its components are shielded by screens which are connected directly to earth. The detector is coupled to the secondary circuit through an intervalve transformer which is connected to the output terminal of the amplifier. This terminal has therefore to be maintained at earth potential. An auxiliary circuit is provided by the mutual inductance m. Any slight difference in phase angle of the two mutual inductances M and m is remedied by means of the 4-terminal sliding resistances. Two blocking condensers are introduced into the two detector leads in order to avoid any interference with the maintenance of the necessary steady voltage conditions in the amplifier circuits.

The earthing system operates in the following manner. The Wagner detector may be brought to silence by adjusting m and s. When this adjustment is made, the common point of both detectors and the screened amplifier is brought to earth potential. M and S need to be adjusted alternately with m and s before a state of balance is obtained simultaneously in both circuits. The attainment of this state is indicated by simultaneous silence in both detectors.

In practice it is advisable to keep the resistances R and S low in magnitude. The capacitance of the amplifier screen to earth is thrown in series with the secondary winding of the auxiliary mutual inductance m across the resistance S. At the higher frequencies this tends to introduce an error into the phase determination of the bridge which is larger the greater the value of S. For values of S below 1 000 ohms this effect is negligible up to frequencies of the order of 8 000 cycles per sec. In a normal case the inaccuracy of the bridge at this frequency is estimated to be less than 1 per cent from measurements made under different conditions.

The bridge has found applications in the measurement of the characteristics of intervalve transformers, resistance-capacitance stages, and complete amplifiers; and in the measurement of the amplification factors and anode-filament resistances of valves under working conditions with resistances of any magnitude in the anode circuit. The latter measurement is of considerable interest since it utilizes measurements of phase angle. Considering a resistance-capacitance stage with external anode resistance $R_e$ and internal resistance $R_a$, if a capacitance $C$ be connected across the resistance $R_e$ the values of the amplification of the stage before and after making the connection will be as follows:—

Condenser off, $\mu_1 = \mu R_e/(R_e + R_a)$

Condenser on, $\mu_2 = \mu R_e/[R_e + R_a(1 + jR_eC\omega)]$

neglecting the small initial phase angles of the resistors $R_e$ and $R_a$. The phase displacement $(\Delta\phi)$ observed is given by

$$\tan(\Delta\phi) = -[R_eR_a/(R_e + R_a)]C\omega = \Delta M\omega/S$$

$$\mu = \mu_1 CSR_a/\Delta M$$

and $\quad 1/R_a = -(CS/\Delta M) - (1/R_e)$

$R_a$ being known, $\mu$ is immediately deducible.

These measurements present no difficulties at frequencies below 800 cycles per sec. Even dispensing with the earthing arrangement would not incur errors of more than a few per cent in $R_a$, and less in $\mu$.

The bridge unfortunately does not lend itself to measurements of large output amplitudes, since the current in the coils become excessive or the mutual inductances unduly large. For accurate measurements of smaller magnitude the bridge has proved satisfactory.

## APPENDIX 2.

(*Received 6th January*, 1934.)

The author has recently discovered that in some screen-grid valves the capacitance between anode and screen may be as high as 10 $\mu\mu$F. This capacitance ($K$) is thrown across the resistance ($R$) in the input-capacitance measurements described in the first section of the paper. The pure-resistance-load condition may therefore be violated in such cases, and some provision needs to be made to render the anode load non-reactive. This can be most conveniently done by inserting a high-frequency choke in series with the resistance, at the anode-battery end, of an inductance $L$ such that $L = R^2K$.

A check on the suitability of the value of inductance which has been introduced is furnished by the measurements themselves. When the load is practically non-inductive, the deflections observed on the voltmeter are identical for the two conditions of resonance in the input circuit, (*a*) with the anode load $= R$, and (*b*) with the anode load $= 0$ (i.e. there is no resistive component of input impedance due to the load). The inductance should therefore be chosen to satisfy this condition, or it should be slightly greater than the appropriate value; a final adjustment being made by the addition of a minute capacitance across the resistance $R$ itself. The amplification measurements will correspond to the correct non-reactive anode load, and the results should not be in error. Neglect to compensate for the factor is now considered to account in great measure for the slight disparity between the results obtained by the two methods in the case of valves in which the insulation is good.

# DESCRIPTION OF THE QUARTZ CONTROL OF A TRANSMITTER AT 1 785 KILOCYCLES PER SECOND.*

By L. ESSEN, B.Sc.

[From the National Physical Laboratory.]

(*Paper first received* 12*th September, and in final form* 13*th December*, 1933.)

SUMMARY.

The paper describes the quartz-crystal control arrangements for a radio transmitter designed to radiate a standard frequency of 1 785 kilocycles per sec.

The special feature of the quartz oscillator is its mode of support. It rests on four pins screwed into holes drilled into the edges of the oscillator in its nodal plane. By the use of this method of mounting the damping of the oscillator is decreased, and frequency-changes due to movement between the electrodes are nearly eliminated. The quartz plate was tested as a resonator, and its edge was ground until one strong resonance was obtained well removed from neighbouring resonances. It was then found to give a good performance as an oscillator. Tilting the mounting produced a maximum frequency-change of 2 parts in $10^6$, and the frequency varied linearly with temperature and smoothly with air-gap. The frequency was adjusted to 1 785 kilocycles per sec.

The feeble oscillations of the crystal were amplified in two transformer-coupled stages and were then fed to the power valves of a transmitter. Changes of 5 per cent in the tuning of the amplification and power stages produced frequency-changes of only a few parts in $10^7$.

The frequency of the transmitter showed a stability of 1 part in $10^7$ over short periods and 1 part in $10^6$ from day to day.

A small condenser connected between the grid and the plate of the valve driving the crystal enabled the frequency of oscillation to be adjusted so that it could be maintained at a value within 1 part in $10^7$ of that of the standard tuning-fork.

## (1) INTRODUCTION.

It was decided in 1930 to emit from the National Physical Laboratory radio waves of a standard frequency of 1 785 kilocycles per sec., in order that by the use of this frequency and its harmonics amateur experimenters might be able to calibrate their transmitters and wavemeters in the various frequency bands allocated to them. For this purpose provision was made for controlling the then existing standard-frequency transmitter by means of a quartz-crystal oscillator, the work being carried out by the late Dr. D. W. Dye. These standard emissions were commenced on the 3rd March, 1931. The early transmissions were successful in that the frequency was well within the required limit of 1 part in $10^4$ of the nominal value, but the quartz oscillator showed a tendency to oscillate in two modes and difficulty was also experienced in stabilizing the amplification stages. The author, aided by the experience gained, therefore ground another quartz plate, mounted it in a new type of holder, and improved the stability of the amplifier by the introduction of an intermediate screen-grid stage. Since these improvements were effected the transmitter has given a very satisfactory performance with a high degree of frequency stability. As the quartz plate and the circuit arrangements are very simple and such as could be incorporated in any transmitter of this type, it is thought that a short description of them and their performance will be of general interest.

## (2) THE QUARTZ OSCILLATOR.

The quartz oscillator consists of a plate about $2 \cdot 5$ cm square, cut in a plane perpendicular to the electric axis. It vibrates in its fundamental longitudinal mode. Four conical holes of 1 mm depth were cut, one at the

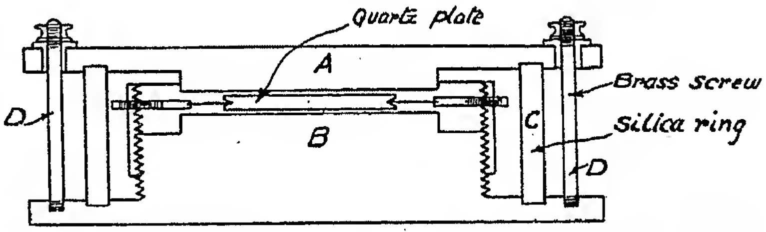



FIG. 1.—Section showing quartz plate mounted between the electrodes.

mid-point of each edge of the plate. The holder consists of two brass electrodes A and B (Fig. 1) separated by a silica ring C and clamped by the screws D. A thin brass cylinder screws on a fine thread cut on the base of the electrode B. The cylinder carries four light metal adjustable pins which screw into the conical holes so that the plate is not quite gripped. The plate therefore rests on the four pins, being free to move only by very small amounts; and frequency-changes, which usually occur on account of the movement of the plate between the electrodes, are thus very nearly eliminated. The plate is, moreover, supported at points in its nodal plane, which should tend to decrease the damping due to the supports. The cylinder is screwed down and clamped so as to leave an air-gap of $0 \cdot 05$ mm between the plate and the lower electrode; and the silica ring is so ground as to make an equal air-gap between the plate and the upper electrode.

When the plate had been mounted in the electrodes and an approximate frequency-adjustment had been made, it was ground so as to have only one mode of vibration in the neighbourhood of the required frequency.

* The Papers Committee invite written communications, for consideration with a view to publication, on papers published in the *Journal* without being read at a meeting. Communications (except those from abroad) should reach the Secretary of the Institution not later than one month after publication of the paper to which they relate.

It is well known that the nature of the vibrations can be considerably affected by grinding the edge of the plate. The method of procedure developed at the Laboratory is to test the plate as a resonator, as described by Dye.* The resonances occurring in the region of frequency corresponding to the thickness of the crystal are detected as crevasses in a resonance curve. It has been found that as the edge of the plate is ground a regular and uniform change occurs in the crevasses, those of lower frequency gradually becoming more pronounced, while the frequencies of all the crevasses become greater. It is usually possible to obtain a condition in which there is one deep crevasse, free from irregularities and well removed from the neighbouring crevasses.

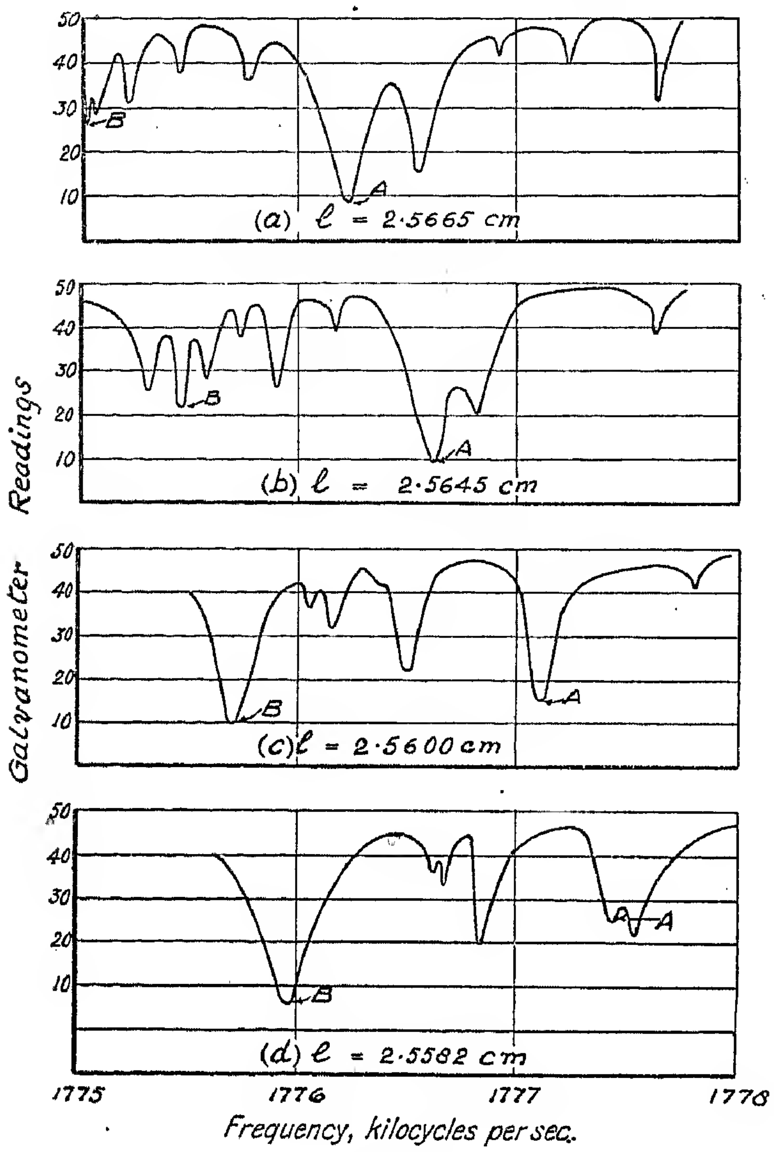



FIG. 2.

Four stages in the grinding of this plate are shown diagrammatically in Fig. 2. The deep crevasse A corresponds to the mode in which oscillations initially occurred. The state of the plate as depicted in Fig. 2(*a*) was considered unsatisfactory, and it was ground along the edge parallel to the optic axis. The length $l$ of the side perpendicular to the optic axis is given in the diagrams. The grinding was performed in small stages in order that the crevasse patterns might in all cases be correlated. As the grinding proceeded, the low-frequency crevasses became deeper until (Figs. 2*c* and 2*d*) B became the main crevasse. Meanwhile the mode of oscillation had changed from A to B. It was found that the quartz plate had now a very good performance as an oscillator. There was no suspicion of oscillations in more than one mode, the frequency varied smoothly as the air-gap was varied from 0·05 to 0·5 mm, and the temperature coefficient was linear between the limits investigated (15° C.–30° C.). The final frequency-adjustment to 1 785 kilocycles per sec. was made by grinding the face of the crystal. For small adjustments this does not materially affect the nature of the crevasse curves.

The temperature coefficient of the plate was $-20$ parts in $10^6$ per deg. C. rise in temperature, and it was therefore decided to control its temperature by means of a thermostat and to aim at an accuracy of 1 part in $10^6$.

### (3) THE ELECTRICAL CIRCUIT.

The electrical circuit diagram is shown in Fig. 3. The quartz-controlled oscillations are first amplified in two stages and then passed on to the power valves of the transmitter, which is described fully in another paper.* The tuned inductances $L_1$, $L_2$, and $L_3$, in the anode circuits of the valves, consist of toroidal coils, the secondaries of which are wound turn for turn over the primaries. The first stage of amplification is a screen-

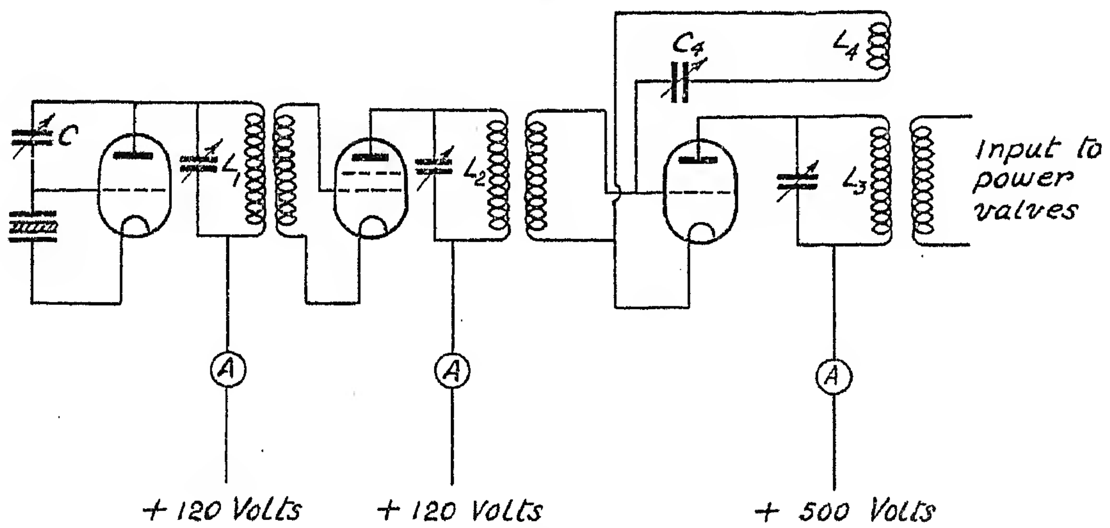



FIG. 3.

* D. W. DYE: *Proceedings of the Physical Society*, 1926, vol. 38, p. 399.

* H. A. THOMAS: "The Emission of Special Radio Signals for the Study of the Ionosphere" (to be published in a later issue of the *Journal*).

grid stage, which serves to prevent reaction on the quartz oscillator and also provides sufficient amplification to enable the vibrations of the quartz to be reduced to a small amplitude, thus increasing their frequency stability. The second stage consists of a 50-watt valve operating with an anode voltage of 500 volts. The stability of this and the power stage was increased by the inclusion of a neutrodyning coil $L_4$ and condenser $C_4$. A small condenser C was connected between the plate and grid of the valve driving the crystal, to provide a simple means of making small frequency-adjustments. A total frequency-change of 70 cycles per sec. was obtained by changing the capacitance C from 15 to 50 $\mu\mu$F.

### (4) Performance.

To test the frequency stability of the emitted wave, a direct comparison was made with the 1 785th harmonic of the standard tuning-fork. The effect of moving the quartz holder was first investigated. It was found that the holder could be tilted through any angle, completely inverted, and shaken, without causing frequency-changes of more than 2 parts in $10^6$. The effect of detuning the amplification stages was then studied. A change of 25 $\mu\mu$F, equivalent to 5 per cent of the total capacitance in the anode circuit of the screen-grid stage, produced a frequency-change of less than 4 parts in $10^7$; and 5 per cent changes of capacitance in the following stage and the power stage produced frequency-changes of about 1 part in $10^7$. Measurements were then made while the transmitter was left untouched for a period of 30 minutes. The results are given graphically in Fig. 4. It is seen that the points lie on a smooth curve with an accuracy of 1 or 2 parts in $10^8$. This is the order of frequency stability of the standard tuning-fork. The general drift in frequency is due to a change in the temperature of the quartz plate. At the time of the observations the temperature in the transmitting hut was increasing rapidly at the rate of about 5 deg. C. per hour; and this caused an increase of 0·05 deg. C. in the temperature of the quartz during the course of the observations. The temperature coefficient of the quartz being $-20$ parts in $10^6$, the corresponding frequency-change is $-1 \cdot 8$ cycles per sec. The sudden decrease in frequency which occurs at regular intervals takes place when the heating current is switched on by the relay.

During future transmissions it is intended to keep the frequency within 1 part in $10^7$ of that of the standard tuning-fork by a fine adjustment of the condenser C.


Fig. 4.

It has not yet been possible to obtain much information concerning the frequency stability of the transmitter over long periods of time; but, the temperature being constant, the frequency has been reproducible from day to day within 1 part in $10^6$.

### (5) Conclusion.

It is to be concluded from these experiments that a quartz plate, suitably supported in its electrodes and thermostatically controlled at a constant temperature, is capable of operating in its longitudinal mode with a short-period stability of 1 part in $10^7$ and a day-to-day stability of 1 part in $10^6$. The feeble oscillations of the quartz oscillator may be amplified to transmitter strength in such a manner that the frequency is nearly independent of the tuning of the amplification stages.

The development of the quartz oscillator was carried out as part of the programme of the Radio Research Board, and this paper is published by permission of the Department of Scientific and Industrial Research. The author's thanks are also due to Dr. R. L. Smith-Rose for reading the manuscript of the paper, and to Mr. Hatcher for assisting in the measurements.

# METER AND INSTRUMENT SECTION: CHAIRMAN'S ADDRESS

By W. LAWSON, Member.

(*Address delivered 3rd November*, 1933.)

## INTRODUCTION.

My first duty is to tender my thanks to the members for electing me their Chairman. It is an honour which I value the more because it represents the high-water mark of my career as a meter engineer, and I shall do my utmost to justify the confidence placed in me.

In the task of putting together this Address I was faced with the usual difficulty of selecting a subject or subjects. The choice was decided on the assumption that those matters with which I had recently been personally associated, and in which, therefore, I have been specially interested, might be the most appropriate to the occasion. I propose to deal with two subjects of technical interest, namely, the insulation and earthing of meters and apparatus, and the measurement of load conditions on supply systems; but before I embark on these I feel that I must refer to the more important question of the progress and welfare of our Section.

I entertain a special interest in and regard for this Section, not only because I had a hand in its foundation but also because, for a long time prior to its advent, I realized the dire need for some organization which would establish testing and metering—and especially metering—in their rightful place and prominence in electrical work. As a proof of this I will quote the concluding words of my paper* on "Defects of Electricity Meters": ". . . it is to be deplored that meter engineers themselves do not come together to compare methods, discuss difficulties, and exchange information. Such co-operation could not fail to bring to light a vast amount of knowledge which would be of mutual assistance not only to themselves but also to those engaged in the manufacture of meters."

That was said 13 years ago and, as I look back on all that has been accomplished by this Section and on the sustained interest of its members, I seem to be witnessing the realization of a dream. Although it was left to others to take the all-important first steps in that realization, I may claim to be specially privileged to refer to past history, as I am the only member of the old M.E.T.A. Council on the Committee of the Meter and Instrument Section at the present time.

There is no question that up to the time the M.E.T.A. was formed there existed a very real need for raising the status of meter engineers, many of whom felt acutely the lack of interest in their work which frequently characterized the managements of undertakings. There is, perhaps, nothing that warps a man's nature and tends to impair his efficiency more than purblind neglect from those who stand to lose by such an attitude. It is pleasing to reflect, however, that through the activities of this Section the standard of metering practice has undoubtedly been raised, and the qualifications demanded of the meter engineer are as high as those of any other; no more will he be recruited "ready made" from the ranks of the wireman and meter reader—meter engineers must now be "made to measure."

If only because of that happy result, the formation of this Section has been justified, and no one would deny that the Institution showed wisdom in absorbing the M.E.T.A., whose precious, enthusiastic spirit is, I hope, by no means spent. We must not forget, however, that metering is only one phase of the activities of our Section; and whilst it will, I think, always keep a prominent place in our proceedings, there will, doubtless, be a stronger tendency in the future to go much further afield in the sphere of electrical measurement generally. In particular, this Section ought to attract —there is much evidence that it does attract—those engaged in research and educational work, and should thus form a common meeting-ground for the scientific worker and the practical technical man.

As to the future, our usefulness and prestige as a Section are largely in the hands of the members, but are of course also bound up in the fortunes of the whole electrical industry, about which I, for one, have few misgivings. In the supply industry and, doubtless, in many others, the fundamentals on both the engineering and economic sides have been laid down, if not widely comprehended, and it now remains for us to solve the problem of spreading the benefits of electrical service throughout the entire population. In this, the scientist, the inventor, the designer, the commercial and technical expert, and others, will be called upon to make their contribution, but, notwithstanding all current theories to the contrary, I am convinced that the only serious obstacles are of a technical order and that only the technical man is fitted to surmount them. In any field of activity the truly successful are those who have complete mastery of the medium with which they work; in our field there is no better aid to such mastery than the art of measurement.

## INSULATION AND EARTHING OF METERS AND APPARATUS.

There is still almost unlimited scope for research in the production of insulating materials, and little prospect of the appearance of a cheap material which combines the mechanical properties of the cheaper metals with the relatively stable insulating characteristics of mica or the best porcelains. That may never eventuate, but in the meantime a better appreciation of the correct uses for the available material is demanded. The selection of insulation material for any apparatus should be governed by the worst conditions which it

* *Journal I.E.E.*, 1920, vol. 58, p. 747.

will be liable to be called upon to withstand in service. Moisture is the enemy, and it is questionable whether mere immersion is of much value as a test of insulating materials for water absorption. Also, high-pressure tests on apparatus prior to installation, when such apparatus is in sound condition, are liable to establish confidence in the safety of the apparatus which may be quite illusory.

Electricity meters are too frequently installed in unsuitable places, one of the very worst being a damp, unventilated cellar. In such conditions direct-current meters suffer especially owing to osmotic action, which results in the accumulation of moisture at the negative pole; and electrolysis engenders corrosion and sometimes produces complete breakdown of the insulation. The effect, of course, is more pronounced with some materials than with others. A comparison between a sound meter terminal-block and one which has been so affected makes one wonder what new problems might arise if the much-talked-of super-tension d.c. transmission were to be adopted. It is fortunate that with alternating current osmotic action is absent, because an a.c. meter case which is live is a much more serious matter than a live d.c. case; but, even without osmosis, absorption of moisture will result in a breakdown if the insulating material used is unsuitable. In view of this fact, some years ago I instituted a method of testing the ability of meters to resist damp conditions. The method was described in the *Journal*,* but I propose to repeat the description here and also to say something of how the method was made applicable to the wider field of domestic appliances.

Clearly my experiments were to be directed to reproducing the damp, unventilated cellar condition. A box was therefore constructed with a well-fitted lid to prevent the flow of air in and out. Inside, a shelf was provided on which to stand the meters, and beneath the shelf was a shallow bowl containing water. By this means it was to be expected that a damp atmosphere would be created and maintained. The results were startling, for after a week or so the insulation resistance fell from something of the order of 15 megohms or more to 0·25 megohm or less when tested after removal of the surface moisture. The insulation resistance of an a.c. meter dropped to zero in less than a month. This led to the manufacturers' adopting a different material for insulating the current coils, and there were other instances where the insulating material in the meter had to be improved. The action which takes place appears to be due to the penetration of water vapour, which in time completely permeates the material, followed by condensation with fall of temperature, the action being repeated until complete saturation is reached. Needless to say, tests are only applied to samples of meters submitted for approval, and not as a routine.

As regards the wider application to domestic appliances, this test forms the most important detail in a scheme which has been adopted by the Birmingham Electricity Department for protecting consumers against the possibility of using apparatus that may be dangerous. This scheme, which was devised and instituted by the Birmingham city electrical engineer, is, I believe, unique in many respects, and I take this opportunity of describing it, not only on account of its unquestionable importance, but because the principal test that it employs originated in the Meter Department and, moreover, forms an example of a laboratory device which was found capable of being much more extensively applied.

The scheme originated partly as a result of the change of system from direct current to alternating current and the conviction that supply authorities, contractors, and manufacturers, should abandon the d.c. outlook which rather tended to make light of safety measures as regards domestic installations. Mr. F. Forrest, the city electrical engineer, therefore decided to form a Committee consisting of the installation engineer, the commercial engineer, and myself as Chairman, whose duty was to set up a standard of safety to which all domestic appliances and apparatus exhibited and sold by the Department should comply. As there were then no regulations applicable to domestic installations similar to those in force in factories, it was laid down that Regulation 13 of the Home Office Regulations under the Factory and Workshop Act should be worked to as closely as practicable. I need not repeat this Regulation, for it will suffice to say that it refers to measures to be taken to safeguard persons handling portable apparatus.

Among the first decisions arrived at were that all a.c. prepayment meters must be earthed and that all apparatus received by the Department must be subject to an insulation resistance test at 500 volts. The main work of the Committee is to examine every piece of apparatus submitted for exhibition in the showrooms of the Department, and to draw up a list of such as are found to be unsuitable from a safety point of view. They have also to examine the design and construction in detail, to point out to the manufacturer any defect, and to make suggestions for improvements in order that the apparatus may comply with the Department's standard. Special attention is given to apparatus of the portable type. The liability to danger in use is estimated not only from the character of the apparatus but also from the conditions under which it may be used and the frequency with which it would ordinarily be handled and put into operation.

On the question of earthing, the Committee arrived at views which may be of interest. They consider that apparatus with exposed and ineffectually-guarded conductors, such as fires and toasters, should not be earthed. On the other hand they consider that the geyser type of water heater, which is solidly earthed in virtue of its metallic connection with the cold-water supply-main, should in addition be independently earthed, as it might be disconnected from the supply pipe (say by a plumber repairing the tap) without being electrically disconnected.

The Committee devoted a good deal of attention to the question of better guards for electric fires, but reached the conclusion that if such guards were to be effective they would substantially reduce the radiating efficiency of the fire and spoil its appearance. They could suggest no improvement on the present practice, but decided that whenever a fire is sold or hired the consumer should be warned that a guard fixed to the radiator would not comply with the legal requirements

* *Journal I.E.E.*, 1928, vol. 66, p. 777.

for guards to be placed in front of fires for the protection of children.

A great deal of attention was given to cookers, it being decided that oven elements of the totally enclosed type should be made standard in the future, and 800 of the Department's cookers having oven elements protected by covers which could readily be removed were modified in accordance with the recommendation of the Committee.

Electric irons also were very critically examined, chiefly to ensure that sufficient clearance is provided between the live parts and the case, in view of the fact that shocks have been obtained owing to iron filings bridging across the clearance in the interior of irons. It was noted that an undesirable feature in some irons was the exposure of the live connecting-pins owing to the absence of shielding. In dealing with the vital question of insulation, the Committee invariably proceed on the plan of endeavouring to estimate the factor of safety when the apparatus is handled or operated under the worst possible conditions, namely when there is no means of earthing or when the earth wire is temporarily disconnected, combined with an excessively damp situation.

In connection with this, the application of the damp-chamber method of trying-out insulation has proved very useful. The equipment is designed on the same lines as that used for trying-out meters, but it is of larger dimensions. Under this test a number of pieces of apparatus failed and some went completely to earth, but in most cases there was no difficulty in getting the manufacturers to replace the insulating material by other material which would stand up to the conditions. The test is very severe and, where extremely low values of insulation resistance are reached, the apparatus is not rejected if it is capable of recovering rapidly under dry conditions. In some instances recovery has not taken place after several weeks, and this has gone against acceptance.

In addition to the work of the Committee the outside men who visit consumers' premises for various purposes are instructed to take every opportunity of inspecting consumers' appliances. They make a note of frayed, kinked, and twisted flexible cords, loose connections, broken connectors, and apparatus that has been badly treated. The meter inspectors also carry with them instructions for ascertaining whether single-pole switches are on the live part of the circuit.

It must be admitted that when they first undertook the investigation the Committee concluded that there must be an appreciable number of unsafe appliances in use on domestic installations and also some in unfavourable conditions, but that a "scare" attitude in the matter is totally unnecessary. It is clear that with very little trouble, and at no expense, all possible danger in use can be eliminated from domestic appliances if these are properly treated.

Obviously the core of the problem is the insulation: earthing should be regarded only as a standby or a second line of defence. The all-insulated idea has its advantages, but where it has been put into practice important considerations have been overlooked. The architecture of the job, so to speak, has been deficient, inasmuch as the design has followed the lines of similar structures built of more robust material than the bakelite which is used. The thinness of the material and the weakness of lugs and similar projections, also of sharp corners, etc., are some of the bad features. Then, again, bakelite is a doubtful material, if not made to a standard specification and process, and consequently not of uniform quality. It has, in fact, gained a confidence and popularity not entirely deserved. I found that some samples warped badly, although this is exceptional. Notwithstanding these drawbacks, very real progress has been made; the all-insulated prepayment meter has arrived, and one can venture to prophesy the ultimate if not rapid disappearance of the metal-clad meter.

## The Measurement of Load Conditions on Supply Systems.

It is a well-known fact that a great change has taken place in recent years in the system of electrification in this country and, whilst we are all familiar with the stages of that change and the forces responsible for it, the problems arising from it are not so self-evident. Briefly, what has happened is that whilst there has been a considerable contraction numerically of the centres of generation, the areas of distribution and the number of points of supply have expanded. Generating stations are not now necessarily in the happy position of being at the centres of gravity of their loads. Whether the present order of things is the most economical arrangement that could have been contrived may be a legitimate question, but it is not at the moment a practical one. It must, however, be conceded that the attainment of the lowest possible cost of generation is a fundamental requirement, as the low-priced unit made available to all consumers of electricity is also a fundamental necessity on the retail side. Between the two there is the vastly intricate problem of the most economic loading of the system.

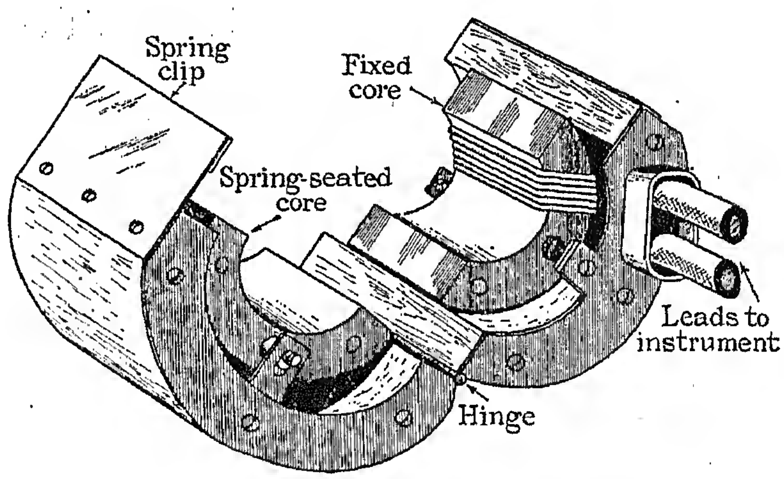
I believe that analysis of load conditions will play an important part in the future progress of the supply industry, and that a proper understanding of general and local variations on systems will make all the difference between the adoption of a timid or a bold policy in development. For example, in heating schemes based on heat storage such knowledge is essential in order to work out the periods of supply most favourable to quoting the low rates per unit that are necessary.

It is from considerations such as these, and because I am of the opinion that all possible aids should be available for ascertaining the load conditions on various circuits, that I have investigated the means of measurement. I am not specially concerned here with the measurement of maximum demand for the purpose of charging on demand tariffs, although the information so obtained is of the utmost value, but with the measurement of loads at substations and different centres of distribution, and possibly also on consumers' premises. It is remarkable how scanty is the information given in this respect in up-to-date textbooks, but the recent appearance of new devices for such measurement is evidence of the attention now being given to the subject. Naturally the instrument that first suggests itself is the recording ammeter, but at once the question of

cost becomes serious. The chart record does provide the complete story, but it also tells a good deal of what is already known and much that is immaterial, and is therefore cumbersome to handle. These instruments we must have for the few cases that cannot otherwise be dealt with satisfactorily, but generally it is the maximum reading that is vital. I decided as a start to explore the possibilities of maximum-load measurement on e.h.t. consumers' supplies. We have about 600 of these, but none with kVA demand indicators and few with kW indicators. The idea of connecting thermal indicators in the secondaries of the meter current-transformers was considered and abandoned. Each meter panel has an ammeter of the moving-iron type, and it was decided to attempt to make an attachment having a pointer which would be actuated by the pointer of the instrument and would remain at the position of maximum indication. The requirements were that it should operate with a reasonable amount of lag, have no independent movement, and not affect the accuracy of the ammeter apart from the lag period. It was a difficult problem.

The first experiment was with a device which consisted of a light tube bent to a half circle, with a bracket to enable it to be pivoted at its centre and on which to mount the pointer. The tube was filled with thick oil and the ends were fitted with oil-tight caps; the whole movement was carefully balanced, after which one of the caps was removed and a lead shot inserted. It was thought that if the pointer were held in one position the shot would slowly sink to the lowest part of the tube and the pointer would then remain permanently in that position. In practice it was found that the ball very rarely did reach the lowest position, even when a thin oil was used, so that the indication was generally on the low side, also the torque of the instrument was not equal to the work of moving the pointer at low loads; there was also evidence that the balance of the movement was disturbed by inserting the shot. It was therefore decided to try a tube of smaller bore open at both ends and partially filled with oil. This was an improvement in so far as the maximum reading was accurately indicated and less work was thrown on the instrument, but the lag was very short and, worst feature of all, it was unstable and the pointer would very slowly creep back to zero. As it was evident that some change of tactics was necessary, the tube was put on one side in favour of a small fixed tank of oil into which a paddle attached to the pointer was immersed, the pointer and paddle being extremely light. This arrangement was an improvement in all respects and appeared to be quite stable, but the final arrangement was a combination of the paddles with a tank in the form of a tube. In order, however, to ensure absolute stability a friction brake-disc with a spring adjustable to vary the friction at will was fitted. The disc is so shaped that there is no brake action at the lower portion of the scale where the torque of the instrument is low. Six of these have recently been put into use and have already justified themselves; in one case a quite unexpected state of overload was almost immediately shown up at a colliery. The attachment is also being fitted to a substation voltmeter for the purpose of indicating maximum voltage.

The thermal demand-indicator is being freely used and, during the past year, we have installed nearly 200 in our static substations. Another current-measuring device which is extremely useful where there are no switchboard instruments is the grip tester, and I have found it necessary to make up a special pattern so that it can be applied to the porcelain fuse-carriers on outgoing distributors in substations. This method of measurement has, in fact, entirely displaced switchboard ammeters which could be plugged in on different distributors—an unsatisfactory and dangerous operation. Another device which has been worked out and put into service is a means of measuring the maximum demand on low-tension overhead lines. This takes the form of a self-locking grip tester, which may have the maximum-reading ammeter self-contained or the ammeter may, preferably, be separately mounted on the pole. In the latter arrangement the maximum indicator consists of a lightly constructed metal drum partially filled with oil, which is prevented from flowing rapidly from one side of the drum to the other by means of perforated barriers to produce the lag. The idea was once applied




Clip-on current transformer.

to indicating instruments in order to produce damping, but it has not been used for many years. It is, in fact, unstable, but to overcome this a brake like that previously described is fitted.

A number of tests carried out on overhead lines with this device disclosed a liability of the frame and cores of the grip to be affected by bad-weather conditions, and owing to its angular shape it was found difficult to provide suitable protective covering. Another design has therefore been produced which, to a large extent, overcomes this handicap. The details are illustrated in the Figure. This shows the current transformer arranged to clip on to the insulated portion of the overhead line, with leads for connecting up to the instrument (not shown), which would be separately mounted on the pole. The split core of the transformer is housed in a split, hinged wooden bush; one half of the core is fixed in the bushing, but the other is spring-seated so that, when the transformer is clipped on, pressure is exerted on the two halves of the core and a good magnetic joint is thereby maintained. A spring clip enables the transformer to be self-locking. When fastened in position the apparatus takes a simple cylindrical form which can readily be wrapped with waterproof material.

### Conclusion.

In conclusion, there is some evidence of a falling-off in the output of papers from the members of this Section; and this falling-off, if only temporary, is yet difficult of explanation. It might be suggested that it is due either to lack of fertility in ideas or to paucity of subjects. I cannot admit that it is the former, and, as to the latter, it does not require much reflection for numerous subjects to suggest themselves.

In the problems with which I have here dealt there is much room for amplification and discussion. Then a paper is badly wanted on the maintenance of relays, and the recent developments in synchronous motor-driven clocks call for a reassessment of the applications of time switches. A paper on the technique of testing and measurement in electrical research would be of great interest. One dealing with the art of measurement would usefully reveal the many pitfalls which, from time to time, result temporarily in our undoing. Instruments have failings, idiosyncrasies, even personalities, and I have no doubt that among the members there are many who could compile a treatise on a single instrument.

The field is therefore boundless. If there be any falling-off of interest, I can only suggest that our minds are unconsciously becoming sterilized by a false sense of finality in electrification, induced by the completion of the grid. However that may be, I feel sure that the members will do their utmost to see that the Section is well provided with the contributions which are necessary to continue its activities and which are its life-blood.

# INSTITUTION NOTES.

## Council's Nominations for Election to the Council.

The following have been nominated by the Council for the vacancies which will occur in the offices of President, Vice-Presidents, Honorary Treasurer, and Ordinary Members of Council, on the 30th September, 1934:—

**President.** (*One Vacancy.*)

Professor W. M. Thornton, O.B.E., D.Sc., D.Eng.

**Vice-Presidents.** (*Two Vacancies.*)

W. E. Highfield.
Lieut.-Col. A. G. Lee, O.B.E., M.C.

**Honorary Treasurer.** (*One Vacancy.*)

F. W. Crawter.

**Ordinary Members of Council.**

Members. (*Four Vacancies.*)

N. Ashbridge, B.Sc.(Eng.) V. Watlington, M.B.E.
J. R. Beard, M.Sc. W. J. H. Wood.

Associate Members. (*Two Vacancies.*)

A. H. M. Arnold, Ph.D. C. L. J. B. Nadaud.

Companion. (*One Vacancy.*)

Brig.-General R. F. Legge, C.B.E., D.S.O.

## Premiums.

The Council have made the following awards of Premiums for papers read during the session 1933–34 or accepted for publication:—

*The Institution Premium* (*value* £25).

W. Kidd and J. L. Carr, B.Sc. — "The Application of Automatic Voltage and Switch Control to Electrical Distribution Systems."

*The Ayrton Premium* (*value* £10).

B. A. G. Churcher, A. J. King, B.Sc.Tech., and H. Davies, M.Eng. — "The Measurement of Noise, with special reference to Engineering Noise Problems."

*The Fahie Premium* (*value* £10).

T. S. Skillman, M.A. — "Development in Long-Distance Telephone Switching."

*The John Hopkinson Premium* (*value* £10).

W. G. Thompson, Ph.D., B.Sc. — "The Application of a Gas-Cooled Arc to Current Conversion, with special reference to the Marx-type Rectifier."

*The Kelvin Premium* (*value* £10).

B. L. Goodlet, B.A. — "Electromagnetic Phenomena in High-Voltage Testing Equipment."

*The Paris Premium* (*value* £10).

J. L. Miller, Ph.D., B.Eng. — "The Influence of Certain Transmission-Line Associated Apparatus on Travelling Waves."

*and*

J. L. Miller, Ph.D., B.Eng., and J. E. L. Robinson, M.Sc. — "The Design and Operation of a High-Speed Cathode-Ray Oscillograph."

*The Webber Premium* (*value* £10).

G. Shearing, O.B.E. — Address as Chairman of the Wireless Section.

*An Overseas Premium* (*value* £5).

S. P. Chakravarti, M.Sc.(Eng.) (India). — "Audio-Frequency Constants of Circuits and Telephone Lines."

*Premiums (each value £5).*

| | |
|---|---|
| M. A. B. Brazier, Ph.D., B.Sc. | "A Method for the Investigation of the Impedance of the Human Body to an Alternating Current." |
| B. S. Cohen, O.B.E. | "Research in the British Post Office." |
| C. W. Marshall, B.Sc. | "The Lower-Voltage Sections of the British Grid System." |
| P. D. Morgan, M.Sc. (Eng.). | "Reinforced Concrete Poles for Overhead Lines." (E.R.A. Report.) |
| W. G. Radley, B.Sc. (Eng.), and S. Whitehead, M.A., Ph.D. | "Recent Investigations on Telephone Interference." (E. R. A. Report.) |
| H. Rissik, B.Sc.(Eng.). | "Some Aspects of the Electrical Transmission of Power by means of Direct Current at Very High Voltages." |

*The Willans Premium (value £14 6s.)*

The triennial award of the Willans Premium, which falls to the Institution on this occasion, has been made to Mr. D. B. Hoseason for his paper entitled "The Cooling of Electrical Machines," read before the Institution on the 6th November, 1930, and published in Volume 69 (page 121) of the *Journal*.

### Wireless Section Premiums.

*The Duddell Premium (value £20).*

| | |
|---|---|
| T. Walmsley, Ph.D. | "An Investigation into the Factors controlling the Economic Design of Beam Arrays." |

*Premiums (each value £10).*

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|---|---|
| L. H. Bedford, M.A., and O. S. Puckle. | "A Velocity Modulation Television System." |
| E. B. Moullin, M.A., and H. D. M. Ellis, B.A. | "The Spontaneous Background Noise in Amplifiers due to Thermal Agitation and Shot Effects." |
| A. H. Reeves. | "The Single Side-Band System applied to Short-Wave Telephone Links." |

### Meter and Instrument Section Premiums.

*The Silvanus Thompson Premium (value £10).*

| | |
|---|---|
| A. H. M. Arnold, Ph.D. | "Current-Transformer Testing" and "Leakage Phenomena in Ring-Type Current Transformers." |

*Premiums (each value £5).*

| | |
|---|---|
| J. B. Lees. | "The Equipment and Operation of a Modern Meter and Test Department." |
| E. Mallett, D.Sc. (Eng.). | "A Valve Wattmeter." |

*Premiums (each value £5)—continued.*

| | |
|---|---|
| G. F. Shotter. | "Experience with, and problems relating to, Bottom Bearings of Electricity Meters." |

The awards for papers read before the Students' Sections will be announced later.

## Elections and Transfers.

At the Ordinary Meeting held on the 26th April, 1934, the following elections and transfers were effected:—

### Elections.

*Associate Members.*

Butterworth, Hubert.
Chadwick, Albert Thomas.
Cook, Frederick.
Crocker, William Gordon.
Goodall, Sidney Edward, M.Sc.(Eng.).
Hall, Eric Spencer.
Holdsworth, Thomas Clifford.
Joyce, Reginald Montague.
Long, Rupert Basil M., Lieut.-Commdr.
McCullagh, Gordon Ralph, B.Sc.
Metcalf, Alfred Whitley, M.Sc.Tech.
Read, Frank William.
Robinson, Douglas Harry.
Roe, William Francis.
Swan, Robert John J.
Windross, Frederick Edward.
Young, John Edward.
Young, Thomas Maclennan.

*Companion.*

Murray, Ian Christian A.

*Associates.*

Burgin, Walter Edward.
Butcher, Harry.
Duncan, K. Venour.
Eckersley, Reginald.
Ezelarab, Abdelaziz, B.Sc.
Haugaard, Frederick Bertleson.
Hodder, Joseph.
Isaacs, Sydney George.
Sykes, Carl Herrmann.

*Graduates.*

Beckley, John David.
Bromwich, William Albert.
Buglass, Stanley Richard.
Cleur, Charles William.
De, Lalitmohan.
Egan, Patrick Joseph.
Fairfield, Christopher Leonard G., B.A.
Jackson, Reginald Henry.
Joshi, Mahadeo Sakharam, B.E.
Kay, Jack.
Kennion, Wilfrid Roger.
Kouyoumdjian, Kerop K., B.Sc.
Lautier, Vincent.
McCabe, Ernest.
McLeod, Percival Hector.
Manton, William Joseph W.
Miller, William Leslie E.
Modi, Jayantilal Thakordas.
Nag, Dhirendra Chandra.
Potts, Frank, M.Eng.
Pritchard, John Neville H.
Richards, Charles Graham, M.Sc.Tech.
Sahiar, Jehangir Hormusji.
Sinnadurai, Sinnathamby.
Smith, Donald Sinclair, B.A., M.A.Sc.
Thompson, Henry Lawrence.
Wainscott, Percival Darville.
Wallace, Ranald Hamilton.
Watson, Daniel Stewart, B.Sc.(Eng.).

*Students.*

Akers, Bernard Horace.
Allen, Philip Weston.
Annandale, James.
Attwood, Charles Ernest, B.Sc.
Ayyar, Sesha Jagannatha.
Bacon, James Laurence.
Batten, Charles John.
Bax, Claude Harry J.
Bennett, Frederick Ithell.
Bright, John Reginald.
Bulger, Reginald Frederick.
Clayton, Kenneth Bernarr.
Cowper, Anthony Alexander T.
Crawford, Alexander Gillies.
Crocker, Frederick William.
Croker, George Leslie.
Davis, John Hancock.
Dhruva, Manubhai Maganlal.
Dixon, Walter Harry.
Eaton, John Reginald.
Edwardes, John.
Flack, Reginald.
Franklin, William Alfred.
Goodger, John Francis.
Hall, James Edward.
Hamilton, James Lawrie.
Harrison, Frederick Anthony.
Heath, Charles Livingstone.
Heywood, Cyril Phillip E.
Hix, Kenneth William.
Howard, John Lawrance.
Hughes, John.
Kistner, Albert Francis.
Knowles, William.
Lall, Ram Behari.
Lang, Leonard Charles.
Long, Henry James.
Lorkin, Terence Edward, M.A.
McKie, Norman Alan M.
Marklew, Howard Murray, B.Sc.
Maskell, Leslie Horace.
Metcalfe, Harold.
Millar, John Boyer.
Morgan, Hugh Gabriel B.
Nurse, Raymond Aubrey.
Oxley, Jesse.
Packer, Johnson Lewis.
Patel, Noor Mohamed H.
Paton, Ian Keith.
Raban, Harry Denys P.
Rennie, David.
Ritchie, David Malcolm.
Rowbotham, Francis Joseph.
Ryan, Joseph Gerard.
Shallcross, John, B.Sc. Tech.
Shohet, Joseph I.
Smith, Charles Cholerton.
Smith, John Hugh.
Smith, Thurston Meiggs W.
Standing, Colin Frederick.
Subrahmanyam, Periyapatna Narasimhayya.
Taylor, Duncan Philip.
Taylor, John, Jun.
Thistlewood, Geoffrey Charles.
Weatherley, Walter Charles.
Weeks, Herbert Derrick.
Wilson-Haffenden, Philip Allen.
Wood, Edward Kemp.

TRANSFERS.

*Associate Member to Member.*

Arthur, James William.
Gordon, Andrew Howard.
Henniker, Harry Vincent.
Hutton, Leslie Bertram, B.Eng.
Kemsley, Alfred George.
Kill, Ernest Frederick.
Lucas, George Sail C.
McCaffery, James, O.B.E.
MacDonald, George John.
Moore, Will.
Murphy, Frank, B.Sc. (Eng.).
Reiseger, Johan Herman.
Riley, Edgar Strettell.
Winfield, Frederick Cavendish, M.Eng.

*Associate to Associate Member.*

Whittick, Robert Baker.

*Graduate to Associate Member.*

Armitage, Geoffrey Lockhart, B.Sc.
Belliss, William Howard A., B.Eng.
Birch, Stanley Harold.
Boyd, William.
Burton, Colin Rupert, B.Sc.
Collis, William Blow G., B.Sc.(Eng.).
Donkin, John, B.Sc.(Eng.).
Follett, Samuel Frank, B.Sc.(Eng.).
Gay, Harold.
Langford, William Lawson.
McCandless, Joseph, M.Sc.
McPherson, William Lindsay, B.Sc.(Eng.).
Mayes, Guy Noel H.
Metcalf, Herbert Eustace L.
Mitchell, James William E.
Pack, Stanley Walter C., Lieut.-Commdr., R.N.
Plummer, Reginald.
Rees, Thomas John.
Rostron, Frank.
Salisbury, John Edward.
Sanderson, William Troutbeck, B.A.
Vincent, Russell Swale, B.Sc.
Wade, Robert Capel, B.Sc.
Wakerley, Wallace Reginald, B.Sc.(Eng.).
Williamson, David Blair, B.Sc.

*Student to Associate Member.*

Hollingsworth, Philip Massey, B.Eng.

In addition the following transfers have been effected by the Council:—

*Student to Graduate.*

Allen, Maurice, B.Eng.
Bache, Derick John, B.Sc.
Bamford, Thomas.
Bowyer, Frederick Paget, B.Sc.(Eng.).
Bradford, Warren Ernest.
Bray, William John, B.Sc. (Eng.).
Bridge, Hugh John.
Browell, John Lowther.
Chandler, Henry Charles.
Clouston, Charles Edward.
Connock, Sidney Henry G., B.Sc.(Eng.).
Copland, John Eric M., B.Sc.
Fahey, George, B.Sc.
Ghose, Suresh Chandra.
Gilbert, Geoffrey Egerton.
Godden, Alec William, B.Sc.(Eng.).
Haslam, John Robert, B.Sc.(Eng.).
Hodges, Philip George L.
Howie, Alexander Smellie.
King, George Alfred T.
Legate, John Noel M., B.Sc.
Milne, Archibald George.
Pickup, Harry, B.Sc.Tech.
Pressey, Brion George, B.Sc.(Eng.).
Price, Harry Tomkinson.
Prout, Leslie Ronald.
Radcliff, Richard Hamilton, B.E.
Razdan, Suraj Narayan, B.Sc.
Revell, Hedley James, B.Sc.
Rogers, Joseph William, B.E.
Scoles, Graham John, B.Sc. (Eng.).
Scott, Leonard Thompson.
Sharma, Ram Kumar.
Sinclair, Allan Terence.
Smith, Maurice Joy.
Somerville, Robert Dennis, B.Sc.
Tucker, Edward William, B.Sc.(Eng.).
Walker, Frederick Alfred S.
Walton, James William W.
Wheeler, Alan.
White, George Edward B., B.Sc.
Wood, Leslie Bertram, B.Eng.

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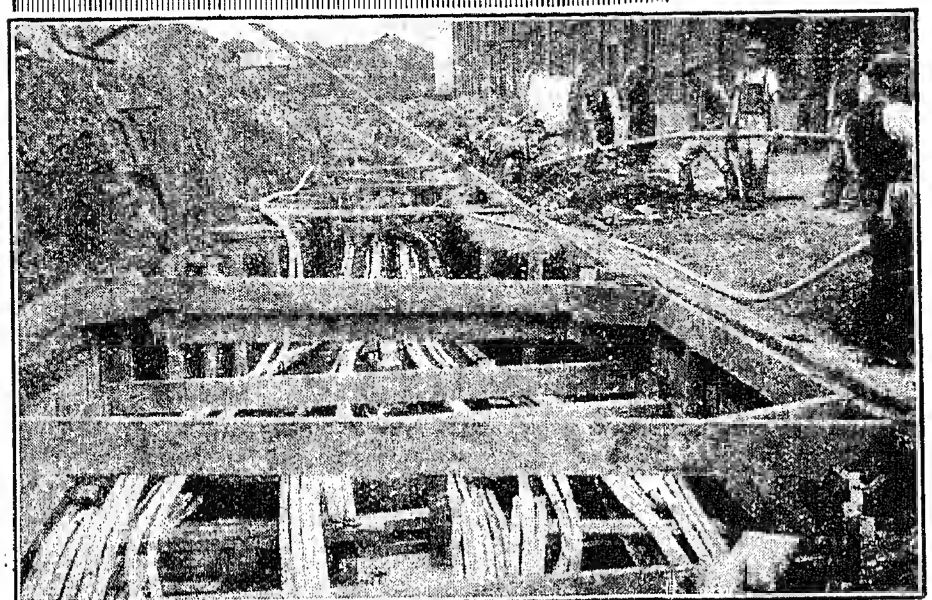
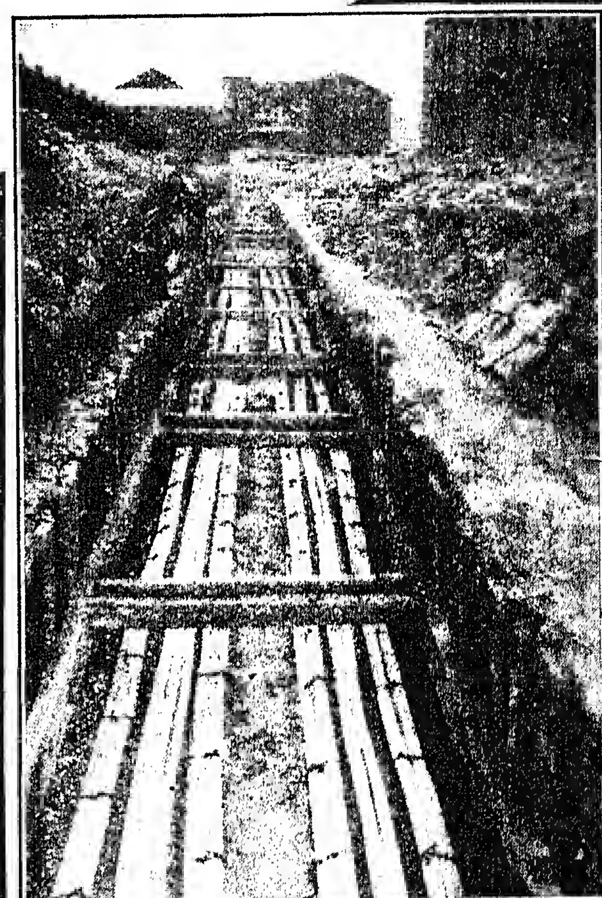
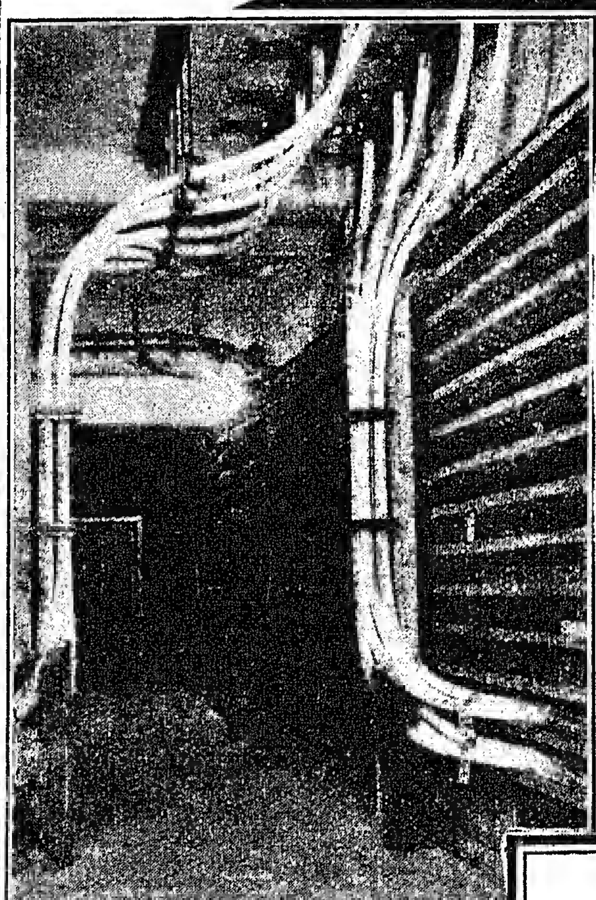
As may have been expected, twenty-nine cables occupied too much street space, and therefore four of the circuits plus five auxiliaries follow one route, and the remainder take a parallel route.



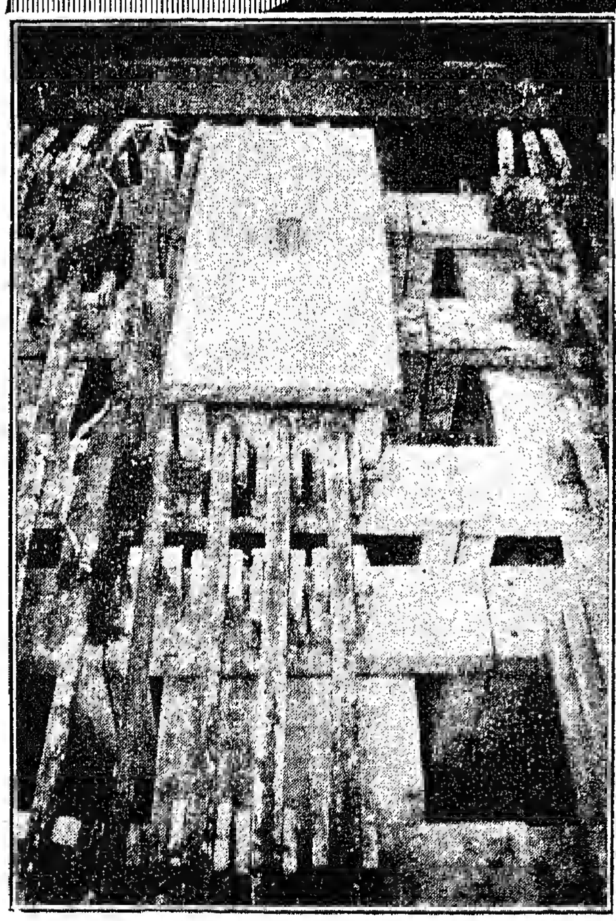
A joint bay (twelve single core cables and five auxiliaries) being prepared during cable laying.



Four circuits (with five auxiliaries) during installation—each circuit consists of three cables in delta formation.



Four circuits (twelve cables) rising from the cable subway at Clarence Dock to their switchgear terminations.



Completed joints (protected by concrete coffins) arranged in tiers in bay.

*Photographs above by permission of Mr. P. J. Robinson, M. Eng., M. I. Mech. E., M.I.E.E., City Electrical Engineer.*

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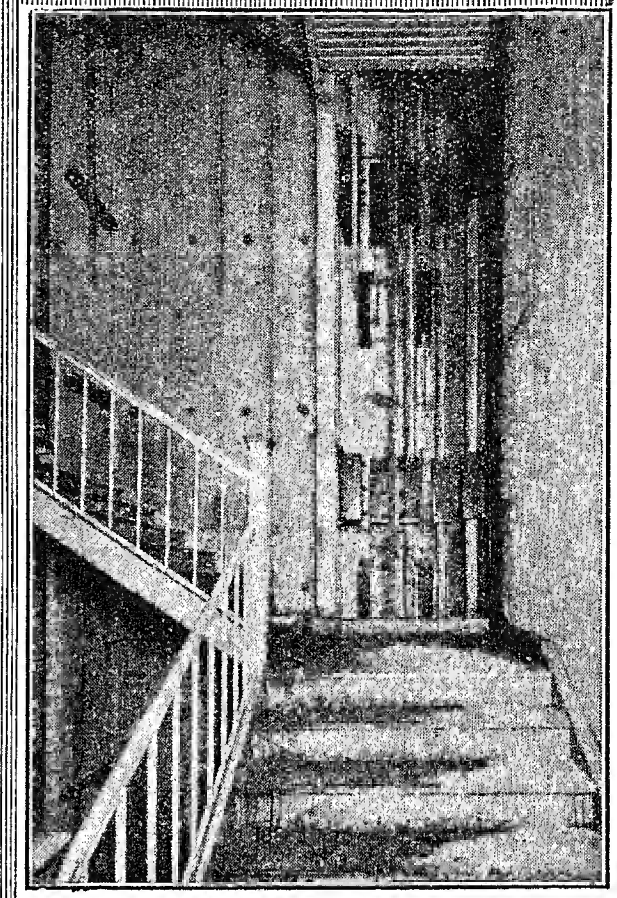
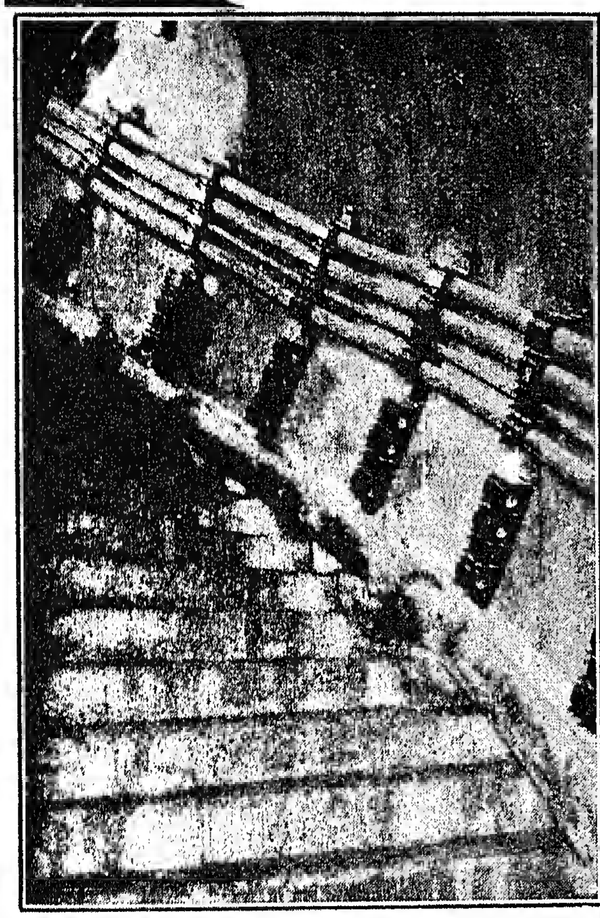
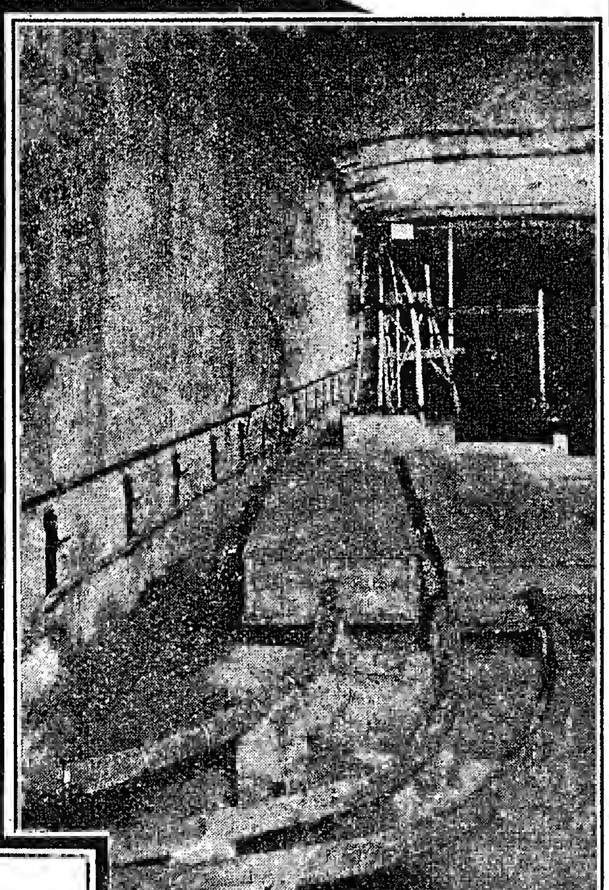
The Central Electricity Board System connecting Clarence Dock Power Station (Liverpool) with Birkenhead Power Station consists of three 0·25 sq. in. 3 core HSL 33kV cables (with one auxiliary cable). These enter the new road tunnel under the Mersey at St. George's Dock, as illustrated below, and emerge in similar fashion at Morpeth Dock (Birkenhead).



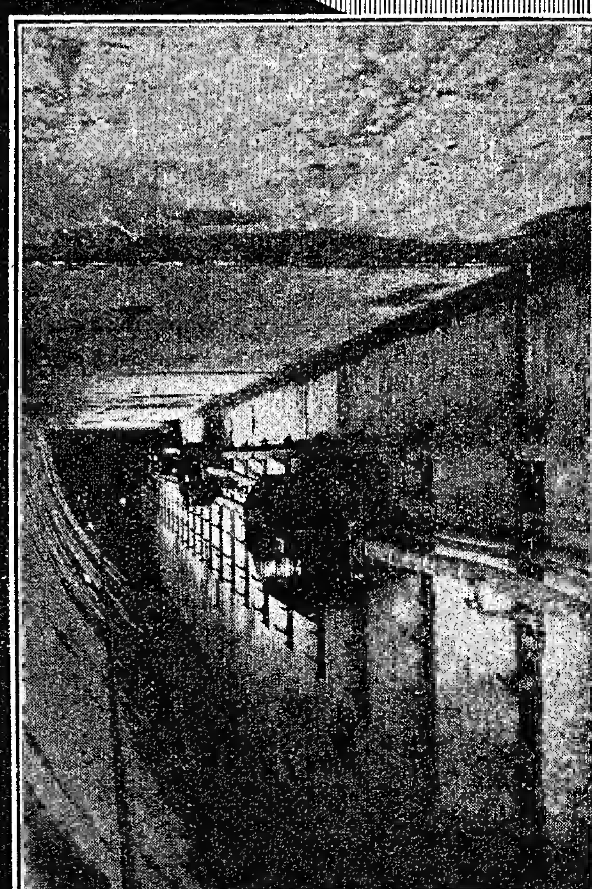
Cables descending vertically in ventilating shaft at St. George's Dock.



Cables descending staircase leading off from base of ventilating shaft at St. George's Dock.



Under-road section of Tunnel at St. George's Dock—main barrier joints at base of shaft.



Main run of cables on racks in South Air Duct (showing joints).

*By courtesy of the Mersey Tunnel Joint Committee. Photographs by Stewart Bale.*

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